Chapter 6

Random Weighting Estimation of One-Sided Confidence Intervals in Discrete Distributions

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ABSTRACT

This paper presents a new random weighting method for estimation of one-sided confidence intervals in discrete distributions. It establishes random weighting estimations for the Wald and Score intervals. Based on this, a theorem of coverage probability is rigorously proved by using the Edgeworth expansion for random weighting estimation of the Wald interval. Experimental results demonstrate that the proposed random weighting method can effectively estimate one-sided confidence intervals, and the estimation accuracy is much higher than that of the bootstrap method.

INTRODUCTION

Random weighting method is an emerging computing concept in statistics (Gao & Zhong, 2010). It has received great attention in the recent years, and has been used to solve different problems (Gao & Zhong, 2010; Gao et al., 2008; Barvinok & Samorodnitsky, 2007; Fang & Zhao, 2006; Xue & Zhu, 2005; Gao, Zhang & Yang, 2004; Gao, Zhang & Zhou, 2003). However, there has been little research to investigate random weighting estimation of one-sided confidence intervals. Currently, Bootstrap is a commonly used method for estimation of confidence intervals and distribution errors (Karlis & Patilea, 2008; Mandel & Betensky, 2008; Mantalosa & Zografos, 2008; Chakraborti & Li, 2007; Dogan, 2007; Haukoos & Lewis, 2005). This method can determine confidence intervals for an
unknown parameter of the underlying distribution function. Although both the random weighting method and Bootstrap method conduct statistical computations and analyses in a parallel manner, the random weighting method has advantages in comparison with the Bootstrap method (Gao & Zhong, 2010; Gao et al., 2008; Gao, Zhang & Yang, 2004; Gao, Zhang & Zhou, 2003; Zheng, 1987). The random weighting method is simple in computation and suitable for large samples. It does not need to know the distribution function. It can also be used to calculate a statistic with a probability density function, since the resultant statistical distribution provides a probability density function. Therefore, the random weighting method provides a promising solution for confidence interval estimation.

One-sided confidence intervals such as the commonly used Wald and Score intervals play an important role in many applications (Montgomery, 2001; Duncan, 1996). The one-sided interval estimation problem significantly differs from the two-sided problem, although there are some common features. In particular, despite the good performance of the Score interval in the two-sided case, the one-sided Score interval does not perform well for three important distributions, i.e., the binomial, negative binomial and Poisson distributions, in terms of coverage probability and expected length. Studies have shown that both the one-sided Wald and Score intervals suffer a pronounced systematic bias in the coverage, although the severity and direction are different (Kott & Liu, 2009; Cai, 2005).

Focusing on the problems of one-sided confidence intervals in the binomial, negative binomial and Poisson distributions, this paper adopts the random weighting method to estimation of one-sided confidence intervals in discrete distributions. Random weighting estimations are constructed for the Wald and Score intervals. Based on this, the coverage probability for the random weighting estimations is also studied. A theorem of coverage probability is rigorously proved for random weighting estimation of the Wald interval. Experiments and comparison analysis have been conducted to comprehensively evaluate the performance of the proposed methodology for estimation of one-sided confidence intervals.

**RANDOM WEIGHTING METHOD**

Assume that \( X_1, X_2, \cdots, X_n \) is a sample of independent and identically distributed random variables with common distribution function \( F \). Let \( x_1, x_2, \cdots, x_n \) be the corresponding observed realizations of \( X_1, X_2, \cdots, X_n \). Further, we shall denote \( \tilde{X}_n = (X_1, X_2, \cdots, X_n) \) and \( \tilde{x}_n = (x_1, x_2, \cdots, x_n) \). Then, the random weighting process can be described as follows:

- **Construct the sample (empirical) distribution function** \( F_n \) from \( \tilde{x} \), i.e.

\[
F_n = \frac{1}{n} \sum_{i=1}^{n} X_i.
\]

- **The random weighting estimation of** \( F_n \) **is**

\[
H_n(x) = \sum_{i=1}^{n} V I_{X_i \leq x}
\]

where \( I_{X_i \leq x} \) is the characteristic function, and random vector \( (V_1, \cdots, V_n) \) obeys Dirichlet distribution \( D(1, \cdots, 1) \), that is, \( \sum_{i=1}^{n} V_i = 1 \) and the joint density function of \( (V_1, V_2, \cdots, V_n) \) is

\[
f(V_1, V_2, \cdots, V_n) = \Gamma(n),
\]

where

\[(V_1, V_2, \cdots, V_{n-1}) \in D_{n-1}\]

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