Chapter 13


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ABSTRACT

The authors prove that the standard least action principle implies a more general form of the same principle by which they can state generalized motion equation including the classical Euler equation as a particular case. This form is based on an observation regarding the last action principle about the limit case in the classical approach using symmetry violations. Furthermore the well known first integrals of the classical Euler equations become only approximate first integrals. The authors also prove a generalization of the fundamental lemma of the calculus of variation and we consider the application in electromagnetism.

1. INTRODUCTION

The least action principle is widely applicable in any field of physics and is particularly fruitful in classical mechanics (Goldstein et al., 2008). It is well known that almost all the physical equations can be derived by such a principle, so that a critical analysis about this principle is certainly useful.

The aim of this paper is to prove that its standard form implies a generalized variational principle by which we can derive generalized Euler equations: these latter are more general than the classical Euler equation and include them as a special case.
In stating our results we make use of a generalization of the fundamental lemma of the calculus of variations.

An attempt is made to interpret as a relativistic correction the difference between the generalized equations and the classical equations. Generally the new equations lead to approximate first integrals which become exact only in limiting case of the classical equations. The results of this analysis can be extended to classical field theory and to Relativistic physics and probably to Quantum mechanics and Quantum field theory (Massaro, Transworld Research Network, 2011). Another application of this theory is in electron optics and, in particular, in the evaluation of the light velocity in a liquid.

2. THE ACTION FUNCTIONAL IN DIFFERENT SETS OF ADMISSIBLE FUNCTIONS

Let us consider the action functional for a single particle moving along x-axis

\[ S[x] = \int_{t_i}^{t_f} L(t,x,\dot{x})dt \]  

(1)

where \( L(t,x,\dot{x}) \) is the Lagrangian. If \( x(t) \) is the real physical curve described by the particle (trajectory), such that \( x(t_i)=a, x(t_f)=b \), we consider the functional (1) in the following sets of admissible functions

\[ \Gamma_1 = \left\{ x(t) = x(t) + h(t) \mid x(t) \in D_1(t_i,t_f), h(t) \in D_1(t_i,t_f), h(t_0) = 0 \right\} \]

\[ \Gamma_2 = \left\{ x(t,\alpha) = x(t) + \alpha h_0(t) \mid x(t) \in D_2(t_i,t_f), h_0(t) \in D_2(t_i,t_f), h(t_0) = 0, |\alpha| \leq \alpha_0 \right\} \]

\[ \Gamma_3 = \left\{ x(t,\alpha) = x(t) + \alpha h_1(t) + \alpha^2 h_2(t) \mid x(t) \in D_3(t_i,t_f), h_1(t) \in D_3(t_i,t_f), h_2(t) \in D_3(t_i,t_f), h_0(t_0) = 0, h(t_0) = 0, h(t_f) = 0, \int_{t_i}^{t_f} h_1(t)dt = 0, |\alpha| \leq \alpha_1 \right\} \]

(2)

Here \( D_i(t_i,t_f) \) is the space of all functions defined on the interval \([t_i,t_f]\) which are continuous and have continuous first derivatives; \( h(t) \) is an arbitrary variable function, \( h_1(t) \) and \( h_2(t) \) are arbitrary but fixed functions; \( \alpha \) is a real variable parameter. In Figure 1 is reported an example of \( x_2 \)-trajectory.

Figure 1. Example of the trajectory \( x_2(t,\alpha) \)