Chapter 3

An Introduction to Wavelet-Based Image Processing and its Applications

Mahesh Kumar H. Kolekar
Indian Institute of Technology, India

G. Lloyds Raja
Indian Institute of Technology, India

Somnath Sengupta
Indian Institute of Technology, India

ABSTRACT

This chapter gives a brief introduction of wavelets and multi-resolution analysis. Wavelets overcome the limitations of Discrete Cosine Transform and hence found its application in JPEG 2000. In wavelet transform, the scaling functions provide approximations or low-pass filtering of the signal and the wavelet functions add the details at multiple resolutions or perform high-pass filtering of the signal. Applying Discrete Wavelet Transform to an image decomposes it into LL, LH, HL, and HH subbands. The low frequency LL band carries most of the significant information in the image. Wavelet transform allows us to analyse the local properties of a signal or image by shifting and scaling operations. The inherent properties of wavelets makes it useful in image denoising, edge detection, image compression, compressed sensing and illumination normalization. The wavelet coefficients at various levels of decomposition follows a parent-child relationship.

INTRODUCTION TO WAVELETS AND MULTI-RESOLUTION ANALYSIS

In the family of transforms, Discrete Cosine Transform (DCT) is the most popular choice for image compression because of several advantages. However, DCT has performance limitations such as blocking artifacts at very low bit rates. In recent years, a new transformation technique has emerged as popular alternative to sinusoidal transforms at very low bit rates. Unlike DCT and Discrete Fourier Transform (DFT), which use sinusoidal waves as basis functions, wavelet transform use small waves of varying frequency and of limited extent, known as wavelets as basis. The wavelets can be scaled and shifted to analyze the spatial

DOI: 10.4018/978-1-4666-4558-5.ch003
frequency contents of an image at different resolutions and positions. A wavelet can therefore perform analysis of an image at multiple resolutions, making it an effective tool in multi-resolution analysis of images.

Furthermore, wavelet analysis performs what is known as space-frequency localization so that at any specified location in space, one can obtain its details in terms of frequency. It is like placing a magnifying glass above a photograph to explore the details around a specific location. The magnifying glass can be moved up or down to vary the extent of magnification, that is, the level of details and it can be slowly panned over the other locations of the photograph to extract those details. A classical sinusoidal transform does not allow such space-frequency localizations. If we consider the spatial array of pixels, it does not provide any spatial frequency information. On the other hand, the transformed array of coefficients contains spatial frequency information, but it does not give us any idea about the locations in the image where such spatial frequencies appear. The space-frequency localization capability of wavelets makes multi-resolution image analysis, representation and coding more efficient (Gonzalez, 2011; Haidekker, 2011).

MultiResolution Analysis (MRA) deals with analyzing the signal at different frequencies with different resolutions as follows: Good time resolution and poor frequency resolution at high frequencies, Good frequency resolution and poor time resolution at low frequencies. MRA is more suitable for short duration of higher frequency; and longer duration of lower frequency components. It is our common observation that the level of details within an image varies from location to location. Some locations contain significant details, where we require finer resolution for analysis and there are other locations, where a coarser resolution representation suffices. A multi-resolution representation of an image gives us a complete idea about the extent of the details existing at different locations from which we can choose our requirements of desired details. Multi-resolution representation facilitates efficient compression by exploiting the redundancies across the resolutions. Wavelet transform is one of the popular, but not the only approach for multi-resolution image analysis. One can use any of the signal processing approaches to sub-band coding such as Quadrature Mirror Filters (QMF) in MRA (Gonzalez, 2011), (Haidekker, 2011). The main objective of this chapter is to make the reader aware of the basic aspects of wavelets and the current trends in wavelet based research. Wavelet transform has many applications in image processing, out of which few applications are discussed in this chapter to give the reader a feel of wavelet. In the case of Haar wavelet transform (Radomir and Bogdon, 2003), the scaling and wavelet basis functions are nothing but the rows of the \( N \times N \) Haar transformation matrices.

**Wavelet Basis Function**

Wavelets are generated from a mother wavelet function \( \varphi(x) \) by shifting it (by a value ‘\( l \)’) and scaling (by a value ‘\( s \)’) as follows:

\[
\varphi_{s,l}(x) = \frac{1}{\sqrt{s}} \varphi \left( \frac{x - l}{s} \right)
\]

(1)

The different wavelet families make different trade-offs between how compactly the basis functions are localized in space and how smooth they are. We will discuss Daubechies wavelet and Haar Wavelet.

**Daubechies Wavelet**

Daubechies (Marc, Michel, Pierre, & Daubechies, 1992) is a family of popular wavelet filters having four or more coefficients. Coefficients of the low pass filter \( h_0(n) \) for the four-coefficient Daubechies filter are as follows: