Chapter 9
Mathematics of Probabilistic Uncertainty Modeling

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ABSTRACT

This chapter presents the uncertainty modeling using probabilistic methods. Probabilistic method of uncertainty analysis is due to randomness of the parameters of a model. Randomness of parameters is characterized by specified probability distribution such as normal, log normal, exponential etc., and the corresponding samples are generated by various methods. Monte Carlo simulation is applied to explore the probabilistic uncertainty modeling. Monte Carlo simulation being a statistical process is based on the random number generation from the specified distribution of the uncertain random parameters. Sample size is generally very large in Monte Carlo simulation which is required to have small errors in the computation. Latin hypercube sampling and importance sampling are explored in brief. This chapter also presents Polynomial Chaos theory based probabilistic uncertainty modeling. Polynomial Chaos theory is an efficient Monte Carlo simulation in the sense that sample size here is very small and dictated by the number of the uncertain parameters and by choice of the order of the polynomial selected to represent the uncertain parameter.

INTRODUCTION

Uncertainties associated with any physical system are essential to quantify for identifying the real behavior of the system. Uncertainties are basically two types: (a) aleatory uncertainty in which the uncertainty is modeled on the basis of probability theory and (b) epistemic uncertainty in which the uncertainty is quantified on the basis of subjective consensus or expert opinion. Aleatory uncertainty cannot be reduced and epistemic uncertainty can be reduced. Uncertainty quantification also leads to a decision making theory. For example, decision making in any environmental measurement...
system always addresses the actual sample size to predict the confidence in measurement and identifies the true mean of the population. Uncertainties are typically classified as aleatory and epistemic. Aleatory uncertainty (also called probabilistic uncertainty) arises from randomness in the system whereas epistemic uncertainty arises due to the lack of knowledge (or ignorance). Epistemic uncertainties may also arise from assumptions introduced in the mathematical models and it can be possible to reduce them using inference from experimental observations. Uncertainty that is explicitly recognized by a stochastic model is categorized as aleatory. Aleatory uncertainty being based on probability theory addresses the uncertain variable as random variable and accordingly the quantification process depends on the large sample size of the random variable. Therefore, mathematics behind the modeling of aleatory uncertainty is focused on the characteristics of the random variable and its generation or simulation. Characteristics of the random variable are defined in terms of the corresponding probability density function. Therefore simulation of the random variable addressing the uncertainty will depend upon its probability density function. The mathematical procedure of generating random variable from the corresponding probability density function is basically known as Monte Carlo simulation. Therefore simulation of the random variable addressing the uncertainty will depend upon its probability density function. The mathematical procedure of generating random variable from the corresponding probability density function is basically known as Monte Carlo simulation. Uncertainty of the model parameters and the model itself is epistemic. Hence, the aleatory/epistemic split of the total uncertainty is model-dependent. The steps involved in the uncertainty quantification of a model generally include (a) estimation of uncertainties of model inputs, (b) estimation of uncertainty of the model output, and (c) propagation of uncertainty in the model output. Monte Carlo methods are the most widely used techniques for statistical/probabilistic uncertainty analysis, with diverse applications. Given the input uncertainty distributions (frequency or probability density data), these methods involve repeated generation of pseudo-random instantiations (sampling) of inputs followed by application of the model to these instantiations to yield a set of model responses. These model outputs are then analyzed statistically.

Monte Carlo simulation needs a large sample size of the random variable. Therefore, the objective of the fundamental mathematical algorithm is to make an efficient Monte Carlo simulation in which sample size is substantially reduced and that is why the method is labeled as efficient Monte Carlo.

BACKGROUND

Monte Carlo simulation is the basic instrument for quantifying uncertainty of any system having random parameters (Mckay et al., 1979; Steck et al., 1976). Sampling scheme to generate random numbers is described in detail in (Iman et al., 1981a; Iman et al., 1981b; Iman et al., 1980). The Latin hypercube sampling technique has been applied to many different computer models since 1975. Uncertainty quantification in this mode is expressed in terms of percentiles computed from the cumulative distribution of the output of the system under quest. Polynomial Chaos theory, an efficient Monte Carlo method used to quantify the probabilistic uncertainty by various researchers (Wiener, 1938; Ghanem & Spanos, 1991; Xiu & Karniadakis, 2002; Wan et al., 2004; Isukapalli, 1999; Papoulis, 1991). Uncertainty associated with any differential equation can be quantified by Polynomial Chaos theory (Wan et al., 2004; Isukapalli,1999).

MATHEMATICS OF MONTE CARLO SIMULATION

Monte Carlo, a branch of experimental mathematics, is a method of directly simulating mathematical relations by random processes. Alternatively, Monte Carlo method is a numerical solution to a problem that models objects interacting with other
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