An Improved Firefly Algorithm-Based 2-D Image Thresholding for Brain Image Fusion

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ABSTRACT

In this article, an attempt is made to diagnose brain diseases like neoplastic, cerebrovascular, Alzheimer's, and sarcomas by the effective fusion of two images. The two images are fused in three steps. Step 1. Segmentation: The images are segmented on the basis of optimal thresholding, the thresholds are optimized with an improved firefly algorithm (pFA) by assuming Renyi entropy as an objective function. Earlier, image thresholding was performed with a 1-D histogram, but it has been recently observed that a 2-D histogram-based thresholding is better. Step 2: the segmented features are extracted with the scale invariant feature transform (SIFT) algorithm. The SIFT algorithm is good in extracting the features even after image rotation and scaling. Step 3: The fusion rules are made on the basis of an interval type-2 fuzzy set (IT2FL), where uncertainty effects are minimized unlike type-1. The novelty of the proposed work is tested on different benchmark image fusion data sets and has proven better in all measuring parameters.

KEYWORDS

Firefly Algorithm, Image Fusion, Image Thresholding, Interval Type-2 Fuzzy, Renyi Entropy, Scale Invariant Feature Transform

INTRODUCTION

Diagnosis of a disease is an important step to cure an illness. There are many diagnostic techniques and one of them is medical imaging. It is a technique to generate a visual representation of the desired part of the human body including tissues, bones and brain. Many multi-modal medical imaging techniques were introduced to achieve it. The main solution among them is Image fusion. Gathering required information from various images and fusing them together into a single image without any distortion and loss of information. Image fusion has many advantages, they include less ambiguity, improved reliability, image sharpening, feature enhancement, improved classification, diagnosing disease in early stages, easy to interpret, low in cost, reduces data transmission and fused image is true in color. Image fusion can be applied in medical diagnosis, satellite imaging, object detection and recognition, military, astronomy.

Many techniques and classifications have been originated in image fusion. The main techniques are spatial domain techniques (Ghassemian, 2016), transform domain techniques (Jin et al., 2017),

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contrast pyramid technique, ratio pyramid technique, morphological pyramid technique (Petrovic & Xydeas, 2004), Laplacian pyramid technique, PCNN (Yang et al., 2009), pixel level image fusion technique (Naidu & Rao, 2008), feature level image techniques (Nirmala & Vaidehi, 2015, March), decision level image fusion techniques (Al-Tayyan et al., 2017). The spatial domain image fusion technique directly deals with the pixels of the input images. This technique can be analyzed in four ways namely, principal component analysis (PCA) (Wan et al., 2013), Intensity hue and saturation (IHS), Simple averaging, Simple maxima. Although they are very important techniques, they have certain limitations. Simple maxima produces highly focused images but it causes blurring (Xu, 2014). Local contrast can be affected by blurring. Simple averaging cannot give clear images of the object, but it is the simplest method for image fusion. Spectral degradation is produced due to principal component analysis technique. Intensity hue saturation is only suitable for color image fusion so it cannot be applied in medical imaging (Haddadpour et al., 2017).

In transform domain techniques, initially the transform of the function is analyzed and the resultant coefficients of transforms are fused. Then, the inverse transform is estimated to get fused images. In this technique, images are converted into multi-resolution or multi-scale representations before fusion. This technique can be analyzed using discrete wavelet transform (DWT), DT-CWT, Curvelet transform (Choi et al., 2015), additive wavelet transforms, non-sub sampled Contourlet transform (NSCT). Contourlet transform is used for catching complex contours, edges and textures (Yang et al., 2008). Few methods in the transform domain are far better than spatial domain methods in many aspects. Discrete wavelet transform cannot handle curved edges. Therefore, it fails in fusing images of the brain in a required manner. It has poor directionality, shift sensitivity, destruction of phase information, less spatial resolution. It has Pseudo-Gibbs effect due to the down-sampling process. Curvelet transform is specially designed for curved edges and capturing curvilinear properties. Therefore, it is beneficial in fusing the images of brain easily and clearly this method is very complex and it takes very high processing time. DT-CWT technique has minor drawbacks as compared to spatial domain analysis and transforms domain analysis. DT-CWT technique has high directionality, better edge representation, approximate shift invariant property and takes less time for processing (El-Hoseny et al., 2018).

The Pyramid techniques in image fusion are suitable only for specific type of images. Therefore, only one bandpass image is generated by pyramid level image fusion techniques and they often cause blocking effects. Apart from this, Contrast pyramid technique will lose much information while Morphological pyramid technique has bad edges. The pulse coupled neural network (PCNN) technique is very useful for image segmentation, image enhancement and pattern recognition. It has an important biological background. PCNN is used to produce images with high contrast, clarity and information. M-PCNN is very useful when several images are fused at a time and depending upon the actual requirements, number of channels can be easily changed. Still it has drawbacks, it is very difficult and complex to set the design constraints, and they have to be adjusted manually or they can be estimated through large amount of training, every parameter is important. The performance of PCCN image fusion is completely dependent on those parameters. To overcome this drawback intelligent optimization algorithms are used. They are GA-PCCN, PSO-PCCN, QPSO-PCCN, MDE-PCNN and FOA-PCCN (Lang & Hao, 2014). The pixel level can hold large amount of information and it also has high precision. The area may approach by which you can achieve pixel level image fusion. They are, intensity hue saturation, principal component analysis and multi-resolution analysis which is widely used.

To decide a technique which is rightly suitable for our application we should consider certain parameters like strength, orientation, offset of the image edges, ridges and other singularities. After deciding the technique, we move for global optimization techniques. To achieve our objective, global optimization is a task for finding the best set of admissible conditions. It is used to find the maxima and minima for continuous and differential functions. The techniques are firefly algorithm (FA), gray wolf optimization (GWO) (Karri & Jena, 2016), Genetic Algorithms (GA) Symbiotic Organisms

Search (SOS), particle swarm optimization (PSO) (Chiranjeevi & Jena, 2017), Honeybee mate optimization (HBMO) (Chiranjeevi & Jena, 2018). Among them, improved firefly algorithm (pFA) has more advantageous like simple mathematics, ease of implementation, high processing time and less convergence speed. Soon after completion of image fusion techniques we have to face two main challenges image contrast and visual quality.

PROPOSED IMAGE FUSION

In this paper, a region-based feature level image fusion is proposed, the regions of both the images which are to be fused are obtained by perfect image segmentation. The images are segmented for extraction of similar regions or areas in both the images by optimal thresholds. The optimal thresholds are obtained using suitable optimization techniques by assuming Renyi entropy as an objective function. These thresholds are optimized based on the 2-D histogram of image to be fused. The task of optimization technique is to maximize the objective function which leads to better threshold, henceforth improved segmentation of input images. The thresholds are optimized with the improved firefly algorithm and compared the results with some other state of art optimization techniques. After successful segmentation of both the images, the features of the input images are extracted with a suitable feature extraction technique. In general, the features of an image are texture, edge and contour. In this paper, image features are extracted with Scale Invariant Feature Transform (SIFT) algorithm, it was invented by (Lowe, 1999) and the working principal of the algorithm starts with random extraction of objects from reference image and new object from other image is recognized on the basis of minimum Euclidean distance between the reference and object which is to be detected. The detailed description about the algorithm is found in reference number (He et al., 2018). The extracted features of both the images are fused based on type - 2 fuzzy fusion rules. Theory of fuzzy set was created in 1965 by Zadeh (Zadeh, 1965). However, it is found that type-1 fuzzy set is not adaptive and flexible, therefore, similarly proposed a type -2 fuzzy set, and in which membership function is uncertain because of uncertainty in membership relationship. So, it is adaptable for both certain and uncertain problems. Figure 1 shows the block diagram of proposed algorithm.

Image thresholding: Image thresholding is one of the image clustering technique. It is used in many applications like image processing, data mining, data clustering etc., because of its simple and flexible nature. Image thresholding may be bi-level or multi-level, in first case two thresholds are used for segmentation of the given image into three regions. Whereas, in second case, number of thresholds is fixed by the user depending on his requirement. If number of thresholds are 'c' then the number of clusters or segments are 'c+1'. The segmented result depends on perfect placement of thresholds which segment the input image. Placing these thresholds is difficult task for different images. So, in this, these thresholds are optimized with improved firefly algorithm (pFA) by considering Renyi entropy as objective function.

MATERIALS AND METHODS

In this section a brief review of Renyi entropy, improved firefly algorithm (pFA), Scale-Invariant Feature Transform, Type-2 Fuzzy Set and Fusion Rules are discussed.

Concept of Renyi Entropy

Let's assume an 'n' array finite discrete probability distributions (pdf) such as $(F_1, F_2, F_3, \dots, F_n) \in \Delta_n$ where $\Delta_n = \{(F_1, F_2, F_3, \dots, F_n), F_i \geq 0, i = 1, 2, 3, \dots, n, n \geq 2, \sum_{i=1}^n F_i = 1\}$ with random variables

Figure 1. Block diagram of proposed technique

$(X_1, X_2, X_3, \dots, X_n)$ then Renyi entropy for independent and additive random events is given as (Rényi, 1960)

$$H_{\alpha} = \frac{1}{1-\alpha} \log_2 \left(\sum_i^n F_i^{\alpha} \right) \quad (1)$$

where ' α ' is greater than zero and it is called as entropy order. When ' α ' tends to one then Renyi entropy becomes Shannon entropy. In general image is clustered in to two clusters, one carries object information (cluster C_1) and another carries background (cluster C_2) then Renyi entropy is:

$$H_{\alpha} [C_1] = \frac{1}{1-\alpha} \left[\log_2 \left(\sum_{i=0}^t \left(\frac{F(i)}{F(C_1)} \right)^{\alpha} \right) \right] \quad (2)$$

$$H_{\alpha} [C_2] = \frac{1}{1-\alpha} \left[\log_2 \left(\sum_{i=t}^{L-1} \left(\frac{F(i)}{F(C_2)} \right)^{\alpha} \right) \right] \quad (3)$$

where $F(C_1) = \sum_{i=0}^t F(i)$, $F(C_2) = \sum_{i=t+1}^{L-1} F(i)$, Here F_i is the normalized one dimensional histogram of the image and 'L' is highest intensity level of gray scale image. The overall renyi entropy for a given image with one threshold 't' is given as:

$$\varphi_{\alpha}(t) = \operatorname{argmax} \left(\left[H_{\alpha} [C_1] + H_{\alpha} [C_2] \right] \right) \quad (4)$$

Multi-Level Thresholding

Let image is divided into 'N' number of clusters $C = (C_1, C_2, C_3, \dots, C_N)$ with N number of threshold values $t = (t_1, t_2, t_3, \dots, t_N)$ then Renyi entropy for each individual cluster is defined as:

$$H_{\alpha} [C_1] = \frac{1}{1-\alpha} \left[\log_2 \left(\sum_{i=0}^{t_1} \left(\frac{F(i)}{F(C_1)} \right)^{\alpha} \right) \right] \quad (5)$$

$$H_{\alpha} [C_2] = \frac{1}{1-\alpha} \left[\log_2 \left(\sum_{i=t_1+1}^{t_2} \left(\frac{F(i)}{F(C_2)} \right)^{\alpha} \right) \right] \quad (6)$$

$$H_{\alpha} [C_N] = \frac{1}{1-\alpha} \left[\log_2 \left(\sum_{i=t_{N-1}}^{L-1} \left(\frac{F(i)}{F(C_N)} \right)^{\alpha} \right) \right] \quad (7)$$

where $F(C_1) = \sum_{i=0}^{t_1} F(i)$, $F(C_2) = \sum_{i=t_1+1}^{t_2} F(i)$, $F(C_N) = \sum_{i=t_{N-1}}^{L-1} F(i)$, the overall Renyi entropy or objection function for a given image for N thresholds is given as:

$$\varphi_{\alpha}(t) = \operatorname{argmax} \left(\left[H_{\alpha} [C_1] + H_{\alpha} [C_2] + \dots H_{\alpha} [C_N] \right] \right) \quad (8)$$

For simplifying the calculations, two dummy thresholds are introduced t_0 and $t_N = L-1$ which satisfy the condition $t_0 < t_1 \dots < t_{N-1} < t_N$. The optimal thresholds are obtained by maximizing the above equation with any soft computing technique.

Two-Dimensional Renyi Entropy

Let $I(m,n)$ is an image intensity at spatial location (m,n) . In digital image $[I(m,n)]$ $m \in \{1, 2, 3, \dots, M\}$, $n \in \{1, 2, 3, \dots, N\}$, where 'M' and 'N' are size of the image and its 1D-histogram $h(x)$ for $x \in \{1, 2, 3, \dots, L-1\}$, where 'L' is 256 for gray scale image. Let denote elements in histogram $\{1, 2, 3, \dots$

255} as G. In literature, optimal thresholds selection is based on 1D-histogram and are obtained by optimizing the objective function/entropy.

The 2-D histogram of an image is obtained by defining a local average of pixel, $I(x,y)$, as the average intensity of its nine neighbors denoted as $g(x,y)$ (Sarkar & Das, 2013):

$$g(x,y) = \frac{1}{9} \sum_{i=-1}^1 \sum_{j=-1}^1 f(x+i, y+j) \quad (9)$$

For example let us take an image of size 4*4 as shown in Figure 2 (a) and its average intensity $g(x,y)$ is calculated by padding required number of zeros at edges as shown in Figure 2 (b). First table

Figure 2. Example for 2-D histogram calculation

I(x,y)			
126	137	152	113
118	121	152	128
108	95	118	155
101	86	149	126

0	0	0
0	126	137
0	118	121

g(x,y)			
56	90	89	61
78	125	130	91
70	116	126	92
43	73	81	61

in Figure 2 is image and first element i.e 126, $g(x,y)$ is calculated by padding zero's at edges as in Figure 2 and last tables shows $g(x,y)$ for entire image $I(x,y)$. 2-D histogram of Lena image at marked area is shown in Figure 3, where diagonal quadrants carry much information.

The 2-D histogram of tested images as shown in Figure 3 and are divided into four clusters by a single threshold (t,s). Where t is threshold for original image intensity $I(x,y)$ and s is threshold for average intensity image $g(x,y)$. The divided cluster area is not same. The diagonal quadrant 1st represents object and 3rd represent background and 2nd and 4th quadrants are neglected because does

Figure 3. Lena image and 2-D histogram

not carry any information (pair occurrence is less) as show in Figure 3. The Renyi entropy for object and background is given as:

$$H_{object}^{\alpha}[t, s] = \frac{1}{1 - \alpha} \left[\log_2 \sum_{i=0}^t \left(\sum_{j=0}^s \left(\frac{F(i, j)}{F_{D1}(t, s)} \right)^{\alpha} \right) \right] \quad (10)$$

$$H_{background}^{\alpha}[t, s] = \frac{1}{1 - \alpha} \left[\log_2 \sum_{i=t+1}^{L-1} \left(\sum_{j=s+1}^{L-1} \left(\frac{F(i, j)}{F_{D2}(t, s)} \right)^{\alpha} \right) \right] \quad (11)$$

where:

$$F_{D1}(t, s) = 1 - \sum_{i=0}^t \sum_{j=0}^s F(i, j) \text{ and } F_{D2}(t, s) = 1 - \sum_{i=t+1}^{L-1} \sum_{j=s+1}^{L-1} F(i, j)$$

The final objective function which is to maximized for better threshold (t,s) selection is:

$$\varphi_{\alpha}(t) = \operatorname{argmax} \left(\left[H_{object}^{\alpha}[t, s] + H_{background}^{\alpha}[t, s] \right] \right) \quad (12)$$

Proposed Renyi 2D-Histogram Based Multi-Level Thresholding

Multilevel thresholding with 1-D histogram deliver inferior results because of incorrect selection of thresholds, so recent study proved that thresholding with 2-D histogram deliver superior results especially in multilevel thresholding. Multilevel thresholding gained lots of popularity over bi-level thresholding because, it clusters the image into several useful clusters, helps for accurate analysis and interpretation of the image. In this paper, we proposed a 2-D Renyi entropy based multilevel thresholding for image segmentation by incorporating the advantage of 2-D histogram. If the 2-D histogram of an image is cluster into 9 clusters with two thresholds (t_1, t_2) and (s_1, s_2) as shown in Figure 4 (a). Then the diagonal quadrants 1st, 5th and 9th represents objects(s), intermediate regions and background respectively as illustrated in Figure 4 (a) and reset of the regions are noise and edges and are ignored. The Renyi entropy of diagonal quadrants are given as:

$$H_{object}^{\alpha}[t, s] = \frac{1}{1 - \alpha} \left[\log_2 \sum_{i=0}^{t_1} \left(\sum_{j=0}^{s_1} \left(\frac{F(i, j)}{F_{D1}(t, s)} \right)^{\alpha} \right) \right] \quad (13)$$

$$H_{intermediate}^{\alpha}[t, s] = \frac{1}{1 - \alpha} \left[\log_2 \sum_{i=t_1+1}^{t_2} \left(\sum_{j=s_1+1}^{s_2} \left(\frac{F(i, j)}{F_{D2}(t, s)} \right)^{\alpha} \right) \right] \quad (14)$$

Figure 4. 2-D histogram for a) 3- level b) 4- level

	(0,0)	t_1	t_2	(0,L-1)
s_1	1	2	3	
s_2	4	5	6	
	7	8	9	
	(L-1,0)		(L-1,L-1)	

(a)

	(0,0)	t_1	t_2	t_3	(0,L-1)
s_1	1	2	3	4	
s_2	5	6	7	8	
s_3	9	10	11	12	
	13	14	15	16	
	(L-1,0)			(L-1,L-1)	

(b)

$$H_{background}^{\alpha}[t, s] = \frac{1}{1 - \alpha} \left[\log_2 \sum_{t_2+1}^{L-1} \left(\sum_{s_2+1}^{L-1} \left(\frac{F(i, j)}{F_{D3}(t, s)} \right)^{\alpha} \right) \right] \quad (15)$$

where:

$$F_{D1}(t, s) = 1 - \sum_{i=0}^{t_1} \sum_{j=0}^{s_1} F(i, j), \quad F_{D2}(t, s) = 1 - \sum_{i=t_{l+1}}^{t_2} \sum_{j=s_{l+1}}^{s_2} F(i, j) \text{ and}$$

$$F_{D2}(t, s) = 1 - \sum_{i=t_{2+1}}^{L-1} \sum_{j=s_2+1}^{L-1} F(i, j)$$

The final objective function which is to maximized for better threshold (t, s) selection is:

$$\varphi_{\alpha}(t) = \operatorname{argmax} \left(\left[H_{object}^{\alpha}[t, s] + H_{intermediate}^{\alpha}[t, s] + H_{background}^{\alpha}[t, s] \right] \right) \quad (16)$$

Above equation can be extended for 'N' threshold values as given below:

$$\varphi_{\alpha}(t) = \operatorname{argmax} \left(H_1^{\alpha}[t, s] + H_2^{\alpha}[t, s] + H_3^{\alpha}[t, s] \dots H_{N+1}^{\alpha}[t, s] \right) \quad (17)$$

where:

$$H_k^{\alpha}[t, s] = \frac{1}{1 - \alpha} \left[\log_2 \sum_{i=t_{k-1}+1}^{t_k} \left(\sum_{j=s_{k-1}+1}^{s_k} \left(\frac{F(i, j)}{F_{Dk}(t, s)} \right)^{\alpha} \right) \right] \quad (18)$$

For simplifying the calculations, two dummy thresholds are introduced t_0 and $t_{N+1} = L-1$ which satisfy the condition $t_0 < t_1 \dots < t_{N-1} < t_N + 1$. Similarly, two dummy variable s_0 and $s_{N+1} = L-1$ which satisfy the condition $s_0 < s_1 \dots < s_{N-1} < s_N + 1$. The 2-D histogram of four standard images are shown in Figure 5 and from these it is observed that most of the information/energy is concentrated on

Figure 5. Input images and corresponding 2-D histogram

diagonal quadrants. Multilevel thresholding is a time-consuming process and is proportional to the number of thresholds 'N'. So, soft computing techniques play a significant role in this contest by assuming Equation (17) as an objection function, which leads to reduction in the computational time.

FIREFLY ALGORITHM

In FA, it mainly imitates the flashing behavior among fireflies. Each firefly is considered as a potential solution in search space, and the move behavior among firefly's stands for solution's upgrading to find a better solution. Moreover, in order to guarantee the algorithm on the right road, it is idealized by 3 hypotheses which are depicted as follows:

1. Fireflies are asexual; namely, they can be attracted with each other regardless of gender;
2. The size of the attraction is proportional to the brightness of the flashing. For two random fireflies x_i and x_j , if x_i gloomier than x_j , then x_i is attracted by x_j . If x_i is the best one among all fireflies, it will move randomly in search space;
3. The brightness of fireflies is relative with the value of objective function.

Basic Parameters

In FA, both brightness (I) and attractiveness (β) play an important role. In general, for maximum problem, the brightness of a firefly is positively relative with the value of objective function, and for minimum problem, they are negatively relative with each other. Prior to defining the attractiveness (namely β), firstly, the Euclidean distance of x_i and x_j should be calculated as follows:

$$r_{ij} = \sqrt{\sum_{i=1}^D (x_{it} - x_{jt})^2} \quad (19)$$

where D is the dimension of the problem. Then, attractiveness is described as follows:

$$\beta = \beta_0 \times e^{-\gamma \times r_{ij}^2} \quad (20)$$

The Movement of the Firefly

Supposed that x_i is attracted by x_j , and then, x_i will move as following:

$$x_i^{t+1} = x_i^t + \beta \times (x_j^t - x_i^t) + \alpha \times (\text{rand} - 0.5) \quad (21)$$

where x_i^t represents the position of x_i at the t -th iteration. $\text{rand} \in [0, 1]$, α is the random step, and $\alpha \in [0, 1]$.

The Detailed Step of the FA

Step 1: Initialize population NP , and set parameters, such as β_0 and α .

Step 2: Calculate r_{ij} and β with Formula (19) and (20), respectively.

Step 3: Upgrade solution with Formula (21).

Step 4: Generate a better solution by comparing the previous solution with the current solution.

Step 5: For the new solution, calculate its value of objective function.

Step 6: Judge whether the algorithm satisfies the terminate condition (up to the max number of iterations ($ItMax$)).

If satisfied, the algorithm end, meanwhile, output optimal solution; if not, return to Step 4.

The Improved FA: pFA

In FA, each firefly can be attracted by too many other fireflies, which caused severe oscillation. Thus, in order to alleviate this disadvantage, we proposed an improved algorithm abbreviated as pFA. In pFA, the attraction among fireflies is reduced greatly, and it can also lower computational time complexity and increase the accuracy of solution. The detailed description is as follows.

In pFA, firstly, we calculate the value of fitness (f_{it}) of each firefly x_i Formula (6):

$$\text{fit} = \begin{cases} \frac{1}{1 + f(x_i)}, & \text{if } f(x_i) \geq 0 \\ 1 + |f(x_i)|, & \text{else} \end{cases} \quad (22)$$

where $f(x)$ is objective function. And then, those values of fitness are bigger than x_i 's (namely i -th firefly's) are put into a set K . Finally, we utilize the roulette wheel selection to choose its neighbor, the formula is shown as the following:

$$p_{\text{fit}} = \frac{f_{it}(x_k)}{\sum_{x_{ik}} \text{fit}(x_i)} \quad (23)$$

For X_i randomly select a firefly x_k as its neighbor with p_{fit} in set K , not in all fireflies. By this way, the worse fireflies are sifted out, and the remainders are better than x_i . Furthermore, from (21), we can know, the better the firefly is, the more chances to be chosen it has. Thus, no matter which one is selected, the chosen one will always give a good direction to lead x_i rightly. Evidently, this strategy can not only speed up the convergence and guarantee the diversity of population, but also reduce selection pressure for fireflies. However, there exists such case that no firefly is better than x_i , that is to say x_i is the best one in the whole population. Thus, it falls into local optimum easily. To deal with this case, the opposite learning skill is adopted by utilizing the information of x_i . Based on the above discussion, we can rewrite Formula (3) as follows:

$$X_i^t = \begin{cases} x_i^t + e^{-\gamma r_{ij}^2} (x_k^t - x_i^t) + \alpha (rand - 0.5), & \text{if } k = \Phi \\ l + u - x_i^t, & \text{else} \end{cases} \quad (24)$$

As far as the step α , this paper employees the following formula to update:

$$X_i^{t+1} = \begin{cases} \alpha = 0.25, & \text{if } t = 1 \\ \alpha_t = \alpha_0 \alpha^{t-1}, & \text{else} \end{cases} \quad (25)$$

Here $\alpha_0 = 0.7$. And the α_0 's scope should be between 0 and 1.

The detailed description of pFA is given in Algorithm 1.

Algorithm 1: Pseudo-code of pFA

```

ep 01: Initialize the population size  $NP$ ,  $x_i | i = 1; 2; \dots; NP$ , the
maximum number of iterations  $ItMax$ 
Step 02: while  $t \leq ItMax$ 
Step 03: for  $i = 1$  to  $NP$  do
Step 04: Calculate the value of fitness of  $x_i$ .
Step 05: better fireflies are chosen to form the set  $K$ .
Step 06: Choose a firefly  $x_k$  from set  $K$  with probability  $p_{fit}$ 
Step 07: Move  $x_i$  according to (6).
Step 08: Compute the fitness value of the new  $x_i$ .
Step 09: end;
Step 10: Rank the fireflies and find the current best;
Step 11:  $t = t + 1$ ;
Step 12: end

```

Time Complexity

For the optimization problem (f), suppose that $O(f)$ is computational time complexity of evaluating its function value. Thus, the computational time complexity of FA is $O(ItMax * NP^2 * f)$, while the computational time complexity of firefly algorithm with random attraction (RaFA) (Wang et al., 2016) and firefly algorithm with neighborhood attraction (NaFA) (Wang et al., 2017) are $O(ItMax * NP * f)$ and $O(ItMax * k * NP * f)$, respectively, k is the number of neighbor in NaFA. In this paper, for pFA, its computational time complexity is $O(ItMax * NP * f)$. Apparently, when it comes to computational time complexity, NaFA is equal to pFA, however, considering the accuracy and convergence speed of the algorithm, pFA is still better than NaFA.

Experiments on Benchmark Functions

In this section, the performance of pFA is proved on 21 common benchmark functions derived from CEC 2005 and compared with FA, RaFA, and NaFA. And these functions derived from CEC 2005, where f_1-f_9 are unimodal functions, f_{10} is noise function, $f_{11}-f_{18}$ are multimodal functions, f_{19} and f_{20} are orthogonal functions and f_{21} is shifted sphere function. The specific descriptions of these functions are shown in Table 1.

Parameter Setting

To make it fair, for all algorithms, the population size NP is set to 40, and the maximum iterations (ItMax) is set to 2500. Both the initial β_0 and γ are set to 1, and except for pFA, $\alpha = 0.25$. The α in pFA is discussed in Experiment 1: Moreover, for NaFA, K is set to 3 (the number of neighbors). Furthermore, to deeply analyze the performance of pFA statistically, all algorithms run 30 times on 30, 50 and 100 dimension, respectively, and we take minimum, mean and deviation as the evaluation criterion to analyze the statistical results.

Experiment 1: Determine the Value of the α_0 and α_1 in Formula (9)

The better values of α_0 and α_1 in Formula (9) play a vital role in pFA. In the early stages of the iteration, the larger value of α can enhance the global search capability of the algorithm. As the iteration progresses, the smaller value of α can give the algorithm a strong local search ability. Thus, the α should be regressive with iterating. Now, we set α_0 to 0.1, 0.3, 0.5, 0.7, 0.9 and α_1 is set to 0.15, 0.35, 0.55, 0.75, and 0.95. And compare them on 21 benchmark functions with 30 dimension. Through experiment, we determine which value among them is the best for pFA. The results are shown in Table 2 and Table 3.

According to the data given in Table 3, for most functions, the best value of α_0 is chosen in [0.5; 0.7].

Experiment 2: Comparison With Other FAs on Min, Worst, Mean and Standard Deviation

In this part, to prove the superiority of pFA, we compare it with FA, RaFA and NaFA on 21 benchmark functions, which described in Table 1.

When the dimension of the problem is 30, the computational results of FA, RaFA and NaFA and pFA are given in Table 4. Observing the table, we can know, for the mean, minimum and standard deviation, pFA is better than others on f_2-f_{21} . Among them, for f_5-f_9 ; f_{11} ; f_{13} ; f_{17} and f_{20} , pFA can find the minimum value of 0, and even the mean and standard deviation are 0, which demonstrates its high accuracy and better robustness; for f_3 , f_4 , f_{10} , f_{16} and f_{19} , the minimum value found by pFA is close to 0. Moreover, both of FA and pFA can find the minimum value of 0 on f_1 and f_6 , while For f_{18} , $\alpha_0 = 0.9$ is best, and for f_{19} , f_{21} , $\alpha_0 = 0.7$ is better than others. In Table 4, for f_3 and f_{14} , $\alpha_0 = 0.55=0.75$ is best, except them, its better that the value of α_1 is chosen in [0.15; 0.35] based on overall consideration. Thus, it is recommended that this combination, $\alpha_1 = 0.7$; $\alpha_1 = 0.25$, should be selected. RaFA and NaFA fail to find it. Finally, for f_{14} , f_{15} , f_{18} , and f_{21} , pFA is nearly equal to others. The computational results on 50 and 100 dimension are given in Table 5 and Table 6. Comparing with the results in Table 6, we can see, with dimension increasing, the mean, minimum and standard deviation of FA, RaFA and NaFA are all getting worse, while the results of pFA is almost invariable. It shows that pFA is still effective in solving high dimensional problems.

In order to further analyze the performance of pFA, we give some convergent curve graphs of f_3 , f_7 , f_{11} , f_{15} , f_{19} , and F_{21} on 30, 50 and 100 dimension, respectively. The results are shown in Figure 6, Figure 7 and Figure 8. In Figure 6, for f_3 , pFA almost converge to 0 when the iteration ends, and all of FA, RaFA and NaFA present slow increasing trend throughout the iteration process. For f_7 , f_{11} and

Table 1. Benchmark functions

Functions	Range	Optimal Value
$f1 = \sum_{i=1}^D x_i^2$	[-100,100]	0
$f2 = \sum_{i=1}^D x_i + \prod_{i=1}^D x_i $	[-10,10]	0
$f3 = \sum_{i=1}^D \left(\sum_{j=1}^i x_j \right)$	[-100,100]	0
$f4 = \max_i \{ x_i , 1 \leq i \leq D \}$	[-100,100]	0
$f5 = \sum_{i=1}^D i x_i^2$	[-100,100]	0
$f6 = \sum_{i=1}^D i x_i^4$	[-10,10]	0
$f7 = \sum_{i=1}^D x_i ^{(i+1)}$	[-1.28,1.28]	0
$f8 = \sum_{i=1}^D 10^{6 \frac{i-1}{D-1}}$	[-1,1]	0
$f9 = \sum_{i=1}^D (x_i + 0.5)$	[-100,100]	0
$f10 = \sum_{i=1}^D i x_i^4 + \text{random}[0,1]$	[-1.28,1.28]	0
$f11 = \sum_{i=1}^D (x_i^2 - 10 \cos(2\pi x_i) + 10)$	[-5.12,5.12]	0
$f12 = -20 \exp \left(-0.2 * \sqrt{\frac{\sum_{i=1}^D x_i^2}{D}} \right) - \exp \left(\sum_{i=1}^D \cos \left(2\pi \frac{x_i}{D} \right) \right) + 20 + e$	[-32,32]	0

continued on following page

Table 1. Continued

Functions	Range	Optimal Value
$f13 = \frac{1}{4000} \sum_{i=1}^D x_i^2 - \prod_{i=1}^n \cos \left(\frac{z_i}{\sqrt{i}} \right) + 1$	[-600,600]	0
$f14 = 0.5 + \frac{\sin \left(\sqrt{\sum_{i=1}^D x_i^2} \right)^{2-0.5}}{\left(1 + 0.001 \sum_{i=1}^D x_i^2 \right)^2}$	[-100,100]	0
$f15 = \frac{\sum_{i=1}^D (x_i^4 - 16x_i^2 + 5x_i)}{D}$	[-5,5]	0
$f16 = \sum_{i=1}^D x_i \sin(x_i) + 0.1x_i $	[-10,10]	0
$f17 = \sum_{i=1}^D (x_i^2 - 10 \cos(2\pi x_i) + 10) \cdot x_i < 0 $ $\sum_{i=1}^D \left(\left(\frac{\text{random}(2x_i)}{2} \right)^2 = 10 \cos(\pi \text{random} 2x_i) \right) + 10 \cdot x_i \geq 0.5 $	[-5.12,5.12]	0
$f18 = \frac{\pi}{D} 10 \sin^2(\pi y_1) + \frac{\pi}{D} \sum_{i=1}^{D-1} (y_i - 1)^2 [1 + 10 \sin^2(\pi y_{i+1})] + \frac{\pi}{D} (Y_D - 1)^2$ $+ \sum_{i=1}^D u(x_i, 10, 100, 4) y_i = 1 + \frac{x_{i+1}}{4} u(x_i, a, k, m)$ $= \begin{cases} k(x_i - a)^m, & x_i > a \\ 0, & -a \leq x_i \leq ak(-x_i - a)^m, & x_i < -1 \end{cases}$	[-50,50]	0
$f19 = \sum_{i=1}^D \left(20^{\frac{i-1}{D-1}} * z_i \right)^2, z = x * M$	[-100,100]	0
$f20 = (1000x_i)^2 + \sum_{i=2}^D (z_i)^2, z = x * M$	[-100,100]	0
$f21 = \sum_{i=1}^D z_i^2, z = x - 0$	[-100,100]	0

Table 2. Results Obtained by pFA with different values of α_0 and α_1 on 30-dimensions

Fun.	$\alpha_0=0.1, \alpha_1=0.25$	$\alpha_0=0.3, \alpha_1=0.25$	$\alpha_0=0.5, \alpha_1=0.25$	$\alpha_0=0.7, \alpha_1=0.25$	$\alpha_0=0.9, \alpha_1=0.25$
F1	0(0)	0(0)	0(0)	0(0)	$1.03 e^{-224(0)}$
F2	$1.59 e^{-280(0)}$	$6.39 e^{-281(0)}$	$1.80 e^{-281(0)}$	$3.08 e^{-279(0)}$	$4.37 e^{-113(4.58 e^{-113})}$
F3	$1.78 e^{-119(9.76 e^{-119})}$	$1.64 e^{-158(9.03 e^{-158})}$	$6.35 e^{-155(3.48 e^{-154})}$	$1.65 e^{-133(9.06 e^{-133})}$	$3.50 e^{-113(4.58 e^{-113})}$
F4	$2.66 e^{-242(0)}$	$2.64 e^{-240(0)}$	$9.00 e^{-243(0)}$	$1.19 e^{-241(0)}$	$4.46 e^{-113(5.38 e^{-114})}$
F5	0(0)	0(0)	0(0)	0(0)	$1.55 e^{-225(0)}$
F6	0(0)	0(0)	0(0)	0(0)	(0)
F7	0(0)	0(0)	0(0)	0(0)	$1.36 e^{-234(0)}$
F8	0(0)	0(0)	0(0)	0(0)	$3.28 e^{-220(0)}$
F9	0(0)	0(0)	0(0)	0(0)	$2.2396 e^{-63(1.1914 e^{-62})}$
F10	$1.06 e^{-05(1.02 e^{-05})}$	$1.08 e^{-05(9.68 e^{-05})}$	$1.39 e^{-05(1.88 e^{-05})}$	$1.30 e^{-05(1.35 e^{-05})}$	$8.94 e^{-06(9.51 e^{-06})}$
F11	0(0)	0(0)	0(0)	0(0)	0(0)
F12	$3.49 e^{-15(1.52 e^{-15})}$	$3.13 e^{-15(1.22 e^{-15})}$	$3.37 e^{-15(1.44 e^{-15})}$	$3.49 e^{-15(1.52 e^{-15})}$	$8.94 e^{-15(3.45 e^{-15})}$
F13	0(0)	0(0)	0(0)	0(0)	0(0)
F14	$0.0097(6.23 e^{-11})$	$0.0097(3.41 e^{-07})$	$0.0097(3.3167 e^{-10})$	$0.0097(2.16 e^{-07})$	$0.0097(1.01 e^{-10})$
F15	-38.4619(2.0583)	-39.0839(2.3885)	-38.9468(2.2689)	-39.52(2.4386)	-46.8126(4.8447)
F16	$1.78 e^{-278(0)}$	$1.97 e^{-280(0)}$	$2.85 e^{-282(0)}$	$1.21 e^{-282(0)}$	0.0034(0.0064)
F17	0(0)	0(0)	0(0)	0(0)	0(0)
F18	0.7996(0.2319)	0.8171(0.2756)	0.7627(2.2689)	0.8048(0.3094)	0.0468(0.0267)
F19	$1.86 e^{-12(1.02 e^{-11})}$	$1.09 e^{-13(6.00 e^{-13})}$	$5.46 e^{-12(2.52 e^{-11})}$	$9.41 e^{-18(5.14 e^{-17})}$	$4.06 e^{-15(2.18 e^{-14})}$
F20	0(0)	0(0)	0(0)	0(0)	$2.28 e^{-221(0)}$
F21	$0.0094(1.00 e^{-11})$	$0.0110(1.21 e^{-10})$	$0.0064(2.83 e^{-18})$	$0.0108(9.11 e^{-19})$	$0.0115(2.73 e^{-18})$

Table 3. Results Obtained by pFA with different values of α_0 and α_1 on 30-dimensions

Fun	$\alpha_0=0.1, \alpha_1=0.15$	$\alpha_0=0.7, \alpha_1=0.35$	$\alpha_0=0.7, \alpha_1=0.55$	$\alpha_0=0.7, \alpha_1=0.75$	$\alpha_0=0.7, \alpha_1=0.95$
F1	0(0)	0(0)	0(0)	0(0)	0(0)
F2	$7.80 e^{-280(0)}$	$9.41 e^{-282(0)}$	$8.86 e^{-279(0)}$	$1.12 e^{-280(0)}$	$4.37 e^{-280(0)}$
F3	$9.27 e^{-127(5.08 e^{-126})}$	$8.944 e^{-133(4.89 e^{-32})}$	$1.16 e^{-155(6.36 e^{-155})}$	$9.98 e^{-148(5.41 e^{-147})}$	$3.30 e^{-97(1.80 e^{-96})}$
F4	$1.40 e^{-239(0)}$	$7.76 e^{-242(0)}$	$1.36 e^{-238(0)}$	$1.41 e^{-241(0)}$	$4.87 a e^{-242(e^{-242(0)})}$
F5	0(0)	0(0)	0(0)	0(0)	(0)
F6	0(0)	0(0)	0(0)	0)	(0)
F7	0(0)	0(0)	0(0)	0(0)	(0)
F8	0(0)	0(0)	0(0)	0(0)	(0)
F9	0(0)	0(0)	0(0)	0(0)	0(0)
F10	$1.13 e^{-05(1.01 e^{-05})}$	$9.88 e^{-06(1.06 e^{-05})}$	$1.18 e^{-05(9.54 e^{-05})}$	$1.04 e^{-05(9.87 e^{-05})}$	$1.03 e^{-05(1.27 e^{-05})}$
F11	0(0)	0(0)	0(0)	0(0)	0(0)
F12	$3.37 e^{-15(1.44 e^{-15})}$	$3.84 e^{-15(1.70 e^{-15})}$	$3.25 e^{-15(1.34 e^{-15})}$	$3.49 e^{-15(1.52 e^{-15})}$	$3.49 e^{-15(1.52 e^{-15})}$
F13	0(0)	0(0)	0(0)	0(0)	0(0)
F14	$0.0097(3.16 e^{-09})$	$0.0097(5.65 e^{-09})$	$0.0097(1.65 e^{-08})$	$0.0097(8.64 e^{-11})$	$0.0097(1.01 e^{-11})$
F15	-39.0518(3.2473)	-39.4378(2.0107)	-38.7370(2.5830)	-39.6825(2.8559)	-39.1156(2.1769)
F16	$4.01 e^{-282(0)}$	$1.27 e^{-280(0)}$	$2.17 e^{-282(0)}$	$1.38 e^{-282(0)}$	$9.86(e^{-282(0)})$
F17	0(0)	0(0)	0(0)	0(0)	0(0)
F18	0.7197(0.2319)	0.7063(0.2046)	0.7107(0.2567)	0.7918(0.2931)	0.7459(0.2802)
F19	$6.43 e^{-24(3.52 e^{-23})}$	$1.42 e^{-22(7.82 e^{-22})}$	$1.02 e^{-17(5.62 e^{-17})}$	$9.87 e^{-12(5.39 e^{-11})}$	$1.95 e^{-20(1.06 e^{-19})}$
F20	0(0)	0(0)	$1.13 e^{-17(5.62 e^{-17})}$	0(0)	$4.80 e^{-09(9.40 e^{-09})}$
F21	$0.0126(5.86 e^{-18})$	$0.0122(4.89 e^{-18})$	$0.0065(3.47 e^{-17})$	$0.0146(5.06 e^{-18})$	$0.0059(1.51 e^{-18})$

f_{19} , pFA is able to converge to 0 within the iteration process, while others change little. For f_{15} , RaFA is better slightly than others, and for f_{21} , pFA is worse than others at the beginning, with the iteration goes, it quickly caught up with others. As shown in Figure 7 and Figure 8, as the dimension increase,

Table 4. Computational results of FA, RaFA, NaFA and pFA for each function on 30 dimensions

Algorithm	Time	Min	Mean	Std
FA	249	0	0	0
RaFA	4	874.6161	2.11e+03	647.9445
NaFA	6	450.5938	1.17e+03	508.6607
pFA	62	0	0	0
FA	159	9.73e-322	0.8723	1.5287
RaFA	4	13.2200	18.6540	3.3840
NaFA	6	9.3405	15.1840	3.4537
pFA	69	1.26e -292	3.80e-279	0
FA	25	551.8415	1.90e+03	874.7558
RaFA	4	1.17e+03	3.18e+03	1.40e+03
NaFA	8	1.59e+03	2.87e+03	799.3186
pFA	93	2.84e-273	1.65e-133	9.06e-133
FA	25	0.0062	2.9432	2.5806
RaFA	2	15.4290	20.0772	3.4026
NaFA	5	10.3884	16.2469	2.7713
pFA	76	1.57e-254	1.19e-241	0
FA	22	1.76e-04	2.2007	4.2620
RaFA	4	174.1170	325.1527	77.6503
NaFA	6	62.8256	183.4959	69.9538
pFA	55	0	0	0
FA	164	0	0	0
RaFA	5	0.0233	0.1905	0.2088
NaFA	8	0.0025	0.0998	0.0747
pFA	83	0	0	0
FA	33	7.59e-08	1.13e-06	6.33e-07
RaFA	5	5.39e-08	5.39e-05	1.32e-04
NaFA	8	1.01e-08	2.19e-06	2.82e-06
pFA	83	0	0	0
FA	31	1.38e+06	4.91e+06	2.53e+06
RaFA	5	5.80e+06	3.26e+07	1.79e+07
NaFA	8	2.54e+06	9.51e+06	6.17e+06
pFA	107	0	0	0
FA	1	1	2.5330	1.3322
RaFA	0.5	1	2.4333	1.1351
NaFA	0.7	0	1.9000	1.1552
pFA	44	0	0	0
FA	1088	0.0193	0.0552	0.0250
RaFA	31	0.0742	0.3144	0.2032
NaFA	181	0.0705	0.2063	0.0900
pFA	125	3.30e-08	1.30e-05	1.35e-05
FA	63	31.8386	53.5950	12.5248
RaFA	4	101.9936	146.5316	21.3669
NaFA	10	65.0032	99.0897	16.2696
pFA	27	0	0	0
FA	22	6.21e-15	1.67e-14	5.31e-15
RaFA	4	7.3980	9.5945	0.9476
NaFA	6	5.5759	8.2415	1.1312
pFA	30	2.66e-15	3.49e-15	1.52e-15

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Table 4. Continued

Algorithm	Time	Min	Mean	Std
FA	27	5.02e-04	0.8384	0.3028
RaFA	5	0.9594	0.9955	0.0076
NaFA	46	0.7904	0.9476	0.0485
pFA	35	0	0	0
FA	21	0.1270	0.2915	0.0843
RaFA	3	0.4419	0.4735	0.0107
NaFA	59	0.4297	0.4706	0.0127
pFA	77	0.0097	0.0097	2.16e-07
FA	113	-70.7927	-67.3345	2.2962
RaFA	5	-47.4986	-41.3536	2.7055
NaFA	9	-53.9663	-48.5161	3.4484
pFA	123	-45.8318	-39.5200	2.4386
FA	17	30	56.1333	17.7234
RaFA	5	66.8561	121.6540	23.2301
NaFA	17	40.4044	69.3322	13.0052
pFA	38	0	0	0
FA	47	1.58e-27	0.1424	0.3472
RaFA	7	5.3744	530.7299	2.30e+03
NaFA	13	5.9968	15.1325	7.0716
pFA	182	0.3828	0.8048	0.3094
FA	35	2.18e+03	1.03e+04	5.44e+03
RaFA	6	3.07e+04	5.82e+04	2.04e+04
NaFA	11	9.83e+03	3.75e+04	1.45e+04
pFA	135	0	9.41e-18	5.14e-17
FA	27	1.58e+04	2.63e+04	7.21e+03
RaFA	4	2.27e+03	6.05e+03	2.27e+03
NaFA	8	3.44e+03	6.76e+03	2.40e+03
pFA	78	0	0	0
FA	30	0.0173	0.0173	3.34e-18
RaFA	12	0.0131	0.0131	8.38e-18
NaFA	36	0.0137	0.0137	6.54e-18
pFA	78	0.0108	0.0108	9.11e-19

the convergence of pFA becomes slower in f_3 . For f_7, f_{11} and f_{21} all of them remain unchanged, and for f_{15} and f_{19} , pFA becomes better, while others change little.

In summary, the performance of pFA is better than FA, RaFA and NaFA on most functions, and for majority functions, pFA can find the minimum value of 0 and it also effectively for solving higher dimension problem.

Experiment 3: Comparison With Other FA's Based on a Probabilistic Metric

In (Yu et al., 2015), Gomes et al. proposed a new way of comparison among metaheuristic optimization algorithms, in this way, it can be clearly seen which algorithm is better. In this paper, as shown in (Yu et al., 2015), we regard P_{better} as the probability that represents pFA is better than FA, RaFA and NaFA. If pFA is worse than FA, RaFA and NaFA, we regard P_{better} as the probability, and if pFA is equal to FA, RaFA and NaFA, we regard P_{equal} as the probability. Here, the population size, dimension and the maximum iterations are set to 40, 50 and 2500, respectively. Moreover, we select $f_3, f_7, f_{11}, f_{15}, f_{19}$ and f_{21} as benchmark functions, and for each algorithm, it was run 30 times. The results are shown in Table 7. As shown in Table 7, for f_3, f_7, f_{11} , and f_{19} , pFA is the best, and it is 100% likely to find

Table 5. Computational results of FA, RaFA, NaFA, and pFA for each function on 30 dimensions

Function	Algorithm	Time	Min	Mean	Std.
F1	FA	49	3.03e-104	1.60e-04	6.17e-14
	RaFA	5	3.26e+03	5.15e+03	1.83e+03
	NaFA	7	2.67e+03	4.49e+03	1.08e+03
	pFA	68	0	0	0
F2	FA	23	1.0787	13.9119	8.7174
	RaFA	4	29.6556	39.2844	4.1969
	NaFA	6	27.7283	38.6810	6.0088
	pFA	74	3.31e-277	1.47e-266	0
F3	FA	34	5.41e+03	9.71e+03	3.07e+03
	RaFA	6	4.09e+03	1.06e+04	4.58e+03
	NaFA	12	5.60e+03	9.23e+03	2.80e+03
	pFA	129	5.39e-179	5.12e-62	2.80e-61
F4	FA	26	7.2956	14.0661	2.8475
	RaFA	3	18.3396	23.7321	2.9446
	NaFA	6	18.2441	22.5221	2.6046
	pFA	83	4.44e-242	2.41e-234	0
F5	FA	22	3.8608	42.1408	29.4447
	RaFA	4	726.9976	1.47e+03	260.9366
	NaFA	6	585.9786	1.06e+03	316.2513
	pFA	65	0	0	0
F6	FA	41	2.51e-09	1.27e-04	1.63e-04
	RaFA	6	0.2298	0.9258	0.4317
	NaFA	10	0.2569	0.7123	0.3796
	pFA	118	0	0	0
F7	FA	41	2.17e-07	1.34e-06	8.20e-07
	RaFA	6	6.67e-08	8.38e-05	2.55e-04
	NaFA	9	1.70e-09	3.98e-06	4.54e-06
	pFA	109	0	0	0
F8	Fa	38	6.40e+06	1.81e+07	8.36e+06
	RaFA	6	3.92e+07	1.06e+08	5.49e+07
	NaFA	10	1.12e+07	4.31e+07	1.94e+07
	pFA	150	0	0	0
F10	FA	1422	0.0918	0.8179	0.0638
	RaFA	38	0.3719	1.0542	0.5028
	NaFA	230	0.4187	1.0951	0.5019
	pFA	168	3.44e-07	8.09e-06	7.92e-06
F11	FA	44	60.6924	100.9517	21.1218
	RaFA	4	238.4711	300.2502	31.4020
	NaFA	8	180.3972	246.0055	24.5482
	pFA	31	0	0	0
F13	FA	20	1.0000	1.0000	7.26e-10
	RaFA	4	1.0000	1.0000	1.48e-06
	NaFA	39	0.9937	0.9994	0.0014
	pFA	44	0	0	0
F14	FA	73	0.4147	0.4798	0.0177
	RaFA	3	0.4850	0.4930	0.0031
	NaFA	64	0.4888	0.4932	0.0023
	pFA	82	0.0097	0.0097	6.87e-08

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Table 5. Continued

Function	Algorithm	Time	Min	Mean	Std.
F15	FA	38	-70.1781	-66.0138	1.9884
	RaFA	7	-40.9722	-37.7798	1.8800
	NaFA	13	-48.9140	-43.6976	2.6474
	pFA	167	-41.4030	-35.9725	2.5076
F16	FA	28	0.7043	3.0358	1.6669
	RaFA	4	18.9912	25.1443	2.7943
	NaFA	8	16.6327	20.4971	2.5943
	pFA	76	2.20e-277	1.43e-266	0
F17	FA	18	68	111.9750	28.0144
	RaFA	5	193.7580	258.1913	29.0084
	NaFA	23	150.2852	189.0966	23.2016
	pFA	43	0	0	0
F18	FA	54	1.0212	4.6927	2.1361
	RaFA	9	9.7041	6.15e+03	2.20e+04
	NaFA	15	15.5344	1.14e+03	4.19e+03
	pFA	241	0.51454	0.8954	0.2381
F19	FA	45	1.26e+0.4	4.59e+04	2.28e+04
	RaFA	8	8.21e+04	1.73e+05	6.05e+04
	NaFA	16	6.17e+04	1.23e+05	3.69e+04
	pFA	195	0	1.95e-66	1.07e-65
F20	FA	30	2.98e+04	4.84e+04	1.29e+04
	RaFA	5	6.51e+03	1.05e+04	2.43e+03
	NaFA	9	7.90e+03	1.36e+04	3.53e+03
	pFA	92	0	0	0
F21	FA	74	0.0091	0.0091	3.42e-18
	RaFA	19	0.0073	0.0073	4.39e-18
	NaFA	93	0.0062	0.0062	2.24e-18
	Pfa	113	0.0059	0.0059	2.95e-18

the optimal value in a single run. For f_{15} , it can be seen that pFA is worse than FA, RaFA and NaFA with the probability of 100%, 72% and 99%, respectively. And for f_{21} , pFA is better than FA, RaFA and NaFA with the probability of 100%.

Experiment 4: Comparison With De and DeSH

In this part, the validity of the improved algorithm is furtherproved by comparing with DE, chDE (Zhenyu et al., 2006), jDE (Brest et al., 2006), aDE (Noman et al., 2011) and IMMSADE (Wang et al., 2017) on $f_1-f_9, f_{11}, f_{13}, f_{15}, f_{17}, f_{19}$, and f_{21} . These benchmark functions are consistent with literature. For the sake of fairness, all parameter settings are derived from literature. For each algorithm, program runs 30 times independently. The dimension D and the population size NP are set to 30, 100, respectively, and the maximum iterations ($ItMax$) is set to 3000. We utilize the statistics of average value (mean) and standard deviation (std) to evaluate the performance of algorithms after 30 times running. The statistical results are displayed in Table 7. And except for the statistics of pFA, statistics of others are derived from their original papers. For example, the parameter settings of IMMSADE algorithm are the same as literature (Wang et al., 2017). Besides, the parameter settings of DE originate from literature, in Which CR is set to 0.9 and F is set to 0.9, 0.5. For pFA, β_0 , and are set to 1.

As shown in Table 8, for $f_1, f_3 -f_7$ and f_9 , both mean and std of pFA are superior to others'. In particular, both of them are 0 on f_1, f_3, f_6 and f_7 . For f_8 and f_{15} , the values of mean and std of all algorithms are 0. For f_{13} , pFA is as good as aDE and IMMSADE, of which mean and std are 0.

Table 6. Computational results of FA, RaFA, NaFA, and pFA for each function on 100 dimensions

Function	Algorithm	Time	Min	Mean	Std.
F1	FA	28	1.25e+03	3.28e+03	1.61e+03
	RaFA	5	1.05e+04	1.39e+04	2.04e+03
	NaFA	8	1.24e+04	1.73e+04	2.75e+03
	Pfa	79	0	0	0
F2	FA	25	45.7986	93.6254	19.3097
	RaFA	4	75.6588	91.2161	8.1710
	NaFA	7	86.5001	105.3091	9.7488
	Pfa	83	1.51e-265	7.46e-265	0
F3	FA	33	6.76e+03	9.85e+03	2.40e+03
	RaFA	6	5.97e-	9.63e+03	3.22e+03
	NaFA	13	3.91e-	9.63e+03	3.22e+03
	Pfa	129	8.96e-	1.32e-56	6.58e-59
F4	FA	29	19.70	29.5103	3.3598
	RaFA	3	22.96	29.3519	2.7539
	NaFA	6	25.58	30.6770	2.2555
	Pfa	91	6.78e-	2.67e-227	0
F5	FA	25	724.73	2.04e+03	716.0177
	RaFA	4	4.13e-	6.54e+03	1.19e+03
	NaFA	7	5.13e-	7.82e+03	1.22e+03
	Pfa	74	0	0	0
F6	FA	56	0.254	1.1921	0.6860
	RaFA	9	3.013	6.3736	2.0038
	NaFA	16	4.019	9.5038	2.7436
	Pfa	197	0	0	0
F7	FA	60	4.56e-	1.41e-06	7.20e-07
	RaFA	8	1.53e-	2.14e-04	8.78e-04
	NaFA	14	2.94e-	3.41e-06	5.64e-06
	Pfa	177	0	0	0
F8	FA	55	3.44e-	8.77e+07	2.27e+07
	RaFA	9	1.70e-	3.71e+08	1.13e+07
	NaFA	17	9.24e-	2.51e+08	9.16e+07
	Pfa	260	0	0	0
F9	FA	2	9	17.3333	4.1133
	RaFA	0.8	19	24.5667	3.5006
	NaFA	1.3	15	21.2000	2.4691
	Pfa	63	4	8.2000	2.3839
F10	FA	2086	1.1660	3.0710	1.2766
	RaFA	56	3.9583	7.2508	3.0172
	NaFA	349	4.7223	11.4290	4.7380
	Pfa	274	1.044e-07	1.40e-05	1.43e-05
F11	FA	29	182.9112	283.1775	32.5341
	RaFA	5	666.9288	743.7117	43.0929
	NaFA	10	573.5190	643.1476	30.7975
	Pfa	39	0	0	0
F12	FA	28	6.2005	8.4004	1.1137
	RaFA	5	11.2087	12.1025	0.4671
	NaFA	11	11.0521	12.9293	0.6050
	Pfa	38	2.66e-15	4.08e-15	1.77e-15

continued on following page

Table 6. Continued

Function	Algorithm	Time	Min	Mean	Std.
F13	FA	27	1.0000	1.0000	0
	RaFA	1	1.0000	1.0000	0
	NaFA	5	1.0000	1.0000	0
	Pfa	61	0	0	0
F14	FA	115	0.4979	0.4991	4.12e-04
	RaFA	4	0.4980	0.4988	3.49e-04
	NaFA	71	0.4976	0.4987	3.83e-04
	Pfa	93	0.0097	0.0097	1.78e-08
F15	FA	58	-62.4677	-59.1327	1.7.38
	RaFA	12	-38.9661	-34.5243	1.7015
	NaFA	23	-43.6207	-37.4587	2.0631
	Pfa	270	-35.8802	-33.1229	1.4444
F16	FA	28	12.8347	22.8740	4.6885
	RaFA	5	51.0554	61.2341	4.7078
	NaFA	9	52.8338	60.5799	4.9688
	Pfa	90	1.79e-265	2.50e-257	0
F17	FA	27	234.5053	322.2796	46.2017
	RaFA	5	575.2118	672.3144	46.8110
	NaFA	20	477.7268	546.7777	36.9873
	Pfa	55	0	0	0
F19	FA	78	1.29e+05	2.36e+05	5.19e+04
	RaFA	14	3.45e+05	5.29e+05	1.11e+05
	NaFA	26	3.82e+05	5.66e+05	1.22e+05
	Pfa	362	0	1.95e-66	1.07e-65
F20	FA	45	6.43e+04	9.95e+04	2.72e+04
	RaFA	7	1.75e+04	2.45e+04	5.03e+03
	NaFA	15	2.16e+04	2.91e+04	4.63e+03
	Pfa	152	0	0	0
F21	FA	165	0.0038	0.0038	1.62e-18
	RaFA	29	0.0037	0.0037	1.93e-18
	NaFA	217	0.0033	0.0033	1.15e-18
	Pfa	210	0.0031	0.0031	2.02e-18

For f_2 and f_{17} , the performance of pFA is worse than others'. Furthermore, mean and std of pFA is slightly lower than others' in f_{21} . In conclusion, the results show that pFA is reliable and effective.

SCALE-INVARIANT FEATURE TRANSFORM

Harris feature detectors are rotation invariant i.e. same corners in images are detected even after image rotation and before rotation, but it fails to detect corners when image is scaled. After scaling the image, corners in an image may or may not be corners. So, this feature detector is scale variant (Liang & Mendel, 2000). To overcome this drawback, scale-invariant feature transform (SIFT) is introduced in the year 2004. The algorithm follows 4 steps:

Scale Space Peak Selection: Potential Locations for Finding Features

Different sizes of windows are used for different scaled images for detection of key points and this process is known as scale space filtering, difference of Laplacian of Gaussian (DoG) produce windows of different sizes. Difference of Gaussian value is calculated by difference of two Gaussian images.

Figure 6. Convergence curve of $f_3, f_7, f_{11}, f_{15}, f_{19}$ and f_{21} on 30 dimension

Figure 7. Convergence curve of $f_3, f_7, f_{11}, f_{15}, f_{19}$ and f_{21} on 50 dimensions

Figure 8. Convergence curve of f_3 , f_7 , f_{11} , f_{15} , f_{19} , and f_{21} on 100 dimensions

Table 7. Computational results of FA, RaFA, NaFA and pFABased on probabilistic metric

Function	Algorithm	P _{better}	P _{equal}	P _{worse}
F3	FA	1.00	0.00	0.00
	RaFA	1.00	0.00	0.00
	NaFA	1.00	0.00	0.00
	Pfa	1.00	0.00	0.00
F7	FA	1.00	0.00	0.00
	RaFA	1.00	0.00	0.00
	NaFA	1.00	0.00	0.00
	Pfa	1.00	0.00	0.00
F11	FA	1.00	0.00	0.00
	RaFA	1.00	0.00	0.00
	NaFA	1.00	0.00	0.00
	Pfa	1.00	0.00	0.00
F15	FA	0.00	0.00	1.00
	RaFA	0.28	0.00	0.72
	NaFA	0.01	0.00	0.99
	Pfa	1.00	0.00	0.00
F19	FA	1.00	0.00	0.00
	RaFA	1.00	0.00	0.00
	NaFA	1.00	0.00	0.00
	Pfa	1.00	0.00	0.00
F21	FA	1.00	0.00	0.00
	RaFA	1.00	0.00	0.00
	NaFA	1.00	0.00	0.00
	Pfa	1.00	0.00	0.00

Table 8. Computational results of DE and DEs

Function	Algorithm	Mean	Std
F1	DE	6.54e-37	9.42e-37
	chDE	1.02e-56	2.80e-56
	jDE	3.24e-39	3.48e-39
	aDE	2.63e-56	2.23e-56
	IMMSADE	5.68e-165	0
	pFA	0	0
F2	DE	1.27e-05	1.41e-05
	chDE	1.65e-04	1.90e-04
	jDE	2.5800	1.2800
	aDE	1.23 e-01	7.11e-02
	IMMSADE	1.71 e-137	9.20e-137
	pFA	4.86 e-55	2.66e-54
F3	DE	1.08 e-33	1.55e-33
	chDE	7.16 e-53	2.08e-52
	jDE	2.11 e-36	2.58e-36
	aDE	2.93 e-53	2.68e-53
	IMMSADE	5.44 e-161	2.93e-160
	pFA	0	0
F4	DE	4.16 e-18	3.83e-18
	chDE	3.06 e-34	4.40e-34
	jDE	3.40 e-23	2.49e-23
	aDE	6.75 e-33	4.42e-33
	IMMSADE	4.64 e-48	2.50e-47
	pFA	1.10 e-321	0
F5	DE	1.99e-01	3.39e-01
	chDE	1.02e+01	5.0600
	jDE	1.91e-05	1.12e-05
	aDE	2.77e-04	1.10e-03
	IMMSADE	3.14e-88	1.69e-87
	pFA	4.82e-283	0
F6	DE	8.13e-31	8.51e-31
	chDE	9.73e-51	1.97e-50
	jDE	1.57e-33	1.69e-33
	aDE	1.71e-50	1.74e-50
	IMMSADE	3.99e-165	0
	pFA	0	0
F7	DE	1.49e-36	1.93e-36
	chDE	7.19e-56	3.33e-55
	jDE	5.34e-39	4.91e-39
	aDE	5.15e-56	5.00e-56
	IMMSADE	9.09e-125	4.89e-124
	pFA	0	0
F8	DE	0	0
	chDE	0	0
	jDE	0	0
	aDE	0	0
	IMMSADE	0	0
	pFA	0	0

continued on following page

Table 8. Continued

Function	Algorithm	Mean	Std
F9	DE	4.32e-03	1.30e-03
	chDE	3.87e-03	1.02e-03
	jDE	4.88e-03	1.42e-03
	aDE	4.85e-03	1.41e-03
	IMMSADE	1.33e-04	7.24e-05
	pFA	4.32e-06	4.04e-06
F11	DE	0	0
	chDE	2.46e-04	1.33e-03
	jDE	0	0
	aDE	0	0
	IMMSADE	0	0
	pFA	0.0776	0.2426
F13	DE	1.44e+02	2.20e+01
	chDE	2.5200	2.3100
	jDE	4.5900	3.5800
	aDE	0	0
	IMMSADE	0	0
	pFA	0	0
F17	DE	1.35e-19	2.40e-35
	chDE	1.35e-19	3.13e-29
	jDE	1.35e-19	2.41e-35
	aDE	1.35e-19	2.41e-35
	IMMSADE	1.35e-19	2.95e-32
	pFA	0.5411	0.2116
F21	DE	3.41e-01	4.50e-02
	chDE	1.60e-01	4.55e-02
	jDE	3.09e-01	4.01e-02
	aDE	3.18e-01	4.11e-02
	IMMSADE	6.10e-02	1.72e-02
	pFA	0.49e-01	0.98e-01

Key Point Localization

Accurately locating the feature key points. After successful identification of key points in first stage, these points are further refined for accuracy by generating a different window sizes on basis of Taylor series and after doing this, if obtained extreme location intensity is greater than the user defined threshold (contrast threshold) then consider it or else discard. Similarly, any unwanted edges found after difference of Gaussian function, those locations are removed by 2x2 Hessian matrix as like in Harris corner detector. So only utmost edges and key points are obtained after set A and step B.

Orientation Assignment

Assigning orientation to the key points. In this step, orientation is traced between the key points based on their scale, gradient magnitude and direction. In this paper, around 36 bins are used to cover 360 degrees. From histogram of an image the highest intensity 'H' is calculated and if any Key point intensity is above 80% of H, then it will be considered in tracing.

Key Point Descriptor

Describing the key point as a high dimensional vector. Key point descriptor is obtained by local gradient which is generated in above three steps. The obtained gradient information is rotated to line up with

the orientation of the key point and then weighted by Gaussian with variance of 1.5 key point scale. With this data, a collection of histograms is generated which are at center of key point. A total of 16 histograms which are arranged in the form of 4x4 grid and each grid carries 8 orientation bins. This leads to a feature vector of size 128. These vectors are called SIFT keys and used for identification of nearest possible object in an image. If more than three keys support a specific feature, that will be considered for further processes.

TYPE-2 FUZZY SET

The theory called as fuzzy set created by Zadeh. Fuzzy set theory prominently solves the problems related to uncertainty. However, because type-1 conventional membership functions of fuzzy set are determined, the uncertainty effects minimization is difficult by using any kind of algorithm and membership function. In pragmatic applications, usually the membership function is based on intuition empirically with a high degree of subjectivity. A generalized fuzzy set proposed by Zadeh to deal with the problems mentioned above i.e., type 2 fuzzy set. In a second type Fuzzy set the relation between membership function and element is fuzzy and uncertain. Therefore, a type-2 fuzzy set can describe the fuzzy membership function of uncertainty and the elements of uncertainty. Type-2 fuzzy set can be defined as:

$$B = \{((y, u), \mu_B(y, u)) | \forall y \in Y, \forall u \in K_y \subseteq [0, 1]\} \quad (26)$$

where B denotes the type-2 fuzzy set, $\mu_B(y, u)$ denotes its type type-2 membership function with $0 \leq \mu_B(y, u) \leq 1$. K_y denotes a primary membership function $\mu_B(y', u)$ denotes the secondary membership function when $y = y'$. However, the complexity of type-2 fuzzy set is greater than that of type-1, in which more calculations are required. The problem exists while choosing secondary membership functions. In practical applications, a type-2 simplified fuzzy set often used, such as the interval type-2 fuzzy set (Mendel & John, 2002). The membership functions of the element in type-2 fuzzy set is defined as:

$$B = \{((y, u), 1) | \forall y \in Y, \forall u \in K_y \subseteq [0, 1]\} \quad (27)$$

An alternative type-2 fuzzy set is defined as (with the help of membership functions of type-1):

$$B = (y, \mu_M(y), \mu_V(y)) | \forall y \in Y, \mu_M(y) \leq \mu(y) \leq \mu_V(y), \mu \in [0, 1] \quad (28)$$

where $\mu(y)$ denotes an initial membership function of thype-1, B is the interval type-2 fuzzy set, and $\mu_M(y)$ and $\mu_V(y)$ is the upper and lower membership functions with respect to upper and lower envelopes of the interval membership function (type-2) and fuzzy set type-1 membership function can be obtained. Practically a pair to characterize reciprocal parameters i.e.:

$$\mu_M(y) = [\mu(y)]^\alpha, \mu_V(y) = [\mu(y)]^{(1/\alpha)} \quad (29)$$

where α is a fuzzy linguistic hedge ($\alpha \geq 1$). Good output can be obtained when $\alpha \in [1, 2]$. Footprint of uncertainty (FOU) can be represented by relating both type-1 fuzzy set and type-2 fuzzy set (Yang et al., 2016). The shape of type-2 fuzzy set is used in literature is represented by term FOU, therefore the distribution on the top of the shaded area is given by FOU.

FUSION RULE

Let's assume two images M and R which is to be fused. After successful segmentation of both the images its corresponding segmented regions are represented as $M : \{S_{p,q}^M\}$ and $R : \{S_{p,q}^R\}$ where p and q represents the segmented regions and its corresponding pixel positions respectively. The fusion rules play a significant role in performance of the proposed segmentation method. The earlier proposed image fusion rules are sensitivity to noise at high frequency i.e. edges, region boundaries and textures (Li & Li, 2008) so in this paper, type 2 fuzzy logic is proposed which takes local and global information of the images. The shape of the membership function $\mu()$ is given as:

$$\mu_{p,q}^I(i, j) = \frac{1}{1 + \left| \frac{s_{p,q}^I(i, j) - c}{a} \right|^2} \quad (30)$$

where $I = (M, R)$ and (i, j) shows the spatial location of segmented images. Here, c is average of $(s_{p,q}^I)$ and 'a' is minimum of $(s_{p,q}^I)$. Next, at each position of the membership function, lower and upper membership functions $\mu_L(x)$ and $\mu_U(x)$ are calculated as follows:

$$\begin{cases} \mu_L^{I,p,q}(x, y) = [\mu_{p,q}^I(i, j)]^\alpha \\ \mu_U^{I,p,q}(x, y) = [\mu_{p,q}^I(i, j)]^{1/\alpha} \end{cases} \quad (31)$$

Fuzzy set performance is measured on the basis of fuzzy entropy, a high value of fuzzy entropy shows the best fuzzy set and low value of fuzzy entropy shows wicked fuzzy set (Bhateja et al., 2015). Hence a higher value of fuzzy entropy leads to better image fusion rules. The detailed description about the fuzzy entropy already discussed in section 2.

RESULTS AND DISCUSSIONS

The performance of proposed method is evaluated by performing experiments on five pairs of images related to different diseases of brain. In Set-A, CT and T2- Weighted MR images of brain tumor related to neoplastic disease. In Set-B, CT, T2-Weighted MR images of brain stroke due to cerebrovascular disease. In Set-C, depicts the T1-Weighted MR and T2-Weighted MR images of brain related to Alzheimer disease. In Set-D, depicts the T1-Weighted MR and T2-Weighted MR images of brain related to fatal disease, while in Set-E, represents the CT, T1-Weighted MR brain images due to sarcoma disease. The medical images used in the research were obtained from <https://www.imagefusion.org/> and <http://www.med.harvard.edu/aanlib/home.html>. All the simulations are performed on HP Compaq LE1902X personal computer with Matlab version 2016a.

Measuring Parameters: Standard Deviation (STD)

Standard deviation is the estimation of gray contrast in the combined image. At the point when the estimated S is high, the fused image will have high contrast. When $p(k)$ is the probability of gray level k and k_m is the mean of k [33]:

$$STD = \sqrt{\sum_{K=0}^{J-1} (k - k_m)^2 p(k)} \quad (32)$$

Structural Similarity Index (SSIM)

Structural similarity index is used for the estimation of similarity or closeness between two images. It is used to measure quality by comparing one image provided with the other image. It is regarded as an excellent quality. It is the upgraded version of universal image quality index. Consider $D1 = (m_1 N)^2$ and $D2 = (m_2 N)^2$ are the two variables to make the division stable by making the denominator weak. N represents the dynamic range values of pixels. The general values for m_1 and m_2 are 0.01 and 0.03 respectively. If ' f ' is the fused image and r is the reference image:

$$SSIM = \frac{(2r2f + D1)(2r2f + D2)}{(2r2f + D1)(2r + 2f + D2)} \quad (33)$$

Mutual Information (MI)

Mutual information is utilized as a measure to know the performance in multimodal fusion (Bhateja et al., 2015). If the value of MI is high, then it suggests that the combined frame constraints more parental image information. $p(m)$, $p(n)$ are the marginal probability distribution functions of both the images, $p(m,n)$ is the joint probability distribution function:

$$MI(m, n) = \sum_{m \in M} \sum_{n \in N} p(m, n) \log \left(\frac{p(m, n)}{p(m) p(n)} \right) \quad (34)$$

where M and N are number of rows and columns of both the images.

Entropy

It is defined as the amount of information which the image contains. It can be used to measure the quality of the output fused image. The entropy can be known if probability density (p) of pixel levels is known. If the entropy is higher, then fusion is efficient (Bhateja et al., 2015):

$$Entropy = - \sum_{i=0}^{K-1} p(mi) \log p(mi) \quad (35)$$

Edge Based Similarity Measure ($Q^{AB/F}$)

It can be used to measure the edge details in a fused image. The value of Q is close to 1 for a high-quality fused image. A number of ways are introduced to get the edge information of the image like simple edge detection algorithm, local gradients and many more (Bhateja et al., 2015). It is calculated with below equation:

$$Q^{AB/F} = \frac{\sum_{i=1}^x \sum_{j=1}^y Qa(i, j) Wa(i, j) + Q(i, j) Wb(i, j)}{\sum_{i=1}^x \sum_{j=1}^y Wa(i, j) + Wb(i, j)} \quad (36)$$

where:

$$Qa(i,j) = Qa_g(x,y)Qa_a(x,y), Qb(i,j) = Qb(x,y)Qb_a(x,y) \quad (37)$$

Correlation Coefficient (CC)

This parameter is used to measure the closeness between reference image and fused image. If the reference image and fused image are identical then the value of correlation coefficient is nearly equal to 1 (Bhateja et al., 2015). If the difference between the fused image and reference image is additional, then the value of correlation coefficient will be less than 1:

$$C = \frac{\sum_{i=1}^j (m_i - M)}{\sqrt{\sum_{i=1}^j (m_i - M)^2 \sum_{i=1}^j (n_i - N)^2}} \quad (38)$$

where M and N are mean values of corresponding images.

Experiments on CT/MRI Image Fusion

Neoplastic Disease

Neoplastic diseases are conditions that cause tumor growth. It is the kind of tumor due to extreme development of cells in the brain. They can be benign and malignant. Benign tumors are non-cancerous and grow slowly and spread less to other organs. Malignant tumors are cancerous and can spread to other tissues or organs. If not detected in the early stage, it may be life threatening. Identifying such kind of growth at early stage, may not lead to cancer. If such kind of growth is malignant, it will spread to all other parts of brain at quick rate leads to cancer. From Figure 9, such kind of tumor

Figure 9. (a) Input images; (b) Segmented images: Fused images; (c) With PSO; (d) With QPSO; (e) With HBMO; (f) With pFA

growth is identified better with the proposed algorithm as compared to others and Table 9 shows the performance of the proposed algorithm in FSIM, SSIM, Entropy, PSNR, $Q^{AB/F}$ and MI.

Cerebrovascular Disease

Cerebrovascular disease refers to conditions that affect the flow of blood through the brain and can lead to stroke. Hypertension (high blood pressure) is the most important risk factor for stroke

Table 9. Set A: Neoplastic disease

Method	FSIM	ENTROPY	PSNR	SSIM	STD	$Q^{AB/F}$	MI	CC
PSO	0.893427	21.064733	34.32786	0.91319	0.189056	0.620952	3.23917	0.62095
QPSO	0.956501	21.109136	34.36472	0.914872	0.206056	0.663134	3.28259	0.66313
HBMO	0.958995	21.134748	34.38426	0.934355	0.234453	0.683590	3.28660	0.68359
Proposed	0.978974	21.325112	34.38888	0.947851	0.238101	0.696854	3.29874	0.68987

and cerebrovascular disease. This disease is permanently or temporarily effected by some of the reasons like ischemia, area of brain, blood bleeding and in pathological process more than two blood vessels involvement. It includes aneurysms, vertebral stenosis, carotid stenosis and, stroke, vascular malformations and intracranial stenosis. These diseases can be prevented by medication and lifestyle changes. Blood thinners and other modalities, including surgery are used to treat strokes. Figure 10 shows the effectiveness of proposed algorithm in visual quality and Table 10 shows in all measured parameters proposed is better.

Figure 10. (a) Input images; (b) Segmented images: Fused images; (c) With PSO; (d) With QPSO; (e) With HBMO; (f) With pFA

Table 10. Set B: Cerebrovascular disease

Method	FSIM	ENTROPY	PSNR	SSIM	STD	$Q^{AB/F}$	MI	CC
PSO	0.991425	22.640391	34.39331	0.94834	0.137244	0.594891	3.30639	0.59489
QPSO	0.960303	22.679535	34.58154	0.949456	0.152098	0.604217	3.34325	0.60422
HBMO	0.942479	22.689466	34.58518	0.953553	0.183669	0.683029	3.34592	0.68303
Proposed	0.975484	22.754874	34.60245	0.965874	0.112457	0.685874	3.35015	0.69999

Alzheimer Disease

Alzheimer's disease is an irreversible progressive brain disorder that slowly destroys memory and thinking skills eventually, the ability to carry out even simple tasks. It usually appears first in mid - 60s. There is no cure for this disease, but treatment can help slow the progression of the disease and improve quality of life. In general, this disease is diagnosed by testing metal stability manually or eye tracing test or by MRI scan or by CT scan. A small structural difference in brain leads to disease, so identifying such kind of small difference is a big task for the researchers. In this paper, MRI and

CT scan images are fused for better identification of small differences in brain which leads to early diagnosis of Alzheimer. T1-Weighted MR image of 70 years old man and T2-Weighted MR images of 73 years old man is considered for performance test of proposed method. The qualitative result of the proposed algorithm is well explained in the Figure 11. From the Figure 11, the visual quality of Alzheimer disease is better with proposed as compared with other algorithms. The quantitative results of the proposed is as shown in Table 11. It shows the superiority in all aspects.

Figure 11. (a) Input images; (b) Segmented images: Fused images; (c) With PSO; (d) With QPSO; (e) With HBMO; (f) With pFA

Table 11. Set C: Alzheimer disease

Method	FSIM	ENTROPY	PSNR	SSIM	STD	$Q^{AB/F}$	MI	CC
PSO	0.336717	21.390594	32.82359	0.911531	0.777079	0.719411	3.1012	0.71941
QPSO	0.444859	21.405167	32.83409	0.937020	0.444859	0.720572	3.1255	0.72057
HBMO	0.660098	21.485016	32.85253	0.951837	0.660980	0.725686	3.1454	0.72569
Proposed	0.612444	21.489999	32.87854	0.959658	0.325697	0.747844	3.1665	0.73687

Fatal Disease

A fatal disease has no cure and ultimately results in death of the patient. Death can occur within a few hours to several years, depending upon the particular disease. Some examples of fatal diseases include cancer, severe heart disease, AIDS, dementia etc. As there is no cure, the patients can only be given supportive and palliative treatment. In this paper Pick's disease is consider as a fatal disease. Pick's disease is a rare disease which leads to loss of memory, reduction in thinking levels, remembering the language and abnormal behavior of human beings. The reason is some minor damage in brain tissues. This can be identified by fusing CT and MRI images and was successfully done with the proposed method and compared the results with other soft-computing techniques. Form Table 12 and Figure 12 it is observed the proposed method effectiveness in fusing the images.

Sarcoma Disease

Sarcoma is a malignant tumor, a rare kind of cancer. Sarcomas grow in connective tissue cells, for example fat, blood vessels, nerves, bones, muscles and cartilage. They are treated by surgery to remove the tumor and radiotherapy and chemotherapy. Sarcomas are usually incurable and can be deadly. From Figure 13, visual fused image quality obtained with pFA is far better than fused images obtained with PSO, QPSO and HBMO. The green color regions in Figure 13(f) shows the tumor

Figure 12. (a) Input images; (b) Segmented images: Fused images; (c) With PSO; (d) With QPSO; (e) With HBMO; (f) With pFA

Table 12. Set D: Fatal disease

Method	FSIM	ENTROPY	PSNR	SSIM	STD	$Q^{AB/F}$	MI	CC
PSO	0.307900	22.177216	32.56973	0.897339	0.437479	0.742869	3.54065	0.74287
QPSO	0.437479	22.254970	32.63030	0.920576	0.833801	0.756198	3.54868	0.75620
HBMO	0.693950	22.278068	32.78759	0.924099	0.307951	0.768028	3.59584	0.76803
Proposed	0.712457	22.358744	32.89765	0.932547	0.547844	0.77487	3.60124	0.78975

Figure 13. (a) Input images; (b) Segmented images: Fused images; (c) With PSO; (d) With QPSO; (e) With HBMO; (f) With pFA

effected regions which is top highlighted with proposed pFA. The proposed method is better not only in visual quality but also in other fusion measuring parameters as shown in Table 13.

From the above discussions, results and experiments, it is found that proposed pFA gives advanced amount of mutual information (MI) by image thresholding and Scale-Invariant Feature Transform. The experimental results show that the proposed fusion technique has significant quantitative improvement than that of the existing fusion techniques. This fusion algorithm is tested for medical imaging applications and it is also well suited for other applications like remote sensing and development of region of area. The proposed technique can be used for satellite imaging applications such as weather forecasting, forest degradation and object tracking applications. The proposed algorithm overcomes the limitations of PSO, QPSO and HBMO.

Table 13. Set E: Sarcoma disease

Method	FSIM	ENTROPY	PSNR	SSIM	STD	$Q^{AB/F}$	MI	CC
PSO	0.21906	21.23275	33.691	0.89593	0.24321	0.649	2.8722	0.6490
QPSO	0.24321	21.25676	33.708	0.90873	0.32883	0.6583	2.8922	0.6583
HBMO	0.32883	21.39504	33.767	0.92194	0.21906	0.6615	2.8956	0.6615
Proposed	0.396587	21.45878	33.801	0.92878	0.26587	0.6745	2.90124	0.6754

CONCLUSION

In this paper, two MRI images are fused for better diagnosis of diseases like neoplastic disease, cerebrovascular disease, Alzheimer disease, fatal disease, sarcoma disease. The two images are segmented on the basis of 2-D histogram with pFA by assuming Renyi entropy as objective function. The pFA is used to get optimal thresholds for better segmentation. The segmented regions of both the images are extracted with the Scale Invariant Feature Transform (SIFT) algorithm and on the basis of interval type-2 fuzzy set fusion rules both the images are fused. From the experiments, proposed algorithm is better in visual quality for diagnosing diseases and better in mutual information, Edge based similarity measure and correlation coefficient as compared to other algorithms.

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