A New Design of a Functional H Filter for Linear Singular Systems With an Additional Unknown Input and Bounded Disturbances

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ABSTRACT

This paper proposes a design of a H_∞ filter for a functional of the system state that the authors call functional H_∞ filter in time domain for linear singular systems with additional unknown inputs and bounded disturbances. So, this research proposes the estimation of a part of the unknown inputs which will be injected on the system state equation besides the functional state estimation. The design procedure is based on the unbiasedness of the estimation error using Lyapunov-Krasovskii stability theory and H_∞ criterion. The designed filter is characterized by a gain which is the solution of linear matrix inequalities (LMI). Numerical example is given to illustrate the proposed approach.

KEYWORDS

Bounded Disturbances, Functional Filter, H_∞ Criterion, Singular Systems, Unknown Input

1. INTRODUCTION

The reconstruction of the state vector is an inevitable step for control and diagnosis of many processes (Darouach, 2005; Khadhraoui, 2014). This topic has been investigated by many researchers during the last decades (Johnson, 1975; Hostetter, 1973) aiming the synthesis of a control scheme based on an observer gain. Then, the control law is expressed mainly in function of the output measurements. For this purpose, we propose the synthesis of an unknown input functional filter for linear singular systems affected by bounded disturbances and unknown inputs. This is motivated by the importance of such system which brings together three criteria (singular, unknown input and bounded disturbance) often present in most of physical process.

Also, the singularity has, recently, become the focus of several research activities (Dai, 1989; Darouach, 1995; Darouach, 2005) due to its importance in the modeling of physical systems. In fact singular system can be viewed as a generalization of standard linear systems as they contain a dynamical and algebraic part in the systems descriptions. Then the importance of such systems comes from their ability to describe such systems despite their complex structures.

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Moreover, the synthesis of observers becomes more interesting especially when we take into account faults action on the dynamic and the output Equations (Johnson, 1975; Hostetter, 1973). This topic has been recently considered (Hou, 1992; Gaddouna, 1994; Darouach, 1996; Khadhraoui, 2014), and all related works estimate only the state vector. However, the proposed functional unknown input filter reconstructs both the functional state vector and the unknown input vector in the presence of faults. Then we can use these results in the failure detection and the control of systems in presence of disturbances.

Finally, the problem of the state observing of linear singular systems with bounded disturbances has been a great deal of investigation during the last decades (Khadhraoui, 2014; Darouach, 1996; Zasadzinski, 1998). In fact, filtering a state vector, lies on the reconstruction of a linear combination of the system states using only the input and the output measurements.

Two performance measures in filtering problem are extensively used, the H_2 and the H_∞ norms (Khadhraoui, 2014, 2016; Souley, 2006). In this paper, we choose the H_∞ filter due to its performance for minimizing the perturbation effect on the estimation error.

Motivated by these facts, we propose a functional H_∞ filter to estimate both functional state vector and part of the unknown inputs vector based on Lyapunov-Krasovskii approach. And using the unbiasedness condition, we arrange the error expression so that the error dynamics ignores the disturbance derivative ($\dot{w}(t)$) (Souley, 2006). Then a Linear Matrix Inequalities (LMI) approach is developed to find the filter optimum gain.

The paper is organized as follows. Section 2 presents the difference between current work and recent published work. The third section gives assumptions used through this paper and presents the problem formulation that we propose to solve. Section 4 gives a time domain solution for the unknown input functional filter design problem with respect to the H_∞ performance criterion. The next section summarizes the design algorithm presentation. Section 6 gives a numerical example to illustrate our approach and the last section concludes the paper.

2. RELATED WORK

In this paper, we propose to reconstruct; firstly, a functional state vector by reducing mathematical conditions and then reducing time calculate (Ezzine, 2010)). Secondly, in recent references (Khadhraoui, 2014, 2016) the filter of the unknown inputs systems which permit to avoid to have advanced perturbation term in the error dynamics; the authors propose an approach which resolves the H_∞ filtering problem even when the term is present and they can estimate only the state vector. In this present paper we add of these previews results the estimation of the unknown inputs. This contribution is of major importance for faults detection.

3. PROBLEM FORMULATION

Let's consider the following continuous-time linear descriptor system described by:

$$E\dot{x}(t) = Ax(t) + Bu(t) + F_1v(t) + G_1w(t) \quad (1a)$$

$$y(t) = Cx(t) + Du(t) + F_2v(t) + G_2w(t) \quad (1b)$$

$$z(t) = Lx(t) \quad (1c)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ and $z \in \mathbb{R}^r$ are the state, the input, the output and the functional state vectors respectively. $v \in \mathbb{R}^q$ and $w \in \mathbb{R}^k$ are the unknown input and the bounded disturbance vectors. $E, A, B, F_1, F_2, G_1, G_2, L, D$ and C are known matrices of appropriate dimensions.

In the sequel we suppose that:

Assumption 1: (Zasadzinski, 1998):

$$\text{rank}(E) = \bar{n} < n$$

$$\text{rank} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = q$$

$$\text{rank} \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = k$$

$$\text{rank}(F_2) = \bar{q} < q$$

Then, there exist an orthogonal matrix $R \in \mathbb{R}^{p \times p}$ and a non singular matrix $R \in \mathbb{R}^{q \times q}$, such that:

$$R^T F_2 S = \begin{pmatrix} I_{\bar{q}} & 0_{12} \\ 0_{21} & 0_{22} \end{pmatrix} \quad (2)$$

with $0_{12} \in \mathbb{R}^{\bar{q} \times (q-\bar{q})}$, $0_{21} \in \mathbb{R}^{(p-\bar{q}) \times \bar{q}}$ and $0_{22} \in \mathbb{R}^{(p-\bar{q}) \times (q-\bar{q})}$.

Using Equation (2), system (1) is equivalent to:

$$E\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t) + F_{11}y_1(t) + F_{12}v_2(t) + \bar{G}w(t) \quad (3a)$$

$$y_1(t) = C_1x(t) + D_1u(t) + v_1(t) + G_{11}w \quad (3b)$$

$$y_2(t) = C_2x(t) + D_2u(t) + G_{22}w(t) \quad (3c)$$

$$z(t) = Lx(t) \quad (3d)$$

with:

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = R^T y(t) \text{ with } y_1 \in \mathbb{R}^{\bar{q}} \text{ and } y_2 \in \mathbb{R}^{p-\bar{q}} \quad (4a)$$

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = S^{-1} v(t) \text{ with } v_1 \in \mathbb{R}^{\bar{q}} \text{ and } v_2 \in \mathbb{R}^{q-\bar{q}} \quad (4b)$$

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = R^T C, \quad \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = R^T D, \quad \begin{bmatrix} G_{11} \\ G_{22} \end{bmatrix} = R^T G_2 \quad (5a)$$

$$\begin{bmatrix} F_{11} & F_{12} \end{bmatrix} = F_1 S, \quad \bar{A} = A - F_{11} C_1 \quad (5b)$$

$$\bar{B} = B - F_{11} D_1, \quad \bar{G} = G_1 - F_{11} G_{11} \quad (5c)$$

We note that Equations (3b) and (3c) are obtained by multiplying (1b) by R^T and using (2).

3.1. Objective

The objective of this paper is to design in time domain an unknown input functional H_∞ filter for linear singular system. The considered system is affected by a bounded disturbance acting on the dynamic and output Equations. The proposed filter reconstructs both the functional state $z(t)$ and the unknown input vector $v_1(t)$ derived from (3.b). So, we assume that:

$$\bar{z}(t) = \begin{bmatrix} z(t) \\ v_1(t) \end{bmatrix} \quad (6)$$

The functional H_∞ filter estimate $\bar{z}(t)$ described by Equation (6) use only the system known input $u(t)$ and output $y(t)$ as shown in Figure 1.

The system (3) can be written such:

$$E\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t) + F_{11}y_1(t) + F_{12}v_2(t) + \bar{G}w(t) \quad (7a)$$

$$y_2(t) = C_2x(t) + D_2u(t) + G_{22}w(t) \quad (7b)$$

Figure 1. The functional H_∞ filter design

$$\bar{z}(t) = \bar{L}x(t) + \bar{D}_1 u(t) + \bar{G}_{11} w(t) + \bar{I}y_1(t) \quad (7c)$$

with:

$$\bar{L} = \begin{bmatrix} L \\ -C_1 \end{bmatrix}, \bar{D}_1 = \begin{bmatrix} 0_{r \times m} \\ -D_1 \end{bmatrix}, \bar{G}_{11} = \begin{bmatrix} 0_{r \times k} \\ -G_{11} \end{bmatrix}, \bar{I} = \begin{bmatrix} 0_{r \times \bar{q}} \\ \bar{I}_{\bar{q}} \end{bmatrix} \quad (8)$$

and:

$$\bar{L} \in \mathbb{R}^{(r+\bar{q}) \times n}, \quad \text{rank}(\bar{L}) = r + \bar{q} < n \quad (9)$$

In the sequel of the paper we suppose that: (Darouach, 1995):

$$\text{rank} \begin{pmatrix} E \\ C_2 \\ \bar{L} \end{pmatrix} = \text{rank} \begin{pmatrix} E \\ C_2 \end{pmatrix} \quad (10)$$

4. TIME DOMAIN DESIGN

From (10), there exists a non singular matrix T :

$$T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (11)$$

such as:

$$aE + bC_2 = \bar{L} \quad (12)$$

$$cE + dC_2 = 0_{(p-\bar{q}) \times n} \quad (13)$$

with $a \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^{n \times (p-\bar{q})}$, $c \in \mathbb{R}^{(p-\bar{q}) \times n}$ and $d \in \mathbb{R}^{(p-\bar{q}) \times (p-\bar{q})}$.

The designed functional filter is on the form:

$$\dot{\eta}(t) = M\eta(t) + Ku(t) + N_1y_1(t) + N_2y_2(t) \quad (14a)$$

$$\hat{\bar{z}}(t) = \eta(t) + Pu(t) + H_1y_1(t) + H_2y_2(t) \quad (14b)$$

where $\eta(t)$ is the state vector of the filter and $\hat{\bar{z}}(t)$ is the estimate of the functional state $\bar{z}(t)$. The matrices M, K, N_1, N_2, P, H_1 and H_2 will be determined using the LMI approach.

4.1. Conditions of an Unknown Input Filter Synthesis

Using (7c) and (14b) the estimation error $e(t)$ is given by:

$$e(t) = \bar{z}(t) - \hat{\bar{z}}(t) \quad (15)$$

$$= (\bar{L} - H_2C_2)x(t) + (\bar{D}_1 - P - H_2D_2)u(t) + (\bar{G}_{11} - H_2G_{22})w(t) + (\bar{I} - H_1)y_1(t) - \eta(t)$$

We assume that:

$$P = \bar{D}_1 - H_2D_2 \quad (16)$$

$$H_1 = \bar{I} \quad (17)$$

Then the Equation (15) becomes:

$$e(t) = (\bar{L} - H_2C_2)x(t) + (\bar{G}_{11} - H_2G_{22})w(t) - \eta(t) \quad (18)$$

In order to avoid the disturbance time derivative $\dot{w}(t)$ in the error dynamics, we propose that:

$$\epsilon(t) = (\bar{L} - H_2C_2)x(t) - \eta(t) \quad (19a)$$

$$e(t) = \epsilon(t) + (\bar{G}_{11} - H_2G_{22})w(t) \quad (19b)$$

Then, when replacing $\bar{L}$ by its expression (12) in (19a) and using Equation (13) we can write:

$$\begin{aligned}\epsilon(t) &= (aE + bC_2 - H_2C_2)x(t) + \beta(cE + dC_2)x(t) - \eta(t) \\ &= (a + \beta c)Ex(t) + (b + \beta d - H_2)C_2x(t) - \eta(t)\end{aligned}\quad (20)$$

where $\beta \in \mathbb{R}^{(r+\bar{q}) \times (p-\bar{q})}$.

We assume that:

$$H_2 = b + \beta d \quad (21)$$

Then Equations (20) can be written in the following form:

$$\epsilon(t) = (a + \beta c)Ex(t) - \eta(t) = \varphi Ex(t) - \eta(t) \quad (22)$$

with:

$$\varphi = a + \beta c \quad (23)$$

Given the singular system (7) and the functional filter (14), we aim to design the matrices of the filter M, K, N_1, N_2, P, H_1 and H_2 such that:

$$\lim_{t \rightarrow \infty} e(t) = 0, \text{ if } w = 0$$

The filtering error (19) is asymptotically stable and satisfy the H_∞ performance.

The H_∞ criterion is given by:

$$0 < H_{\epsilon w \infty} = \sup_{w \neq 0} \frac{\epsilon_2}{w_2} < \gamma \quad (24)$$

with $H_{\epsilon w}(s) = \frac{\epsilon(s)}{w(s)}$ is a transfer matrix and γ is a positive scalar.

For this, we propose the following theorem.

4.1.1. Theorem 1

The unbiasedness estimation error given by (19) relative to system (7) and functional H_∞ filter (14) is verified such that:

$$\epsilon(t) = M\epsilon(t) + (\varphi\bar{G} - N_2G_{22})w(t) \quad (25a)$$

$$e(t) = \epsilon(t) + (\bar{G}_{11} - H_2 G_{22})w(t) \quad (25b)$$

if and only if the following Equations are satisfied:

$$\varphi \bar{A} - N_2 C_2 - M \varphi E = 0$$

$$\varphi \bar{B} - N_2 D_2 - K = 0$$

$$\varphi F_{11} - N_1 = 0$$

$$\varphi F_{12} = 0$$

4.1.2. Proof 1

The derivative of $\epsilon(t)$ (22) is given as follows:

$$\dot{\epsilon}(t) = \varphi E \dot{x}(t) - \dot{\eta}(t) \quad (26)$$

Now, replacing (7a) and (14a) in (26), we obtain:

$$\begin{aligned} \dot{\epsilon}(t) = & \varphi \bar{A} x(t) + \varphi \bar{B} u(t) + \varphi F_{11} y_1(t) + \varphi \bar{G} w(t) - M \eta(t) - K u(t) - N_1 y_1(t) \\ & - N_2 y_2(t) + \varphi F_{12} v_{-2}(t) \end{aligned} \quad (27)$$

and:

$$\begin{aligned} \dot{\epsilon}(t) = & M \epsilon(t) + [\varphi \bar{A} - N_2 C_2 - M \varphi E] x(t) + [\varphi \bar{B} - N_2 D_2 - K] u(t) \\ & + [\varphi F_{11} - N_1] y_1(t) + [\varphi \bar{G} - N_2 G_{22}] w(t) + \varphi F_{12} v_2(t) \end{aligned} \quad (28)$$

4.2. Determination of Observer Matrices

This method is based on the Lyapunov-Krasovskii stability theory (Krasovskii, 1963) and the LMI approach. So that, the stability conditions of the observer will be independent of the unknown input $v_2(t)$ and subsequently the estimated functional state $\hat{\bar{z}}(t)$ converges asymptotically to $\bar{z}(t)$.

By replacing (23) in conditions i) and iv) of theorem 1 and using (13), we have:

$$a \bar{A} = M a E + J C_2 - \beta c \bar{A} \quad (29)$$

and:

$$aF_{12} = -\beta cF_{12} \quad (30)$$

with:

$$J = N_2 - M\beta d \quad (31)$$

Equations (29) and (30) can be written in matrix form:

$$\Gamma = \Omega\Theta \quad (32)$$

with:

$$\Omega = \begin{bmatrix} M & J & \beta \end{bmatrix} \quad (33)$$

$$\Theta = \begin{pmatrix} aE & 0 \\ C_2 & 0 \\ -c\bar{A} & -cF_{12} \end{pmatrix} \quad (34)$$

$$\Gamma = \begin{bmatrix} a\bar{A} & aF_{12} \end{bmatrix} \quad (35)$$

We note that the general Solution (33) exists if and only if:

$$\text{rank} \begin{bmatrix} \Theta \\ \Gamma \end{bmatrix} = \text{rank}(\Theta) \quad (36)$$

Therefore, by respecting the Condition (36):

$$\Omega = \Gamma\Theta^+ - Z(I - \Theta\Theta^+) \quad (37)$$

where Θ^+ is the generalized inverse of matrix Θ and Z is an arbitrary matrix of appropriate size, that will be determined later using the LMI approach.

The unknown matrix M is defined by:

$$M = \Omega \begin{pmatrix} I \\ 0 \\ 0 \end{pmatrix} = M_1 + ZM_2 \quad (38)$$

By replacing Ω by its expression given by (37) in (38), we obtain:

$$M_1 = \Gamma \Theta^+ \begin{pmatrix} I \\ 0 \\ 0 \end{pmatrix}, M_2 (I - \Theta \Theta^+) \begin{pmatrix} I \\ 0 \\ 0 \end{pmatrix} \quad (39)$$

Similarly, M_1 is obtained such:

$$J = \Omega \begin{pmatrix} 0 \\ I \\ 0 \end{pmatrix} = J_1 + ZJ_2 \quad (40)$$

By replacing Ω by its expression given by (37) in (40), we obtain:

$$J_1 = \Gamma \Theta^+ \begin{pmatrix} 0 \\ I \\ 0 \end{pmatrix}, M_2 (I - \Theta \Theta^+) \begin{pmatrix} 0 \\ I \\ 0 \end{pmatrix} \quad (41)$$

and β is obtained such:

$$\beta = \Omega \begin{pmatrix} 0 \\ 0 \\ I \end{pmatrix} = \beta_1 + Z\beta_2 \quad (42)$$

By replacing Ω by its expression given by (37) in (42), we obtain:

$$\beta_1 = \Gamma \Theta^+ \begin{pmatrix} 0 \\ I \\ 0 \end{pmatrix}, \beta_2 (I - \Theta \Theta^+) \begin{pmatrix} 0 \\ I \\ 0 \end{pmatrix} \quad (43)$$

Using (31), (38) and (42) we can write $N_2 G_{22}$ such that:

$$\begin{aligned} N_2 G_{22} &= (J + M\beta d) G_{22} \\ &= JG_{22} + (M_1 + ZM_2)(\beta_1 + Z\beta_2) dG_{22} \\ &= JG_{22} + M_1(\beta_1 + Z\beta_2) dG_{22} + (ZM_2\beta_1 + ZM_2Z\beta_2) dG_{22} \end{aligned} \quad (44)$$

Since the matrix $N_2 G_{22}$ have a bi-linearity $(Z M_2 Z)$. So we assume that the gain matrix Z satisfies the following relation:

$$Z \beta_2 = 0 \quad (45)$$

So there always exists a matrix Z_1 such that:

$$Z = Z_1 (I - \beta_2 \beta_2^+) = Z_1 \Delta \quad (46)$$

Then M , J and β are defined as:

$$M = M_1 + Z_1 \Delta M_2 \quad (47)$$

$$J = J_1 + Z_1 \Delta J_2 \quad (48)$$

$$\beta = \beta_1 + Z_1 \Delta \beta_2 \quad (49)$$

At this stage, we propose to replace in (25) H_2 , φ and $N_2 G_{22}$ by their expressions given by (21), (23) and (44) respectively, we can write:

$$\epsilon'(t) = A_0 \epsilon(t) + B_0 w(t) \quad (50a)$$

$$e(t) = C_0 \epsilon(t) + D_0 w(t) \quad (50b)$$

where:

$$A_0 = M \quad (51)$$

$$B_0 = (a + \beta_1 c) \bar{G} - (J_1 + M_1 \beta_1 d) G_{22} + Z_1 \Delta [\beta_2 c \bar{G} - (J_2 + M_2 \beta_1 d) G_{22}] \quad (52)$$

$$C_0 = I \quad (53)$$

$$D_0 = \bar{G}_{11} - (b + \beta_1 d) G_{22} \quad (54)$$

Now we can determine all matrix of filter (14) if and only if the matrix Z_1 is determined, for this we use the following theorem.

4.2.1. Theorem 2

The system (14) is a functional filter for the system (1) if there exist matrices $P = P^T > 0$ and Y which satisfy the following matrix inequality:

$$\begin{pmatrix} A_0^T P + P A_0 & P B_0 & C_0^T \\ * & -\gamma^2 I & D_0^T \\ * & * & -I \end{pmatrix} < 0 \quad (55)$$

The gain Z_1 is given by:

$$Z_1 = P^{-1} Y \quad (56)$$

4.2.2. Proof 2

Let $V(\epsilon, t)$ be the Lyapunov-Krasovskii functional of the form:

$$V(\epsilon, t) = \epsilon^T(t) P \epsilon(t) \quad (57)$$

where $P = P^T > 0$.

In order to establish sufficient conditions for existence of (14) according to (24), we should verify the following inequality:

$$H(\epsilon, t) = \dot{V}(\epsilon, t) + e^T(t) e(t) - \gamma^2 w^T(t) w(t) < 0 \quad (58)$$

Using (51b), Equation (59) can be written as:

$$\begin{pmatrix} \epsilon^T & w^T \end{pmatrix} \begin{pmatrix} A_0^T P + P A_0 + C_0^T C_0 & P B_0 + C_0^T D_0 \\ * & D_0^T D_0 - \gamma^2 I \end{pmatrix} \begin{pmatrix} \epsilon \\ w \end{pmatrix} < 0 \quad (59)$$

According to the Schur Lemme (Khadhraoui, 2016) we deduct Equation (55).

5. ALGORITHM FOR FUNCTIONAL FILTER DESIGN

Step 1: Check that conditions 1), 2), 3) and 4) of assumption 1 are satisfied.

Step 2: Compute matrices R and S using (2).

Step 3: Check that condition (10) is verified.

Step 4: Compute matrix T using (12) and (13).

Step 5: Compute Θ and Γ using (34) and (35).

Step 6: Deduce the values of matrices M_1 , M_2 , J_1 , J_2 , β_1 and β_2 from (39), (41) and (43).

Step 7: Verify that the condition (36) is satisfied. Then resolution of the proposed LMI (55), gives the gain matrix Z_1 .

Step 8: Compute, J , and β_1 using (47), (48) and (49).

Step 9: Get matrices P , H_1 , H_2 and N_2 using (16), (17), (21) and (31).

Step 10: Get matrices K and N_1 using ii) and iii) from theorem 1.

6. NUMERICAL EXAMPLES

Let's consider system (1) where:

$$E = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, A = \begin{pmatrix} 1 & -1 & -1 \\ 2 & -1 & 1 \\ 2 & 0 & -1 \end{pmatrix}, C = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$F_1 = \begin{pmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 1 \end{pmatrix}, F_2 = \begin{pmatrix} 0 & 0 \\ 1 & 2 \\ 0 & 0 \end{pmatrix}, G_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, G_2 = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}, L = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$$

The H_∞ criterion is bounded by $= 1,5$.

The input signal $u(t)$, the bounded disturbance $w(t)$ and the unknown inputs $u(t)$ are given by Figures 2-4 respectively.

Following the procedure proposed below, we obtain:

$$S = \begin{pmatrix} -0.2 & -0.89 \\ -0.4 & -0.44 \end{pmatrix}, R = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Figure 2. Input signal $u(t)$

Figure 3. Bounded disturbance $w(t)$

Figure 4. Unknown inputs $v(t)$

By multiplying the unknown inputs $v(t)$ by S^{-1} we have (Figure 5):

The LMI approach application (55) yields:

$$M = \begin{pmatrix} -5.54 & -9.25 \\ 2.74 & 3.79 \end{pmatrix}, K = \begin{pmatrix} -2.44 \\ 1.59 \end{pmatrix}, N_1 = \begin{pmatrix} 0.43 \\ -0.45 \end{pmatrix}, N_2 = \begin{pmatrix} 0.94 & -0.06 \\ -0.56 & 0.43 \end{pmatrix}$$

$$H_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, H_2 = \begin{pmatrix} 0.75 & -0.24 \\ -0.56 & 0.43 \end{pmatrix}, P = \begin{pmatrix} 1.75 \\ -0.56 \end{pmatrix}$$

Figure 6 represents the evolution of the real and the estimated functional state vector $z(t)$ and Figure 7 represents the real and the estimated unknown inputs $v_1(t)$.

We can remark as it shown in Figure 6 and Figure 7 the time domain behavior of the functional H_∞ filter. The proposed filter can estimate the functional state and the unknown inputs vectors independently of unknown inputs with respect to the disturbance attenuation criterion; the effect of the disturbance on the estimation error is evaluated by $H_{ew\infty} = 1, 3 < \gamma$.

Figure 5. Part of unknown inputs $v_1(t)$

Figure 6. Functional state $z(t)$

Figure 7. Unknown inputs $v_1(t)$

Then the estimation error of the functional state $z(t)$ and the unknown input $v_1(t)$ are given by Figure 8 and Figure 9 respectively.

These figures show the disturbance effect on the estimation error of functional state and unknown inputs vectors respectively during the Transient phase, permanent phase and the Transient duration when disturbance is applied ($H_{ew\infty} = 1, 3 < \gamma$). So, we conclude the convergence of the error dynamic which confirm the effectiveness of the proposed approach.

Figure 8. Estimation error of functional state

Figure 9. Estimation error of unknown input

7. CONCLUSION

In this paper, a new method for functional H_∞ filter of linear singular systems with an additional unknown inputs and bounded disturbances has been established in time domain. The output of the proposed filter combines the state and the unknown input vectors.

An LMI solution is proposed to find the optimum gain implemented in the observer design. This solution is based on the unbiasedness of estimation error, the Lyapunov function approach (Krasovskii, 1963) and the disturbance minimizing effect on the estimation error.

Thanks to this filter we can estimate a part of the unknown inputs in order to detect some defaults.

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