A Hybrid Between TOA and Lévy Flight Trajectory for Solving Different Cluster Problems

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ABSTRACT

In data analysis applications for extraction of useful knowledge, clustering plays an important role. The major shortcoming of traditional clustering algorithms is exhibiting poor performance in solving complex data cluster problems. This research paper introduces a novel hybrid optimization technique-based clustering approach. This paper is designed with two main objectives: designing efficient function optimization algorithm and developing advanced data clustering approach. In achieving the first objective, the standard TOA is first enhanced by hybridizing with Lévy flight trajectory and benchmarked on 23 functions. A new clustering approach is developed by conjoining k-means algorithm and Lévy flight TOA. The numerical complexity of the proposed novel clustering approach was tested on 10 UCI clustering datasets and four web document cluster problems. Several simulation experiments were conducted and an analysis of the results was done. The obtained graphical and statistical analysis reveals that the proposed novel clustering approach yields better quality clusters.

KEYWORDS

Clustering Problems, Computational Intelligence, Nature-Inspired Metaheuristics, TOA Levy Flight Trajectory NP-Hard Problems, Web Document Cluster Problems

I. INTRODUCTION

Computational Intelligence (CI) is the fastest growing and promising area of study that have drawn more attention from the researchers over the years (Zadeh, L. A., et al., 2014). CI, also termed as *Soft Computing* in Lotfi Zadeh's terminology (Zadeh, L. A., 1996) is concerned with achieving of pattern recognition in data and structures in data (Wang, Y., 2017; Wang, Y., et al., 2016). CI is the only destination for many design and engineering researchers those who are working on modeling and analyzing complex systems (Pedrycz, W., Et al., 2014). CI embraces Nature-Inspired Metaheuristics (NIMs) like Bio-Inspired Computing (BIC), Swarm Intelligence (SI), Evolutionary Computations (EC), Metaphor-based Metaheuristics (MbM) and other Computational methodologies in solving NP-hard optimization problems (Bouarara, H. A., et al., 2015; Gheraibia, Y., et al., 2015; Kaynak,

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O., et al., 2012; Nouaouria, N., et al., 2014; Schor, D., & Kinsner, W., 2011; Zadeh, L. A., 1973). Numerous successful applications of CI have been found in many design and engineering domains like data mining and knowledge discovery, pattern recognition, clinical decision making system, control system design, Market analysis and forecasting, etc. (Azar, A. T., & Vaidyanathan, S. (Eds.), 2015).

Nature is the key source of rich models for solving computational problems. The algorithms which are designed by mimicking various phenomena existed in the nature for solving complex problems are called NIMs (Nanda, S. J., & Panda, G., 2014). NIMs, a sub-field of CI, are popular, economical approaches for solving various problems, including tough optimization problems and NP-hard problems (Fister Jr, I., *et al.*, 2013). There are several population based meta-heuristic optimization algorithms, for instance Particle Swarm Optimization (PSO) (Eberhart, R., & Kennedy, J., 1995), Whale Optimization Algorithm (WOA) (Mirjalili, S., & Lewis, A., 2016), Grey Wolf Optimization Algorithm (GWO) (Mirjalili, S., *Et al.*, 2014), Tornadogenesis Optimization Algorithm (TOA) (Saidala, R. K., & Devarakonda, N., 2017d), Salp Swarm Algorithm (SSA) (Mirjalili, S., *Et al.*, 2017), New Class Topper Optimization Algorithm (Class TOA) (Das, P., *Et al.*, 2018), etc. were developed and successfully applied to optimization problems. Population based NIMs have several unique features over other algorithms. These include collaborative learning, high exploration ability, decentralized control, and inspiration from the dynamic social behavior. Thus, many global optimization problems which can be observed frequently in many real-life applications like engineering, decision making, machine learning, statistics, optimal control, etc. (James, J. Q., & Li, V. O., 2015; Li, X., *Et al.*, 2014; Mirjalili, S., 2015; Saidala, R. K., & Devarakonda, N., 2017b) are solved with low solution cost. Over the recent years NIMs were successfully dealing with various clustering problems. So, recently the research community has kept their special interests on developing new NIMs and applying them to various data cluster problems (Nanda, S. J., & Panda, G., 2014; Saidala, R. K., & Devarakonda, N., 2017a; Saidala, R. K., & Devarakonda, N., 2018a).

Data analysis and data mining applications, clustering plays a vital role in extracting the useful hidden knowledge from the data (Witten, I. H., *Et al.*, 2016; Xu, W., et al., 2017). In clustering task, the dataset is divided into homogeneous groups called clusters. These clusters must possess a higher degree of closeness (similarity) among objects of each cluster and low degree of closeness (similarity) among objects of the other clusters (Jacques, J., & Preda, C. 2014). Partition-based clustering, Hierarchy-based clustering, density-based clustering, and Grid-based approaches are the most well-known clustering approaches (Bouyer, A., & Hatamlou, A., 2018). The first approach, i.e. partition-based clustering approach, the dataset is partitioned into homogeneous groups using some proximity function (Saidala, R. K., & Devarakonda, N., 2018a; Xu, R., & Wunsch, D., 2005). In case of hierarchic clustering approach the data is grouped over a variety of scales by generating a cluster tree or dendrogram (Murtagh, F., 1983; Saidala, R. K., & Devarakonda, N. R., 2017c). Density-based clustering approach, considers local density conditions instead of proximity between objects. (Kriegel, H. P., *Et al.*, 2011 & Saidala, R. K., & Devarakonda, N. R., 2017c). The last approach, i.e. Grid-based approach, divides the data space into a finite number of cells to construct a grid like structure and form the homogeneous clusters of the grid cells (Nagpal, A., *Et al.*, 2013). Among the other clustering algorithms, Partitional clustering approach is widely used in real-life applications due to exhibiting simplicity and linear-time complexity of data cluster problems. Traditional clustering algorithms are good at clustering simple datasets, but many of real-life application problems are complex in nature. That means traditional clustering techniques effectively solve the data cluster problems which are having less number of attributes and limited number of data instances. When there is an increase in both attribute count and number of instances the data analysis task become complex and traditional clustering algorithms fails to return good clusters (José-García, A., & Gómez-Flores, W., 2016). In the Partitional data cluster approach, the collection of data objects is divided into set of homogeneous group(s) called cluster(s) such that inter-cluster dissimilarity is maximum or intra-cluster dissimilarity is minimum. This can be achieved by minimizing the quantization error which is an optimization problem. The quantization error $\left(J_e\right)$ is expressed as shown in Eq. (1).

Table 1. Notations of Unimodal benchmark functions

Function Description	Range	fmin
$F_1(Dim = 30) = \sum_{i=1}^n x_i^2$	$[-100, 100]$	0
$F_2(Dim = 30) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	$[10, 10]$	0
$F_3(Dim = 30) = \sum_{i=1}^n (\sum_{j=1}^i x_j)^2$	$[-100, 100]$	0
$F_4(Dim = 30) = \max_i \{ x_i , 1 \leq i \leq n\}$	$[-100, 100]$	0
$F_5(Dim = 30) = \sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]$	$[30, 30]$	0
$F_6(Dim = 30) = \sum_{i=1}^n \left[(x_i - 0.5)^2 \right]$	$[-100, 100]$	0
$F_7(Dim = 30) = \sum_{i=1}^n ix_i^4 + \text{random}[0,1]$	$[-1.28, 1.28]$	0

$$J_e = \frac{\sum_{j=1}^{N_c} \left[\sum_{\forall Z_p \in C_{ij}} \frac{d(Z_p, m_j)}{|C_{ij}|} \right]}{N_c} \quad (1)$$

Where $|C_{ij}|$ indicates the number of clusters C_{ij} and d represents the Euclidean distance function as described in Eq. (2).

$$d(D_T, Z) = \sum_{i=1}^n \sum_{j=1}^k \|D_{Tij} - Z_j\|^2 \quad (2)$$

where D_T represents the data point, Z specifies the cluster and $d(D_T, Z)$ represents the distance (Euclidean distance in this work) between D_T and Z .

The ultimate objective of any good clustering technique is to discover the clusters of similar data objects by grouping the unlabeled input data where the similarity should be minimized between the objects of each cluster and maximize the same between objects of other clusters. This leads many researchers to consider the clustering problems as NP-hard problem which can be solved by an optimization algorithm (Das, P., *Et al.*, 2018; Saidala, R. K., & Devarakonda, N., 2017a). Many nature inspired Metaheuristics have been used successfully in solving various clustering problems. The whale

Optimization based clustering approach was proposed by authors Saidala, R. K., & Devarakonda, N., using multiple (Ring, Master-slave and hybrid) cooperative strategies (Saidala, R. K., & Devarakonda, N., 2017c). Authors of (Saidala, R. K., & Devarakonda, N., 2017d) proposed a new tornado formation based meta-heuristic algorithm termed as Tornadogenesis optimization Algorithm (TOA) to solve data cluster problems. The other versions of TOA (variants of TOA) were also proposed in solving data clustering problems (Saidala, R. K., & Devarakonda, N., 2018b; Saidala, R. K., *Et al.*, 2018c). A new optimal data clustering method was proposed for finding optimal clusters of data by introducing the Northern bald Ibis optimization Algorithm (NOA) (Saidala, R. K., & Devarakonda, N., 2018d). Authors of (Saidala, R. K., *Et al.*, 2018e; Saidala, R. K., *Et al.*, 2018f) proposed various chaotic map based NOA for designing new clustering method and successfully applied to UCI and web document cluster datasets. This work presents the design of a new optimal clustering method using k -means clustering technique empowered by chaotic NOA (CNOA) for web document clustering. The key objective of the proposed CNOA based clustering method is to discover the best centroid location and increase the clustering accuracy.

Over the past few years, optimization algorithms have been applied successfully by the large proportion of researchers to solve numerous NP-hard problems. However, a major drawback of the majority of the existing algorithms is lacking of balancing between phases: exploration and exploitation (Mirjalili, S., *Et al.*, 2017; Mirjalili, S., & Lewis, A., 2016). The results with low solution cost are achieved to solve these problems in many cases by invoking hybrid strategies (Wang, G. G., *Et al.*, 2014; Saremi, S., *Et al.*, 2017). The theorem which became more popular, i.e., NFL (No Free Lunch) theorem (Wolpert, D. H., & Macready, W. G., 1997) justifies that the number of new algorithms needed to solve complex problems with low solution cost. This paper presents the hybrid version of TOA in which Lévy flight trajectory is incorporated into the standard TOA to extend its function optimization capabilities and applying it to various cluster problems.

The rest of the research paper is framed as follows: The existing approach along with the pseudo code of the Hybrid Tornadogenesis Optimization algorithm is illustrated in Section 2. The proposed approach along with a neat flowchart and pseudo code is described in Section 3. The Section 4 summarizes the experimental analysis. The conclusions of the whole paper and outline of the future work is presented in Section 5.

II. EXISTING APPROACH

Tornadogenesis Optimization Algorithm (TOA)

Brief Review of TOA

Tornadogenesis optimization algorithm (TOA) is one of the recently developed optimization algorithm inspired by nature. TOA is a population based meta-heuristic algorithm which was developed to address benchmark function optimization and complex data cluster problems (Saidala, R. K., & Devarakonda, N., 2017d). The formation of tornadoes, particularly in Oklahoma State of U.S., paved the path to develop a new algorithm named TOA.

The air around us is a mixture of molecules of warm and cool gases. The warm, humid and cool states of air molecules depend upon temperature, location, and time. The term “air mass” in meteorology is defined as large volume of air whose humidity, temperature and moisture properties are essentially homogeneous. The air in desert zones contains warm air masses wherein ocean zones contains cool or cold air masses. Oklahoma State is the centre place for having flow of warm, humid and cool air masses. The fact is that the unique geographical location of Oklahoma State permitting to have such air masses. Oklahoma State has the source of warm dry air masses from cities of Arizona and New Mexico, warm humid air masses from city of Gulf of Mexico, and cool air masses from the city of Canada. Meteorologists have done lot of study and research on “what makes the Oklahoma State leads the world’s tornadoes?” and confirmed that the geographical location of Oklahoma State

Table 2. Notations of Multimodal benchmark functions

Function Description	Range	fmin
$F_8(Dim = 30) = \sum_{i=1}^n -x_i^2 \sin(\sqrt{ x_i })$	$[-500, 500]$	-418.9829*5
$F_9(Dim = 30) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	$[-5.12, 5.12]$	0
$F_{10}(Dim = 30) = -20e^{\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^n x_i^2}\right)} - e^{\left(\frac{1}{n}\sum_{i=1}^n \cos(2\pi x_i)\right)} + 20 + e$	$[-32, 32]$	0
$F_{11}(Dim = 30) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-600, 600]$	0
$F_{12}(Dim = 30) = \frac{\pi}{n} \{10 \sin(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 [1 + 10 \sin \pi y_{i+1}^2] + (y_n - 1)^2\} + \sum_{i=1}^n u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{x_i + 1}{4} u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$	$[-50, 50]$	0
$F_{13}(Dim = 30) = 0.1\{\sin(3\pi x_1)^2 + \sum_{i=1}^n (x_i - 1)^2 [1 + \sin(3\pi x_i + 1)^2] + (x_n - 1)^2 [1 + \sin 2\pi x_n]^2\} + \sum_{i=1}^n u(x_i, 5, 100, 4)$	$[-50, 50]$	0
$F_{14}(Dim = 2) = \left(\frac{1}{500} \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^2 (x_i - a_{ij})^6} \right)^{-1}$	$[-65, 65]$	1
$F_{15}(Dim = 4) = \sum_{i=1}^{11} \left[a_i - \frac{x_i (b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4} \right]^2$	$[-5, 5]$	0.0003
$F_{16}(Dim = 2) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	$[-5, 5]$	1.0316
$F_{17}(Dim = 2) = \left(x_2 - \frac{5.1}{4\pi^2} x_1^2 + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left(1 - \frac{1}{8\pi} \right) \cos x_1 + 10$	$[-5, 5]$	0.398

continued on following page

Table 2. Continued

Function Description	Range	fmin
$F_{18}(Dim = 2) = \left[1 + (x_1 + x_2 + 1)^2 \left(3x_1^2 - 14x_2 + 6x_1x_2 \right) + 3x_2^2 \right] * [30 + (2x_1 - 3x_2)^2 * (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	$[-5, 5]$	3
$F_{19}(Dim = 3) = -\sum_{i=1}^4 c_i e^{\left(-\sum_{j=1}^3 a_{ij} (x_j - p_{ij})^2 \right)}$	$[1, 3]$	-3.86
$F_{20}(Dim = 6) = -\sum_{i=1}^4 c_i e^{\left(-\sum_{j=1}^6 a_{ij} (x_j - p_{ij})^2 \right)}$	$[0, 1]$	-3.32
$F_{21}(Dim = 4) = -\sum_{i=1}^5 \left[(X - a_i)(X - a_i)^T + c_i \right]^{-1}$	$[0, 10]$	-10.1532
$F_{22}(Dim = 4) = -\sum_{i=1}^7 \left[(X - a_i)(X - a_i)^T + c_i \right]^{-1}$	$[0, 10]$	-10.4028
$F_{23}(Dim = 4) = -\sum_{i=1}^{10} \left[(X - a_i)(X - a_i)^T + c_i \right]^{-1}$	$[0, 10]$	-10.5363

and this state is surrounded by various sources of cool and warm air masses. Typically the formation of tornadoes supervision with the expeditiously revolving columns of mixed cold and warm air masses that configure inside the storms and then extends to the surface of the earth via funnel of the cloud. The formation of tornadoes by collision of cool and warm air masses inspired the authors towards the development of new optimization algorithm.

Formulation of TOA

The mathematical modelling of the TOA is designed in three phases as follows:

Initialization Phase

This phase details the initialization of required parameters and delineation of the problem. The required parameters that are to be initialized are population of air masses (AM_Pop), Count of design parameters (Dim), Cost function: f_{cost} , Maximum number of iterations for terminations (Max_Iter), lower and upper boundaries of the design parameter: $[lb_1, lb_2, \dots, lb_n]$ to $[ub_1, ub_2, \dots, ub_n]$. These boundaries should satisfy the following condition: $lb_i \leq AM_i \leq ub_i$.

Tornado Formation Phase

The tornado formation at Oklahoma State is happened to be occurring in two different ways. They are: (I) the direction of cool and warm air masses is in the same direction and (II) the direction of cool

Table 3. Pseudo code of TOA-BBBC

INPUT: Initial air mass population AM_i, t, p, Maximum number of iterations (Max_Iter), OUTPUT: AM^* Optimal solution	
1.	Initialize population $AM_{(i=1,2,\dots,n)}$
2.	Compute fitness f_{am} for individual candidate
3.	discover position of the best candidate solution AM^*
4.	while ($t < \text{Max_Iter}$)
5.	for1 each AM_i
6.	if1 ($p < 0.5$)
7.	Update the current candidate position with Eq.(4)
8.	elseif1 ($p \geq 0.5$)
9.	Update the current candidate position with Eq. (5)
10.	end if1
11.	end for1
12.	create new population AM_n^j , $j = 1, 2, 3, \dots, k$ using Eq. (9)
13.	amend if any search candidate goes beyond the search space
14.	for2 AM_n^j , $j = 1, 2, 3, \dots, k$
15.	determine fitness f_{am_new} of AM_n^j
16.	New best solution AM_{new}^*
17.	if2 $f_{am_new} > f_{am}$
18.	$AM^* = AM_{new}^*$ and $f_{am} = f_{am_new}$
19.	end if2
20.	end for2
21.	$t = t + 1$
22.	end while
23.	return AM^*

and warm air masses is in the opposite direction. In TOA formulation, it is assumed that the chance of happening of warm and cool air mass collision when they are in the same direction and the opposite direction is 50% and 50% respectively. The formation of tornadoes is expressed mathematically as stated as in Eq. (4) and (5).

If $p \geq 0.5$

$$\overrightarrow{AM}_{(t+1)} \leftarrow AF \cdot bl \cdot \cos(\cot l) \cdot 2\pi - \overrightarrow{AM}_{(t)}^* \quad (4)$$

If $p < 0.5$

$$\overrightarrow{AM}_{(t+1)} \leftarrow E \cdot \cos(\cot l) \cdot \frac{\pi}{2} \quad (5)$$

Where $p = rand()$

$$l = (-1) + 2 \times rand()$$

$$E \leftarrow AF \cdot bl \cdot \cos(\cot l) \cdot 2\pi - \overrightarrow{AM}_{(t)}^* \quad (6)$$

AF is the additional force required to form a tunnel structure in improving phase. AF is defined as follows:

$$AF \leftarrow \left| \overrightarrow{AM}_{(t)}^* - \overrightarrow{AM}_{(t)} \right| \quad (7)$$

where $\overrightarrow{AM}_{(t)}^*$ denotes the position vector of the candidate having the best fitness and $\overrightarrow{AM}_{(t)}$ is the position vector.

Termination Phase

Both Improving and tornado formation phases are repeated until the iteration count reach to the maximum number of iterations. Only after the successful execution of the TOA algorithm by meeting the termination criteria, the optimal position of search candidates in terms of cost function value will be returned.

BB-BC Algorithm

The BB-BC, a novel heuristic approach, was designed by taking inspiration from the well-known evolutionary theory; namely, big bang (BB) and big crunch (BC) theories (Erol, O. K., & Eksin, I., 2006). The mathematical formulation of big bang and big crunch gives rise to develop a new optimization algorithm. In the mathematical formulation of BB-BC, the particles are treated as solution candidates. According to BB-BC algorithm, all of randomly distributed candidate solutions

will be dragged into the center point through the center of the candidate in the BC phase. This can be formulated in an equation form as shown in Eq. (8).

$$x^c = \frac{\sum_{i=1}^N \frac{1}{f^i} x^i}{\sum_{i=1}^N \frac{1}{f^i}} \quad (8)$$

where N depicts the total number of candidate solutions x^i denotes the position of the i^{th} candidate solution. f^i represents the fitness values of the i^{th} candidate solution. The BB phase new candidate solutions will be generated using the formulae as shown in Eq. (9).

$$x_{new}^i = x^c + \frac{lr}{k} \quad i = 1, 2, \dots, N \quad (9)$$

Where k represents the iteration number, r represents the random number from the standard normal distribution in the range $[0, 1]$. In Eq. (9), the parameter l limits search space size.

TOA –BBBC (TOA HYBRIDIZED WITH BB-BC)

The local optima and the premature convergence are the two issues usually observed in many Metaheuristics. To overcome these issues several performance improving methods such as invoking chaotic maps, embedding other algorithms into the standard algorithm etc. are used by many researchers. The hybridization TOA with BB-BC is such type. The standard TOA was enhanced by hybridized it with BB-BC algorithm. The pseudo code of TOA –BBBC is presented in **Table 4**.

III. PROPOSED APPROACH

Brief Review of Lévy Flight Algorithm

Over hundred years ago the random way of movement was given with a name called Brownian motion. As time moved upon based on the real time instances a French mathematician named Paul Levy in the year 1937 has come to another kind of random movement with some mathematical pattern of movement as Levy flight or Levy walk. It is distinguished with tiny movements connected with infrequent long movements. Most of the steps in this pattern are within a small area and occasionally long moves takes place. Levy like patterns usually found in birds, bees, land mammals, sharks, spiders and even bacteria as well. Common examples of levy flight are like Animals when hunting make random clustering movements to find food in small spaces. When they felt they probably got exhausted their possibilities in an area, they bolt in one random direction for a while and clusters again for food. Affirmation regarding levy flight is seen as random movement of people in university campuses, urban environment. In day to day life people stay in certain part of an area and move towards another direction for work which means they move in small areas for some time and jumps long for work shows a levy pattern. Considering the behaviour of sharks looking

Paul Levy made a general statement beyond the Brownian motion examining other probability distributions for which a single jump or N jumps contribute to a common mathematical form. Based on the power law $1 / x^{1+\gamma}$ for large x values where $0 < \gamma < 2$ the levy distributions tend to decreases.

Table 4. Pseudo code of LTOA

INPUT: Initial air mass population AM_i, t, p, Maximum number of iterations, OUTPUT: AM^* Optimal solution	
1.	Initialize population $AM_{(i=1,2,\dots,n)}$
2.	Compute fitness f_{am} for individual candidate
3.	discover position of the best candidate solution AM^*
4.	Update p
5.	For1 each candidate
6.	if1 $p < 0.5$
7.	Update the current candidate position with Eq. (4)
8.	else if1 $p \geq 0.5$
9.	Update the current candidate position with Eq. (5)
10.	amend if any search candidate goes beyond the search space
11.	end if1
12.	end for1
13.	For2 each candidate
14.	Update the current candidate position with the Lévy flight Eq. (12)
15.	end for2
16.	amend if any search candidate goes beyond the search space
17.	compute the fitness f_{am} of each candidate
18.	Update AM^* if found better solution
19.	repeat until Max_Iter
20.	return AM^*

When $\gamma = 2$, the Brownian motion trend to be the utmost case of Levy motion (Jensi, R., & Jiji, G. W., 2016; Kamaruzaman, A. F., et al., 2013). The mathematical formulation of a simple Lévy flight distribution in terms of Fourier transform is presented as shown in Eq. (10) and (11) respectively.

$$L(s, \gamma, \mu) = \begin{cases} \sqrt{\frac{\gamma}{2\pi}} \exp\left[-\frac{\gamma}{2(s-\mu)}\right] \frac{1}{(s-\mu)^{\frac{3}{2}}}, & \text{if } 0 < \mu < s < \infty \\ 0, & \text{if } s \leq 0 \end{cases} \quad (10)$$

where γ represents a scale parameter and $\gamma > 0$

μ denotes the location or shift parameter.

$$F(k) = \exp\left[-\alpha|k|^\beta\right], 0 < \beta \leq 2 \quad (11)$$

where α denotes skewness or scale factor parameter, α lies within $[-1, 1]$ interval. The typical Lévy flight trajectory plot is presented in Figure (1).

TOA-Lévy flight (A hybrid between TOA and Lévy flight)

The standard TOA works well for optimization problems. But for high dimensional and also for multimodal function problems it returns poor results. The existing approach is discussed in Section II,

BBBC was used to achieve enhancement in phases exploration and exploitation, and avoids premature convergence. This paper proposes a hybrid between standard TOA and Lévy flight trajectory termed in this work as Lévy flight Tornadoogenesis Optimization Algorithm (LTOA) for achieving better optimization results for higher dimensional function problems. It is observed by many researchers that the Lévy flight algorithm maximizes the diversification of search candidates. This enables the accomplishment of local minima avoidance in the algorithm. The utilization of Lévy flight algorithm in enhancement of standard optimization algorithm was done by many researchers and confirmed the improved optimization performance. Therefore, in this work Lévy flight is used for enhancing the performance of TOA for high dimensional problems. The position update of candidate positions after the position updating (using Eq. (4) or (5) based on p value) in the standard TOA algorithm can be mathematically formulated as shown in Eq. (12).

$$\overrightarrow{AM}_{(t+1)} \leftarrow \overrightarrow{AM}_{(t)} + \mu \sin\left[rand - \frac{1}{2} \right] \oplus Lévy \quad (12)$$

Where the product $\oplus$ denotes entrywise multiplication. $\overrightarrow{AM}_{(t)}$ indicates the i^{th} air mass or the position vector $\overrightarrow{AM}$ at t (where t denotes current iteration). The parameter μ denotes random number that is consistent with a uniform distribution. $rand$ denotes a random number in the range of $[0, 1]$. The $Lévy$ distribution is presented in Eq. (13).

$$Lévy \sim u = t^{-\lambda}, 1 < \lambda \leq 3 \quad (13)$$

For generating random step sizes, in this work Mantegna algorithm is used for symmetric Lévy stable distribution. According to Mantegna's algorithm (Yang, 2010b), the expression for step length s as shown in Eq. (14).

$$s = \frac{u}{|v|^{\frac{1}{\beta}}} \quad (14)$$

Table 5. Statistical analysis of acquired solution cost for F1 to F23

<i>Alg.</i>	<i>Average</i>	<i>Median</i>	<i>Std. Dev</i>	<i>Best</i>	<i>Worst</i>	<i>Average</i>	<i>Median</i>	<i>Std. Dev</i>	<i>Best</i>	<i>Worst</i>
	<i>F1</i>					<i>F2</i>				
WOA	976.093	1.5e-29	6.1e+3	1.7e-81	6.242e+4	2.921e+7	1.703e-18	4.613e+8	3.423e-52	7.302e+9
TOA	143.231	1.8e-154	2.63e+3	2e-316	5.802e+4	3.945e+7	4.454e-82	8.820e+8	1.179e-165	1.97e+10
HTOA	142.380	7.6e-164	2.18e+3	0	6.622e+4	1.410e+6	3.612e-81	7.153e+7	1.309e-169	2.428e+9
LTOA	70.0512	1.3e-183	2.07e+3	0	5.272e+4	1.625e+6	2.133e-85	6.001e+7	1.539e-143	1.165e+9
	<i>F3</i>					<i>F4</i>				
WOA	5.81e+4	5.3e+4	3.44e+4	1.7e+4	1.269e+5	1.4157	2.208e-35	5.2690	6.357e-38	90.7160
TOA	233.564	2.8e-132	3.89e+3	0	8.310e+4	0.3498	4.467e-67	4.3485	2.531e-135	82.9751
HTOA	224.601	2.5e-34	3.96e+3	0	8.145e+4	0.2498	4.154e-71	4.1126	0	82.4402
LTOA	196.811	2.6e-64	2.86e+3	0	1.304e+3	0.2412	3.342e-83	4.0054	0	81.9015
	<i>F5</i>					<i>F6</i>				
WOA	2.61e+6	27.9527	1.94e+7	27.952	2.54e+8	629.8011	0.3204	4.798e+3	0.3204	5.528e+4
TOA	6.69e+5	26.9503	3.20e+6	28.940	2.12e+8	107.5311	0.4654	2.391e+3	0.0187	6.289e+4
HTOA	5.15e+5	26.0428	3.07e+6	28.166	2.22e+8	131.0568	0.3454	3.465e+3	0.2715	5.66e+25
LTOA	3.24e+3	26.4203	3.15e+6	27.840	2.41e+8	106.6108	0.3145	2.382e+3	0.2604	6.205e+4
	<i>F7</i>					<i>F8</i>				
WOA	1.3233	0.0057	11.144	0.0052	139.6725	-1.15e+4	-1.233e+4	1.749e+3	-1.235e+4	-2.45e+3
TOA	0.2595	4.46e-4	5.4104	9.01e-5	120.7298	-3.17e+3	-2.860e+3	635.6914	-4.374e+3	-1.79e+3
HTOA	0.1862	2.65e-4	4.1831	4.67e-5	120.5418	-2.65e+4	-1.655e+4	1.254e+3	-1.252e+4	-3.11e+3
LTOA	0.1604	1.98e+4	4.6574	0.0099	140.5608	-3.35e+4	-1.163e+4	1.375e+3	-1.206e+4	-1.83e+3
	<i>F9</i>					<i>F10</i>				
WOA	27.121	0	76.7544	0	450.6246	0.4300	6.217e-15	2.5248	4.44e-15	20.4593
TOA	9.1750	0	48.0624	0	435.2462	0.1464	8.882e-16	1.4175	8.882e-16	20.2126
HTOA	9.0452	0	24.6906	0	425.3651	0.1352	8.882e-16	0.9811	8.882e-16	20.2281
LTOA	8.1609	0	46.6115	0	422.8401	0.1286	7.417e-14	1.5048	4.206e-13	20.0461
	<i>F11</i>					<i>F12</i>				
WOA	6.8496	0	51.1150	0	573.1051	6.54e+6	0.0472	5.601e+7	0.0472	7.440e+8
TOA	1.9288	0	27.1083	0	589.0046	1.02e+6	0.0023	2.238e+7	0.7495	5.003e+8
HTOA	1.5153	0	27.0531	0	581.8904	1.05e+6	0.0151	2.168e+7	0.0451	4.005e+8
LTOA	1.3804	0	32.6618	0	571.9568	1.26e+6	0.0876	2.322e+7	0.0307	4.739e+8
	<i>F13</i>					<i>F14</i>				
WOA	9.61e+6	0.2720	8.26e+7	0.2720	9.539e+8	3.4340	2.9821	2.6263	2.9821	43.0259
TOA	2.70e+6	2.9942	4.67e+7	2.9927	1.044e+9	0.0051	0.0046	0.0271	0.0014	0.5577
HTOA	2.05e+6	0.3451	4.83e+7	0.8245	2.551e+9	0.0144	0.1445	0.2557	0.0457	43.5165
LTOA	2.30e+6	0.2891	4.37e+7	0.5051	1.355e+9	4.0912	2.0681	0.6913	0.9241	44.6605

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Table 5. Continued

Alg.	Average	Median	Std. Dev	Best	Worst	Average	Median	Std. Dev	Best	Worst
	F15					F16				
WOA	0.0015	3.3e-4	0.0205	3.29e-4	0.4502	-1.0305	-1.0316	0.0089	-1.0316	-0.8963
TOA	0.0051	0.0046	0.0271	0.0014	0.5577	-1.0298	-1.0316	0.0393	-1.0316	-0.1519
HTOA	0.0094	0.1574	0.0571	0.0564	0.9004	-1.4581	-1.0642	0.0716	-0.0572	0.4786
LTOA	0.0065	0.1681	0.0589	0.0653	0.1663	-1.0751	-1.6804	0.8163	-0.4056	0.9348
	F17					F18				
WOA	0.4256	0.3982	0.1260	0.3979	1.2673	3.5112	3.0000	3.5737	3.0000	34.9664
TOA	0.9949	0.9973	0.0233	0.7631	0.9973	12.4007	17.3028	6.4413	3.0171	17.3028
HTOA	0.2454	0.2564	0.5155	0.3281	0.9541	8.5056	3.5497	0.5548	2.9454	19.5065
LTOA	0.5938	0.6441	0.0844	0.2711	0.7662	4.5905	2.6204	3.4541	3.4651	19.0861
	F19					F20				
WOA	-3.8218	-3.8560	0.0616	-3.8565	-3.4609	-2.9480	-2.9899	0.0695	-2.9900	-2.3866
TOA	-3.8589	-3.8589	2.99e-14	-3.8589	-3.8589	-1.4698	-1.4698	1.756e-14	-1.4698	-1.4698
HTOA	-3.7545	-3.8501	0.2451	-3.5454	-3.4521	-3.5410	-3.1541	0.566e-14	-2.5454	-0.5408
LTOA	-3.9371	-3.5693	0.5414	-3.4305	-3.3565	-3.3187	-3.0815	0.406e-15	0.0615	-1.6504
	F21					F22				
WOA	-4.9423	-5.0547	0.4996	-5.0547	-0.5249	-3.6383	-3.7213	0.3054	-3.7213	-0.5457
TOA	-0.9142	-0.9142	1.04e-14	-0.9142	-0.9142	-0.8127	-0.8127	6.00e-15	-0.8127	-0.8127
HTOA	-4.5485	-0.6581	0.0521	-2.6541	-0.2657	-2.5008	-2.6881	0.6545	-0.8365	-0.6371
LTOA	-4.8905	-4.0619	1.29e-13	-3.3545	-0.3513	-2.3052	-2.9341	0.7805	-0.9705	-0.7385
	F23					F00				
WOA	-5.0517	-5.1273	0.3852	-5.1273	-1.3220	0.0000	0.0000	0.0000	0.0000	0.0000
TOA	-1.4705	-1.4705	1.11e-14	-1.4705	-1.4705	0.0000	0.0000	0.0000	0.0000	0.0000
HTOA	-1.0541	-1.2210	1.54e-13	-1.4485	-0.4165	0.0000	0.0000	0.0000	0.0000	0.0000
LTOA	-1.7451	-1.8904	2.68e+14	-0.0678	-0.6044	0.0000	0.0000	0.0000	0.0000	0.0000

$$\sigma_{\mu} = \frac{\left[\left(1 + \beta \right) \times \sin \left(\pi \times \frac{\beta}{1} \right) \right]^{\frac{1}{\beta}}}{\left[\left(1 + \frac{\beta}{2} \right) \times \beta \times 2^{\left(\frac{\beta-1}{2} \right)} \right]} \quad (15)$$

where $u \sim N(0, \sigma_{\mu}^2)$ and $v \sim N(0, \sigma_v^2)$ are drawn from normal stochastic distributions. $\sigma_v = 1$, the notation for σ_{μ} is presented in Eq. (15). Where “ $\Gamma(\cdot)$ ” denotes the Gamma function. The mathematical expression for “ $\Gamma(\cdot)$ ” is presented in Eq. (16).

$$\Gamma(\beta) = \int_0^{\infty} t^{\beta-1} e^{-t} dt \quad (16)$$

Table 6. Mean and Standard deviation (Std_Dev.) for 10 well-known datasets over 30 independent runs for 500 iterations

<i>Alg.</i>	<i>Iris_Dataset</i>			<i>Glass_Dataset</i>		
	<i>Solutioncost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>	<i>Solution_cost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>
DE	0.279 ± 0.0151	2.241 ± 0.3918	1.922 ± 0.2778	0.058 ± 0.0027	5.211 ± 0.8509	4.774 ± 9.2009
PSO	0.279 ± 0.015	2.470 ± 0.3725	1.970 ± 0.2784	0.043 ± 0.0047	5.022 ± 0.7034	6.002 ± 7.3475
WOA	0.268 ± 0.014	2.390 ± 0.3562	1.929 ± 0.2789	0.044 ± 0.0042	5.021 ± 0.7014	4.781 ± 9.2109
CSAWOA	0.268 ± 0.007	2.226 ± 0.3429	1.962 ± 0.2731	0.044 ± 0.0029	4.878 ± 0.8924	6.008 ± 7.3125
RWOA	0.269 ± 9.6e-4	2.122 ± 0.3522	1.980 ± 0.2189	0.043 ± 0.0017	4.875 ± 0.8957	6.419 ± 6.9702
MSWOA	0.266 ± 0.0067	2.064 ± 0.3434	1.929 ± 0.2789	0.043 ± 0.0031	4.093 ± 0.7134	6.720 ± 9.0704
HWOA	0.264 ± 0.0064	2.018 ± 0.3427	1.980 ± 0.2189	0.041 ± 0.0014	4.371 ± 0.7447	6.761 ± 6.4308
TOA	0.263 ± 9.6e-4	2.192 ± 0.0512	1.980 ± 0.2189	0.041 ± 0.0017	4.874 ± 0.8957	6.419 ± 6.9702
CTOA	0.260 ± 8.7e-4	2.128 ± 0.0503	1.924 ± 0.2403	0.041 ± 0.0021	4.868 ± 0.4907	5.940 ± 2.1643
HTOA	0.260 ± 0.0005	2.116 ± 0.0442	1.974 ± 0.2325	0.040 ± 0.0021	4.360 ± 0.4291	6.452 ± 3.1063
LTOA	0.259 ± 6.4e-4	2.113 ± 0.0545	1.907 ± 0.2605	0.042 ± 0.0045	4.336 ± 0.4587	6.610 ± 4.8853
<i>Alg.</i>	<i>Wine_Dataset</i>			<i>Breast_Dataset</i>		
	<i>Solution_cost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>	<i>Solution_cost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>
DE	359.416 ± 11.105	362.001 ± 9.8672	3 5 9 . 8 2 7 ± 5 4 . 4 6 0	0.062 ± 0.0275	16.972 ± 7.46e-15	13.4654 ± 0
PSO	343.838 ± 3.5681	350.197 ± 7.5448	349.746 ± 46.731	0.056 ± 0.0361	17.035 ± 8.01e-15	13.5438 ± 0
WOA	340.112 ± 10.929	350.195 ± 7.5439	329.246 ± 71.920	0.056 ± 0.0285	17.025 ± 7.95e-15	13.4529 ± 0
CSAWOA	342.838 ± 3.6219	341.001 ± 9.8928	3 6 3 . 7 8 4 ± 4 6 . 7 2 0	0.055 ± 0.0341	17.024 ± 7.89e-15	13.4523 ± 0
RWOA	341.7 ± 6.8e-14	332.792 ± 2.2318	3 5 9 . 8 7 1 ± 5 2 . 3 2 1	0.055 ± 6.8e-4	16.980 ± 7.45e-15	13.5521 ± 0
MSWOA	341.63 ± 1.6702	330.104 ± 7.4414	3 4 5 . 2 1 2 ± 4 7 . 3 0 4	0.055 ± 0.1385	16.042 ± 7.75e-15	13.4521 ± 0
HWOA	339.638 ± 1.6716	330.118 ± 7.4034	3 2 9 . 2 7 1 ± 4 7 . 9 1 6	0.053 ± 0.0234	16.403 ± 7.71e-15	13.4520 ± 0
TOA	336.7 ± 6.8e-14	332.792 ± 2.2318	359.871 ± 52.324	0.052 ± 6.8e-4	16.980 ± 7.45e-15	13.5521 ± 0

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Table 6. Continued

Alg.	<i>Iris_Dataset</i>			<i>Glass_Dataset</i>		
	<i>Solutioncost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>	<i>Solution_cost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>
CTOA	335.2 ± 5.9e-14	330.307 ± 8.1084	345.149 ± 14.335	0.051 ± 0.0248	16.670 ± 7.31e-15	13.3417 ± 0
HTOA	340.2 ± 4.5e-14	330.215 ± 6.5457	345.054 ± 35.740	0.051 ± 0.0182	16.640 ± 7.35e-15	13.4055 ± 0
LTOA	337.36 ± 6.5e-14	331.053 ± 2.2494	360.909 ± 74.601	0.051 ± 5.4e-4	16.600 ± 7.17e-15	13.6782 ± 0
Alg.	<i>Diabetes_Dataset</i>			<i>Haberman_Dataset</i>		
	<i>Solution_cost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>	<i>Solution_cost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>
DE	14.5701 ± 0.1814	713.664 ± 74.3624	36.2192 ± 12.246	5.5629 ± 0.2713	35.108 ± 3.5591	8.736 ± 10.813
PSO	14.4416 ± 0.0548	713.650 ± 74.3512	36.2192 ± 12.240	5.7000 ± 0.3045	35.345 ± 3.6982	7.355 ± 9.8732
WOA	14.2800 ± 0.1654	708.013 ± 74.4521	36.2301 ± 14.773	5.7700 ± 0.2845	35.303 ± 3.6534	7.642 ± 9.8043
CSAWOA	13.8524 ± 0.0459	705.981 ± 74.3714	36.8004 ± 14.454	5.3300 ± 0.3545	35.206 ± 3.0475	7.982 ± 9.9801
RWOA	13.7689 ± 0.0025	705.980 ± 74.3512	36.8425 ± 14.451	5.5689 ± 3.6e-6	34.988 ± 3.6875	8.129 ± 10.422
MSWOA	13.7741 ± 0.0454	705.016 ± 74.6151	36.3051 ± 14.351	5.3655 ± 0.3065	34.372 ± 3.5504	7.305 ± 9.8042
HWOA	13.7453 ± 0.0656	705.004 ± 74.3402	36.3801 ± 14.346	5.2814 ± 0.3061	34.312 ± 3.5530	7.361 ± 9.8342
TOA	14.4523 ± 0.0025	705.980 ± 74.3512	36.8425 ± 14.451	4.9842 ± 3.6e-6	34.988 ± 3.6875	8.128 ± 10.422
CTOA	14.0013 ± 0.1501	705.150 ± 74.3501	36.8183 ± 14.450	5.3901 ± 3.6e-6	34.816 ± 3.6002	8.081 ± 9.9723
HTOA	13.5131 ± 0.4228	705.352 ± 16.4868	36.6105 ± 14.641	5.1901 ± 3.6e-6	34.802 ± 3.6812	7.814 ± 9.5508
LTOA	13.4348 ± 0.5105	705.327 ± 72.9391	36.6008 ± 14.330	5.1884 ± 3.7e-6	34.053 ± 3.0274	7.054 ± 9.3142
Alg.	<i>Hayes-Roth_Dataset</i>			<i>E. Coli_Dataset</i>		
	<i>Solution_cost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>	<i>Solution_cost</i>	<i>Intra-cluster</i>	<i>Inter-cluster</i>
DE	0.3928 ± 0.0346	3.518 ± 0.2735	1.8519 ± 0.7531	0.0663 ± 0.0032	0.6643 ± 0.0735	0.3637 ± 0.1341
PSO	0.3507 ± 0.0254	3.5081 ± 0.4318	1.4189 ± 0.6078	0.0671 ± 0.0030	0.6312 ± 0.0726	0.3145 ± 0.1028
WOA	0.3672 ± 0.0284	3.5492 ± 0.4865	1.4256 ± 0.6845	0.0641 ± 0.0028	0.6745 ± 0.0825	0.3176 ± 0.1517
CSAWOA	0.2681 ± 0.0248	3.5413 ± 0.4716	1.7012 ± 0.6146	0.0666 ± 0.0030	0.6245 ± 0.0689	0.4540 ± 0.1056

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Table 6. Continued

Alg.	Iris_Dataset			Glass_Dataset		
	Solutioncost	Intra-cluster	Inter-cluster	Solution_cost	Intra-cluster	Inter-cluster
RWOA	0.2689 ± 0.0129	3.2508 ± 0.3512	1.8251 ± 0.6702	0.0641 ± 0.0012	0.6012 ± 0.0542	0.4643 ± 0.1728
MSWOA	0.2655 ± 0.0105	3.3415 ± 0.4181	1.4046 ± 0.6451	0.0637 ± 0.0012	0.6015 ± 0.0621	0.3206 ± 0.1410
HWOA	0.2645 ± 0.0108	3.5472 ± 0.4160	1.4053 ± 0.6415	0.0624 ± 0.0011	0.6018 ± 0.0721	0.3603 ± 0.1421
TOA	0.2418 ± 0.0129	3.2508 ± 0.3512	1.8251 ± 0.6702	0.0558 ± 0.0012	0.6012 ± 0.0542	0.4643 ± 0.1728
CTOA	0.2381 ± 0.0148	3.2518 ± 0.3414	1.4108 ± 0.5168	0.0553 ± 0.0031	0.5981 ± 0.0601	0.3501 ± 0.0918
HTOA	0.2514 ± 0.0134	3.2415 ± 0.3251	1.4206 ± 0.6105	0.0545 ± 0.0018	0.6003 ± 0.0567	0.3164 ± 0.1306
LTOA	0.2467 ± 0.0478	3.2574 ± 0.3625	1.5624 ± 0.6701	0.0544 ± 0.0054	0.5997 ± 0.0418	0.3481 ± 0.1907
Alg.	Zoo_Dataset			Vowel_Dataset		
	Solution_cost	Intra-cluster	Inter-cluster	Solutioncost	Intra-cluster	Inter-cluster
DE	0.0220 ± 0.0024	1.6512 ± 1.1e-15	2.6502 ± 2.3e-15	39.2800 ± 2.2481	898.617 ± 102.946	353.3712 ± 218.25
PSO	0.0130 ± 0.0018	1.6396 ± 1.1e-15	2.6421 ± 2.3e-15	38.4700 ± 2.3618	899.254 ± 101.895	350.3465 ± 215.71
WOA	0.0120 ± 0.0022	1.6401 ± 1.1e-15	2.6471 ± 2.3e-15	38.4012 ± 2.2314	899.246 ± 101.871	350.3548 ± 215.65
CSAWOA	0.0090 ± 0.0019	1.6381 ± 1.1e-15	2.6487 ± 2.3e-15	36.8140 ± 2.3456	899.228 ± 102.382	350.8086 ± 215.95
RWOA	0.0094 ± 8.4-04	1.5254 ± 4.5e-16	2.6502 ± 2.3e-15	35.8941 ± 2.2145	898.145 ± 102.982	353.3282 ± 218.25
MSWOA	0.0100 ± 0.0012	1.5332 ± 1.1e-14	2.6451 ± 2.3e-15	34.1553 ± 2.2645	898.224 ± 102.314	350.7301 ± 215.96
HWOA	0.0053 ± 0.0024	1.4301 ± 1.1e-14	2.6432 ± 2.3e-15	32.4501 ± 2.2416	898.138 ± 102.311	350.6126 ± 215.94
TOA	0.0102 ± 8.4-04	1.5254 ± 4.5e-16	2.6502 ± 2.3e-15	30.2145 ± 2.2145	898.145 ± 102.982	353.3282 ± 218.25
CTOA	0.0113 ± 0.0023	1.6506 ± 1.1e-15	2.6142 ± 2.3e-15	30.0018 ± 2.0604	894.627 ± 101.692	353.1527 ± 218.04
HTOA	0.0120 ± 0.3581	1.3418 ± 2.1e-16	2.6105 ± 2.3e-15	29.9971 ± 2.0491	895.455 ± 101.385	353.6101 ± 218.69
LTOA	0.0127 ± 4.3e-04	1.3415 ± 0.4e-16	2.4153 ± 0.6e-15	30.6404 ± 2.4058	895.347 ± 101.870	352.0457 ± 215.67

Table 7. The detailed system configuration

<i>Name</i>	<i>Configuration</i>
<i>Hardware</i>	
CPU	Intel(R) Core(TM)-i5-2400
Clock Rate	3.10 GHz
RAM	8 GB
Hard Disc	500 GB
<i>Software</i>	
OS	Windows 7 Home
Programming tool	MATLAB-R2016a

Summarized the main steps of LTOA in the pseudo code as shown in Table 4. Flowchart of the LTOA is presented in Fig. 2.

IV. EXPERIMENTAL ANALYSIS

This section focuses on the performance evaluation of the Lévy flight TOA (LTOA) algorithm using different complex function optimization problems. In order to obtain an unbiased comparison of acquired results, each and every experiment is done on the same system which have the configuration detailed in Table 8. The primary objective of this work is to design an advanced clustering approach for solving different cluster problems. Before achieving this objective, the function optimization performance of proposed hybrid version of the standard TOA is tested. This was denoted as phase one in this work, wherein the second phase solved data cluster problems.

Phase I: Benchmark Function Problems

Several experiments are conducted to evaluate the performance of the proposed Lévy flight TOA algorithm. The twenty three well-known mathematical functions with a variety of complexity are considered in performance evaluation process. These benchmark functions comprises 7 Unimodal (F1 to F7), 6 Multimodal (F8 to F13) and 10 fixed dimension (F14 to F23) and notations of these functions are presented in Table 1, 2 and 3. The obtained results of proposed LTOA are compared with WOA, TOA, and HTOA (Hybrid TOA using BBBC). Five statistical metrics (Median, Standard Deviation (Std. Dev), Minimum (Best) and Maximum (Worst)) are used in result comparison and these statistical analysis are portrayed in Table 6. From the Table 6, it clearly shows, for unimodal functions F1, F3, F4, F5, F6 and F7 exhibits the improved performance when compared to other existing algorithms. The LTOA exhibits the improved performance for functions F9, F10, F11, F15, F15, F18, F19 and F21 of Multimodal and fixed dimension function problems.

Phase II: Clustering Problems

This phase presents the performance evaluation of the proposed Lévy TOA based clustering approach. Part of performance evaluation, several investigations are carried out in solving 10 UCI data cluster problems and 4 Web Document Cluster problems. Any good clustering approach minimizes the quantization error; presented in Eq. (1) for the taken input data cluster problems. To investigate the performance of the proposed clustering approach in this paper, 10 different clustering problems are taken. The statistical measures like mean (Average) and standard deviation (Std. Dev) are considered

Table 8. Ground-truth validation results

<i>Precision</i>	<i>Dataset</i>	WDC- CSK BBIC	HWOA	TOA	CTOA	HTOA	LTOA
	<i>DMOZ-50</i>	89.51	89.52	90.07	90.11	88.15	90.13
	<i>AMBIENT</i>	77.71	77.69	78.00	78.65	78.33	78.16
	<i>MORESQUE</i>	88.16	88.35	87.91	87.93	87.93	88.84
	<i>ODP-239</i>	59.58	56.07	59.15	60.51	60.68	60.71
<i>Recall</i>	<i>Dataset</i>	WDC- CSK BBIC	HWOA	TOA	CTOA	HTOA	LTOA
	<i>DMOZ-50</i>	75.92	75.35	76.64	68.51	73.68	76.66
	<i>AMBIENT</i>	59.06	56.61	58.45	61.31	61.01	59.85
	<i>MORESQUE</i>	39.99	40.01	40.01	41.24	40.26	41.05
	<i>ODP-239</i>	44.77	44.84	41.65	43.54	45.37	43.55
<i>F-measure</i>	<i>Dataset</i>	WDC- CSK BBIC	HWOA	TOA	CTOA	HTOA	LTOA
	<i>DMOZ-50</i>	80.10	80.01	80.00	81.81	80.64	79.81
	<i>AMBIENT</i>	62.02	61.05	62.61	62.24	60.64	63.02
	<i>MORESQUE</i>	49.89	49.90	50.03	50.03	50.71	51.06
	<i>ODP-239</i>	46.37	46.38	46.41	47.08	47.14	47.14
<i>Accuracy</i>	<i>Dataset</i>	WDC- CSK BBIC	HWOA	TOA	CTOA	HTOA	LTOA
	<i>DMOZ-50</i>	93.45	91.38	94.06	94.17	94.06	94.17
	<i>AMBIENT</i>	82.88	82.96	83.44	82.91	82.30	82.91
	<i>MORESQUE</i>	58.82	57.91	58.60	58.35	58.91	58.46
	<i>ODP-239</i>	81.90	82.51	82.06	82.39	82.26	82.67

in comparisons. Obtained clustering results of proposed LTOA based clustering approach are compared with the results obtained for other optimization algorithm based clustering approaches (DE (Differential Evolution), PSO, WOA, CSAWOA (Clonal Selection Algorithm WOA), RWOA (Ring WOA), MSWOA (Master-Slave WOA), HWOA (Hybrid WOA), TOA, CTOA (Chaotic TOA) and HTOA) for the same datasets. Table 7 presents the Mean and Std_Dev values over number of independent runs: 30 for number of iterations: 500. It is observed from the Table 7 that for the Iris dataset, the proposed LTOA based cluster approach obtains the lower solution cost. The mean and standard deviation of the solution cost for the Iris dataset are 0.2591 and 6.3733e-004 respectively. Proposed method achieves the best clusters for the other datasets wine, Breast, Pima Indian Diabetes, Haberman, Hayes-Roth and E. Coli with the low solution costs. For the remaining datasets the clustering performance is almost close to the other existing clustering approaches. From the obtained statistical and graphical results presented in Table 7 and Fig. 3, the proposed LTOA based clustering approach demonstrated to be superior compared to other well-known algorithms.

Four web document clustering problems also used in this paper to test the numerical complexity of proposed LTOA based clustering approach. Four ground-truth validation techniques *Precision*, *Recall*, *F-measure* and *Accuracy* are used. These validation

results for minimizing the quantization error presented in Eq. (1) are portrayed in Table 8. The comparisons have been made to LTOA with other existing web document clustering problem solving clustering approaches WDC-CSK BBIC, HWOA, TOA, CTOA and HTOA. From the Table 8, the proposed clustering approach is working well for DMOZ-50, MORESQUE and ODP-239. For the dataset AMBIENT, Proposed clustering approach gives the results which are very much closer to the other existing methods.

V. CONCLUSION AND FUTURE WORK

In this research work, as primary goal a novel clustering approach was designed by conjoining the traditional clustering algorithm and Lévy flight TOA for solving different data cluster problems which were taken from various sources. The standard TOA is hybridized with Lévy flight trajectory algorithm for enhancing the standard TOA in global function optimization perspective. The simulation of experiments was done in two phases. In the first phase hybrid version of the TOA is benchmarked on 23 mathematical function optimization problems. The comparative analysis of Lévy flight TOA has been made with well-known meta-heuristic methods. From the obtained statistical and graphical analysis, it is confirmed that the proposed hybrid TOA quickly converges to optimal solutions for the majority of the input test functions. Where in the second phase, the numerical complexity of proposed novel clustering approach was done by assessing on 14 different cluster problems which include 10 UCI cluster problems and 4 Web Document Cluster problems. The performance evaluation metrics: solution cost, intra-cluster and inter-cluster distances were used and compared the obtained graphical statistical results with other existed clustering approaches. For the web document cluster problems the ground-truth validation metrics: precision, recall, F-measure and accuracy were used. The simulation experiments and analysis, and conducted comparisons in the second phase acknowledge that the proposed Lévy flight TOA based novel clustering approach was fast and efficient in obtaining best clusters.

This research work primarily focussed on discovering Lévy flight trajectory based hybrid TOA for solving global function optimization problems as well as different data cluster problems. From the simulation experiments and analysis the proposed clustering approach is a suitable addition to clustering domains for solving complex data clustering problems. The NFL theorem logically proved that there is not any single optimization technique existed that can solve all sorts of optimization problems. In this work Lévy flight trajectory algorithm was used to enhance the standard TOA. In future work, other performance boosting up methods can be investigated. The future research also can development of new and novel nature-inspired Metaheuristics.

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APPENDIX

Figure 1.

Figure 2.

Figure 3a.

Figure 3b.