Optimal Kernel Selection Based on GPR for Adaptive Learning of Mean Throughput Rates in LTE Networks

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ABSTRACT

Machine learning models and algorithms have been employed in various applications, including prognostic scrutinizing, learning, and revealing patterns in data, knowledge extracting, and knowledge deducing. One promising computationally efficient and adaptive machine learning method is the Gaussian process regression (GPR). An essential ingredient for tuning the GPR performance is the kernel (covariance) function. The GPR models have been widely employed in diverse regression and functional approximation purposes. However, knowing the right GPR training to examine the impacts of the kernel functions on performance during implementation remains. In order to address this problem, a stepwise approach for optimal kernel selection is presented for adaptive optimal prognostic regression learning of throughput data acquired over 4G LTE networks. The resultant learning accuracy was statistically quantified using four evaluation indexes. Results indicate that the GPR training with the matern52 kernel function achieved the best user throughput data learning among the 10 contending kernel functions.

KEYWORDS

4G LTE Network, Adaptive Learning, Gaussian Process, Kernel Function, Measured Throughput, User Equipment

1.0 INTRODUCTION

Telecommunication is an established cutting-edge technology that permits two parties to communicate employing voice and data signals (Rappaport 2002). The evolution, deployment, and application of various cellular radio frequency (RF) based telecommunications systems has orchestrated rapid development in every aspect of human endeavour (Imoize et al. 2021). Starting from the first generation (1G) brand that came into existence in the early 80s to the ubiquitous fourth generation (4G) LTE (Imoize et al. 2019) (Imoize and Oseni 2019), the recently commercialized fifth-generation (5G) (Gupta and Jha 2015), and the envisioned sixth-generation (6G) wireless systems (Dang et al. 2020), the telecommunication industry is progressive globally. The 4G and 5G systems are data communication-centric (Shynu and Al-Turjman 2021). 4G LTE can provide robust data throughput rates in open terrains (Imoize and Adegbite 2018). However, in built-up terrains such as dense urban cities, where all forms of interference and multipath fading impacts are high, 4G LTE may experience low and fluctuating throughput data rates. Hence, there is a need to regularly conduct a measurement-

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based prognostic examination of User Equipment (UE) throughput rates (Ughegbe, Adelabu, and Imoize 2021). These rates help the RF engineers make the necessary optimization decisions, detect and mitigate interference, including other anomalies that could negatively impact 4G LTE network performance (Huang et al. 2013).

Prognostic algorithms are central to a detailed examination of user data throughput rates (Estevez, Orchard, and Kailas 2013). In recent years, Machine Learning (ML) models and their prognostic algorithms have been deployed for diverse applications comprising data scrutinizing, learning, pattern close-fitting, knowledge extracting, knowledge deducing, and others (Alvarez, Louveaux, and Wehenkel 2017), (Song, Ristenpart, and Shmatikov 2017). There exist numerous ML-based prognostic regression models in the literature (Bui and Turner 2014; Cao and Fleet 2014; Chen and Ren 2009; Chen and Wang 2018; Dervilis et al. 2016; Gu and Hu 2012; Skilling 2006; Su, Peng, and Hu 2017; Vanhatalo, Pietiläinen, and Vehtari 2010; Wan and Ren 2015). One of the most promising techniques is the Gaussian Process Regression (GPR) (Rasmussen 2004) (Chalupka, Williams, and Murray 2013) (Wilson and Adams 2013). The GPR is a non-parametric Bayesian modeling technique with a Gaussian probabilistic structure. Some of the critical advantages of the GPR include (Bui and Turner 2014; Cao and Fleet 2014; Chen and Ren 2009; Chen and Wang 2018; Gu and Hu 2012; Vanhatalo et al. 2010): (i) relatively simple parameterization and implementation structure (ii) proficiency in dealing with stochastic processes of varied intricacies and complexities (iii) proficiency in adaptive learning of noisy and non-noisy data (iv) adeptness in handling uncertainties in datasets (v) expert knowledge incorporation (vi) flexible input data probability distributions, (vii) capacity to integrate prior knowledge (viii) precise stationary and non-stationary fitting of input-output data capability (ix) ability to estimate posterior degradation.

Gaussian Process (GP) is a robust data-driven approach that produces efficient solutions to image processing problems. These problems comprise modelling of low-level image features, image denoising and more. GPs find practical applications in high-resolution object reconstruction for lower resolution images. This is achieved by examining the local structures in natural images defined by their pixel spread (Wang et al. 2017). In wireless propagation measurements, GP regression helps fuse multiple datasets from heterogeneous sources (Vasudevan 2012). For example, a multi-task GP technique has been advanced on multi-modal image fusion (Reid, Ramos, and Sukkarieh 2013). In related literature, (Wilson, Knowles, and Ghahramani 2011) and (Nguyen and Bonilla 2013) introduced some complex models capable of handling complex relationships between the outputs and mixing weights with input dependencies.

In kernel function analysis, (Alaa and van der Schaar 2017) projected the ICM kernel in the hidden and output layers of the GPs for multi-task learning for survival analysis. A general formulation of some existing models has been presented to replace the linear mixing of latent processes with another suitable GP. This allows the modeling of more complex relationships between the assigned tasks. Considering the work due to (Requeima et al. 2019) on the Gaussian process autoregressive regression model, an alternative formulation to non-linear multi-output learning in GPs was reported. In this model, an inherit ordering of the outputs is assumed. Here, the current GP output is concatenated with the inputs of the proceeding GP at the specified location. Although this model can be interpreted as a DGP structure with several missing connections, inherit ordering of the tasks is assumed, and the dataset is closed in a downward trend.

The kernel (covariance) function is essential for tuning the GPR model for optimal performance. It helps the GPR articulate the close-fitting similarity between the input vector and the target response. The GPR techniques have been widely employed for diverse regression and functional approximation purposes (Bui and Turner 2014; Cao and Fleet 2014; Chen and Ren 2009; Dervilis et al. 2016; Gu and Hu 2012; Skilling 2006; Su et al. 2017; Vanhatalo et al. 2010). However, no work has presented a detailed assessment of the severe impacts of its kernel (covariance) functions on its performance characteristics. In order to fill this gap, this paper presents an optimal kernel selection approach based on Gaussian process regression for adaptive learning of mean throughput rates over 4G LTE networks.

Our key contributions in this paper are outlined as follows. First, we highlighted ten essential kernel functions and their mathematical derivatives. Second, we proposed a stepwise search algorithm for the best kernel function among the contenders. Last, we presented experimental throughput data acquired using TEMS investigation tools to test and validate the proposed random search algorithm.

The remainder of this paper is organized as follows. Section 2 covers the Gaussian process, kernel functions, the associated hyperparameters, mean user throughput measurements, and the proposed kernel function selection approach comprising the stepwise search kernel selection algorithm. Section 3 presents the results and discussions. Finally, Section 4 gives the conclusion to the paper.

2.0 METHODS

The methodology employed in this paper is described briefly. The Gaussian process, kernel functions, and the associated hyperparameters mean user throughput measurements, and the proposed kernel function selection approach, including the stepwise search kernel selection algorithm, are highlighted.

2.1 Gaussian Process

The Gaussian Process (GP) regression is a powerful state-of-the-art non-parametric Bayesian modeling technique with a Gaussian probabilistic structure (Cheng and Boots 2016); (Zhang, Huang, and Tian 2017). A function $f(\cdot)$ with a set of associated x input random variables is said to trail a GP if the values of the function also follow the texture of the Gaussian distribution given by (1) (Roberts et al. 2013); (Boustati, Damoulas, and Savage 2019):

$$f = GP(m_e(x_i), k(x_i, x_j)) \quad (1)$$

where, $k(x_i, x_j)$ is the kernel function with the allied inputs (x_i, x_j) , which governs the GP and m_e is the conforming mean vector.

Given a training set, $P = \left[(x_i, y_i) \right]_{i=1}^k$ of i.i.d unknown data samples (i.e., identically independent distribution of yet to be identified data samples) such that $(x_i, y_i) \in \mathbb{R}^d \times \mathbb{R}$, the GPR model assumes that the dependence of the output, y_i on the input, x_i satisfies the regression function in (2) given by:

$$y_i = (f_o(x_i) + \varepsilon_i; i = 1, 2, \dots, k) \quad (2)$$

where, ε_i expresses the i.i.d random noise variables, such that $\varepsilon \approx N(0, \sigma^2)$.

The GP predictive description of $f(x_*)$ any given test point x_* can be expressed by (Sun et al. 2018) in (3). A few of the parameters in (3) are defined in (4) and (5), indicating the respective mean and covariance values (Wu, Zhou, and Gao 2019).

$$p(f(x_*) / P) = N(\mu(x_*), \sigma^2(x_*)), \quad (3)$$

where

$$\mu^{(t)} = P(X, X^{(t)}) \cdot (P(X, X) + \sigma^2 .I)^{-1} \bar{y}, \quad (4)$$

$$\sum^{(t)} = P(X^{(t)}, X^{(t)}) + \sigma^2 .I - P(X^{(t)}, X) \cdot (P(X, X) + \sigma^2 .I)^{-1} P(X, X^{(t)}) \quad (5)$$

2.2. Kernel Functions And Their Hyperparameters

In GPR, the kernel function expresses the close-fitting similarity between the input vector and the target response. As mentioned earlier, several GPR kernel functions exist in the literature. Specifically, a detailed description of ten standard GPR kernel functions and their mathematical representations is presented (Chen and Wang 2018; Rasmussen 2004; Roberts et al. 2013; Wilson 2014). These include Exponential Kernel (EK), Automatic Relevance Determination based Exponential Kernel (ARD-EK), Automatic Relevance Determination based Squared Exponential Kernel (ARD-SEK), Squared Exponential Kernel (SEK), Matern Kernel 3/2 (MK 32), Matern Kernel 5/2 (MK 52), Automatic Relevance Determination based Matern Kernel 3/2 (ARD-MK 32), Automatic Relevance Determination based Matern Kernel 5/2 (ARD-MK 52), Rational Quadratic Kernel (RQK), and Automatic Relevance Determination based Rational Quadratic Kernel (ARD-RQK) (Williams 2007); (Yogatama and Mann 2014); (Zhang et al. 2019); (Palar et al. 2021); (Sharma and Pandey 2021); (Gbémou et al. 2021). A brief description of these standard GPR kernel functions is given as follows.

2.2.1 Exponential Kernel (EK)

The Exponential Kernel (EK) is a robust kernel function with one hyperparameter for boosting Gaussian process regression, and it is given by (6):

$$k(x_i, x_j, l) = \sigma^2 \exp\left(-\frac{u}{l}\right) \quad (6)$$

where l is the hyperparameter, and it is called the characteristic scale length.

2.2.2 Automatic Relevance Determination based Exponential Kernel (ARD-EK)

The Automatic Relevance Determination based Exponential Kernel (ARD-EK) is another vital kernel function with two hyperparameters for Gaussian process regression. It is given by (Gbémou et al. 2021) in (7):

$$k(x_i, x_j, l) = \sigma^2 \exp\left(-\sqrt{\sum_{n=1}^m \frac{(x_{in} - x_{jn})^2}{l^2}}\right) \quad (7)$$

where l is the hyperparameter, and it is called the characteristic scale length.

2.2.3. Automatic Relevance Determination based Squared Exponential Kernel (ARD-SEK)

The Automatic Relevance Determination based Squared Exponential Kernel (ARD-SEK) is a unique kernel function with two hyperparameters for Gaussian process regression. It is given by (Gbémou et al. 2021) in (8):

$$k(x_i, x_j) = \sigma^2 \exp \left(-\frac{1}{2} \sum_{n=1}^m \frac{(x_{in} - x_{jn})^2}{\sigma^2} \right) \quad (8)$$

where l is the hyperparameter, and it is called the characteristic scale length.

2.2.4. Squared Exponential Kernel (SEK)

The Squared Exponential Kernel (SEK) is a popular kernel function with two special hyperparameters, and it is given by (Yogatama and Mann 2014); (Sharma and Pandey 2021); (Gbémou et al. 2021) in (9):

$$k(x_i, x_j, l) = \sigma^2 \exp \left[-\frac{(x_i - x_j)^T (x_i - x_j)}{2l} \right] \quad (9)$$

where l and σ are the special hyperparameters with l being the characteristic scale length and σ the signal standard deviation.

2.2.5 Matern Kernel 3/2 (MK 32)

The Matern Kernel 3/2 (MK 32) is another popular kernel function with two special hyperparameters, and it is given by (Zhang et al. 2019) in (10):

$$k(x_i, x_j, l) = \sigma^2 \left(1 + \frac{\sqrt{3} (x_i - x_j)^T (x_i - x_j)}{l} \right) \exp \left(-\frac{\sqrt{3} (x_i - x_j)^T (x_i - x_j)}{l} \right) \quad (10)$$

where l and σ are the special hyperparameters with l being the characteristic scale length and σ the signal standard deviation.

2.2.6. Matern Kernel 5/2 (MK 52)

The Matern Kernel 5/2 (MK 52) is also a popular kernel function with two special hyperparameters, and it is given by (Palar et al. 2021) in (11):

$$k(x_i, x_j, l) = \sigma^2 \left(1 + \frac{\sqrt{5} (x_i - x_j)^T (x_i - x_j)}{l} + \frac{\sqrt{5} (x_i - x_j)^T (x_i - x_j)}{3l^2} \right) \exp \left(-\frac{\sqrt{5} (x_i - x_j)^T (x_i - x_j)}{l} \right) \quad (11)$$

where l and σ are the special hyperparameters with l being the characteristic scale length and σ the signal standard deviation.

2.2.7. Automatic Relevance Determination based Matern Kernel 3/2 (ARD-MK 32)

The Automatic Relevance Determination based Matern Kernel 3/2 (ARD-MK 32) is a unique kernel function with two special hyperparameters, and it is given by (Zhang et al. 2019) in (12):

$$k(x_i, x_j, l) = \sigma^2 \left(1 + \sqrt{3 \sum_n \frac{(x_{in} - x_{nj})^2}{l_n^2}} \right) \exp \left(-3 \sum_n \frac{(x_{in} - x_{nj})^2}{l_n^2} \right) \quad (12)$$

where l and σ are the special hyperparameters with l being the characteristic scale length and σ the signal standard deviation.

2.2.8. Automatic Relevance Determination based Matern Kernel 5/2 (ARD-MK 52)

The automatic relevance determination based Matern Kernel 5/2 (ARD-MK 52) is a unique kernel function with two special hyperparameters, and it is given by (Palar et al. 2021) in (13):

$$k(x_i, x_j, l) = \sigma^2 \left(1 + \sqrt{5 \sum_n \frac{(x_{in} - x_{nj})^2}{l} + \frac{5}{3} \sum_n \frac{(x_{in} - x_{nj})^2}{l^2}} \right) \exp \left(-5 \sum_n \frac{(x_{in} - x_{nj})^2}{l^2} \right) \quad (13)$$

where l and σ are the special hyperparameters with l being the characteristic scale length and σ the signal standard deviation.

2.2.9. Rational Quadratic Kernel (RQK)

The Rational Quadratic Kernel (RQK) is a unique kernel function with two special hyperparameters, and it is given by (Mohammadzadeh et al. 2018) in (14):

$$k(x_i, x_j, l) = \sigma^2 \left(1 + \frac{(x_i - x_j)^T (x_i - x_j)}{2\alpha l} \right)^{-\alpha} \quad (14)$$

where l and σ are the special hyperparameters with l being the characteristic scale length and σ the signal standard deviation.

2.2.10. Automatic Relevance Determination based Rational Quadratic Kernel (ARD-RQK)

The Automatic Relevance Determination based Rational Quadratic Kernel (ARD-RQK) is a unique kernel function with two special hyperparameters, and it is given by (Mohammadzadeh et al. 2018) in (15):

$$k(x_i, x_j, l) = \sigma^2 \left(1 + \frac{1}{2\alpha} \sum_n \frac{(x_{in} - x_j)^2}{l^2} \right)^{-\alpha} \quad (15)$$

where l and σ are the special hyperparameters with l being the characteristic scale length and σ the signal standard deviation.

2.3. Mean User Throughput Measurements

Mean user throughput expresses mean data transfer “speed” (bits/second) through a communication channel during wired or wireless data connection (Błaszczyszyn, Jovanovicy, and Karray 2014). More technically, it is the successful average bits number sent (or received) for every data request to the practical data transfer or delivery period (Isabona and Ojuh 2014; Isabona and Samson 2013). An efficient way to collect throughput data in cellular telecommunication networks is to use appropriate professional measurement and monitoring tools (Isabona and Konyeha 2013; Isabona and Ojuh 2014; Isabona and Samson 2013). The ASCOM TEMS investigation tools were employed in this work to collect three types of downlink throughput data. These include the RLC throughput (Chen et al. 2003; Paul et al. 2016; Shreevastav and Carbajo 2016), PDSCH throughput (Padaganur and Mallapur 2018), and PDSCH throughput data rates (Favraud and Nikaein 2017). These throughput types were obtained via FTP file downloading 1GBytes at the User Equipment (UE) terminal. Field measurements were conducted using ASCOM TEMS investigation tools over commercial 4G LTE networks. The measurement routes comprise the major roads and open streets in Port Harcourt Garden City, Nigeria. Three measurements, namely; location 1, location 2, and location 3, were used as case studies for the throughput data collection. The 4G LTE network uses a 2×2 MIMO antenna configuration, operates at 10MHz system bandwidth, and transmits at 2600MHz.

2.4. Proposed Kernel Function Selection Approach

This study proposes a stepwise approach for optimal kernel selection for adaptive GPR optimal prognostic regression learning on live throughput data acquired over operational 4G LTE networks. Specifically, we collected three data types: PDSCH, RLC, and PDCP throughput through field test measurement around three LTE eNodeB transmitters. The measurement is required to investigate the performance of the proposed stepwise kernel selection algorithm for optimal GPR predictive learning. PDSCH throughput is the physical channel throughput, and it comprises the transmitted user application data in the payload, signaling messages, paging, and System Information Blocks (SIB) in the downlink (Huang et al. 2012). It is the foremost downlink bearing channel data throughput allotted to user equipment on a resourceful and dynamic basis. The RLC throughput data represent the aggregate data bits transferred over the reverse link per unit time. The RLC throughput quantifies the amount of appropriately received data at the user equipment in bits per second. PDCP throughput is the user internet protocol based data throughput comprising user integrity protection, ciphering, header compression, control plane data, and plane data transferred from the base station.

In order to accomplish the objectives of the study, the stepwise selection program scripts were written in MATLAB and implemented successfully. The algorithm was designed to spontaneously go through a pool of the ten kernels highlighted in Section 2 and then select the best for the GPR model during predictive learning of the throughput data. The degree of the resultant prognostic learning accuracy of each investigated GPR kernel function on the acquired throughput data was statistically quantified using four evaluation indexes. The indexes include the Root Mean Square Error (RMSE), Sum of Absolute Mean Error (SAE), Standard Deviation Error (STD), and Mean Absolute Error (MAE) (Okakwu et al. 2019; Popoola et al. 2019; Wu et al. 2020; Yi et al. 2020). Lower values of each of the four different evaluation indexes indicate high learning accuracy. Additionally, the stepwise search kernel selection algorithm is adopted to achieve optimal GPR performance, leveraging the Bacharach criteria, which defines the way theories fit together to present a clearer picture of an empirical reality in a broad sense (Bacharach 1989). As shown in algorithm 1, after initiating and imputing the data training set, the next step focuses on an intuitive selection of arbitrary kernel. This procedure is followed by performing the training set using the selected kernel and then evaluate the adaptive learning of the kernel function before taking the next step.

Figure 1. Attained throughput data learning accuracy with exponential kernels

Figure 2. Attained throughput data learning accuracy with ardexponential kernels

Figure 3. Attained throughput data learning accuracy with ardsquared exponential kernels

Figure 4. Attained throughput data learning accuracy with squared exponential kernels

Figure 5. Attained throughput data learning accuracy with matern32 kernels

Figure 6. Attained throughput data learning accuracy with matern52 kernels

Figure 7. Attained throughput data learning accuracy with ardmater32 kernels

Figure 8. Attained throughput data learning accuracy with ardmater52 kernels

Figure 9. Attained throughput data learning accuracy with rational quadratic kernels

Figure 10. Attained throughput data learning accuracy with ardrational quadratic kernels

Algorithm 1: Stepwise Search Kernel Selection Algorithm

- i). Input training set (X_{train} , Y_{train}) and other relevant training parameters
- ii). Intuitively choose a kernel function
- ii). Train the kernel with the training set i
- iv). Appraise its accuracy on the training set i (with any set of 1st order statistical index)
- v). For all the kernels do;
- vi). end for
- vii). return kernel with best appraisal score.

4.0 RESULTS AND DISCUSSIONS

This section presents the results and discussions of the study. For brevity in this investigation, we provide only the predictive learning performance attained for PDSCH throughput data along with the measurement points in measurements location 1, as shown in Figures 1 to 10. The figures indicate the prognostic learning results of the ten GPR kernel functions examined. Each figure highlights the subplots showing the RMSE attained over the data point during the learning process and its normalized distribution curve fit. Expressly, both subplots are provided to present a visual representation of the quantified RMSE values. Notably, the frequency distributions fit the collected PDSCH throughput data and the kernel-controlled GPR model at every measurement point.

Figure 11 is a bar chart revealing each kernel's quantified RMSE error values with the GPR model after applying the stepwise selection algorithm. As observed in Figure 11, the matern52 kernel attained the best adaptive PDCP throughput learning performance with 0.00024 RMSE value, and the poorest prediction is with the exponential kernel, which attained 0.0053 RMSE value. This implies that the matern52 kernel provided the best close-fitting similarity between the predicted input and the throughput data-target response. The predictive learning performance summary of the proposed stepwise selection algorithm on PDCSH and RLC throughput data in measurements locations 2 and

Figure 11. A bar chart of the qualified RMSE error attached with each kernel during learning

3 are as shown in Figures 12 to 14, using the sum of SAE, STD, and MAE indexes; 0.010, 0.013, 0.009 SAE values, and 0.00016, 0.00014, 0.00002 MAE values, respectively. This is followed by the squared exponential kernel with 0.010, 0.013, 0.010 SAE values and 0.00020, 0.00015, 0.00021 MAE values, respectively.

As observed in Figures 11-14, the maern52 kernel attained the best adaptive PDCP, RLC, and PDSCH throughput learning capability with 0.010, 0.013, 0.009 SAE values, and 0.00016, 0.00014, 0.00002 MAE values, respectively. The second most promising GPR kernel is the squared exponential kernel with 0.010, 0.013, 0.010 SAE values and 0.00020, 0.00015, 0.00021 MAE values. These results also reveal that the GPR training matern52 slightly outperforms the frequently employed squared exponential kernel in several applications (Bhingé et al. 2014; Nannapaneni et al. 2018; Park et al. 2017). The exponential kernel attained the worst PDCP, RLC and PDSCH throughput learning with 0.1829, 0.2942, 0.1897 SAE values, and 0.0029, 0.0030, 0.00470 MAE values, respectively.

Figure 12. Attained throughput data learning MAE with ten different Kernels

Figure 13. Attained throughput data learning SAE with ten different Kernels

Figure 14. Attained throughput data learning STD with ten different Kernels

4.0 CONCLUSION

This work investigates the weight impacts of ten commonly used kernel functions in Gaussian process regression on throughput datasets acquired over 4G LTE networks. Generally, results reveal that the matern52 kernel function attained the best throughput learning capability among the ten contenders. In comparison, the ardsquared kernel showed the worst throughput learning capability. These results show that the matern52 outperforms the usually acclaimed squared exponential kernel by 20%. Finally, results imply that the matern52 kernel could be a better alternative for practical GPR-based data mining and practical regression analysis. Future work would focus on the optimization of the GPR training with the matern52 kernel function for optimal user throughput data learning.

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APPENDIX A.

List of Abbreviations

Table 1.

Abbreviation	Full Meaning
4G	Fourth Generation
ARD	Automatic Relevance Determination
EK	Exponential Kernel
eNodeB	Evolved Node Base Station
GPR	Gaussian Process Regression
LTE	Long Term Evolution
MAE	Mean Absolute Error
MIMO	Multiple Input Multiple Output
MK	Matern Kernel
ML	Machine Learning
MHz	Megahertz
PDCP	Packet Data Convergence Protocol
PDCCH	Physical Downlink Control Channel
PDSCH	Physical Downlink Shared Channel
RF	Radio Frequency
RLC	Radio Link Control
RMSE	Root Mean Square Error
RQK	Rational Quadratic Kernel
RSRP	Reference Signal Received Power
SAE	Sum of Absolute Mean Error
SEK	Squared Exponential Kernel
SIB	System Information Blocks
STD	Standard Deviation
TEMS	Test Mobile System
UE	User Equipment
UHF	Ultra High Frequency
UMTS	Universal Mobile Telecommunications System

Availability of Data And Material

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Competing Interests

The authors declare that they have no conflicts of interest.

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Authors' Contribution

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