Service Investment and Pricing Strategies in E-Commerce Platforms With Seller Competition

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ABSTRACT

With the development of the two-sided market, many platform enterprises classify their users into different types and cooperate with them with different strategies. The extant literature mainly explores the pricing and investment decisions for the platform, but pays little attention to the classification of sellers when making decisions. This paper investigates the investment of value-added service and pricing strategies for an e-commerce platform with competing sellers. Specifically, this paper considers a two-sided platform that is composed of an e-commerce platform, buyers, and sellers. Sellers with high performance requirement and with low performance requirement compete for the buyers in the platform. This paper assumes that each buyer will choose the sellers' type immediately after entering the platform and buy a unit of product in the platform. Through theoretical analysis, the authors show that the platform will gain more profits by investing in value-added services for type-A sellers, and it will obtain the optimal profit when the transaction fee is moderate.

KEYWORDS
Competing Sellers, E-Commerce Platform, Pricing, Two-Sided Market, Value-Added Service

INTRODUCTION

Recent technological innovations have led to the emergence of online platforms that create two-sided platforms (e.g., eBay, Alibaba, etc.). The typical two-sided platforms serve to facilitate trade and generate revenue by charging from the participants. Two-sided platforms gain competitive advantage and platform performance by adopting better management practices such as various kinds of pricing strategies to maximize profits (e.g., Singh et al., 2014). The pricing strategy is important because it will directly affect the number of participants on the two-sided platforms. For example, eBay.com charges a listing fee plus a per-transaction fee from each seller, while Tmall.com charges a fixed entry fee plus a per-transaction fee.

The growing tendencies in the forecasts of e-commerce services indicate that e-commerce platforms pay attention to the relationship between consumer satisfaction and platform service levels.
Only the consumers are satisfied can two-sided platforms attract more participants and obtain higher profit. Therefore, the platform adopts various service strategies to improve its service level, such as providing value-added services (e.g., Dou et al., 2016). Specifically, the advertising service for online sellers provided by some e-commerce platform improves the transactions effectively; the online application store provides a new payment method to make transactions more convenient, thereby increasing the developer’s potential income; the payment platform of credit card organization provides data mining service, which enables sellers to take better marketing measures. Besides pricing and investment strategies, two-sided platforms also try to gain advantages through participant classification. Such as, Taobao.com, the largest B2C e-commerce platform in China, classifies the sellers into two types (individual vs. professional). Payment system classifies the consumers into two types (ordinary vs. VIP) and Video websites classifies the audiences into two types (general vs. paying).

With the participants’ classification of the two-side platform, the different types of the participants compete in the platform. The success of the platform greatly depends on the balanced investment and pricing for the different types of participants (e.g., Dou et al., 2018). For example, Amazon.com classifies the sellers into individual and professional. The professional sellers are endowed with high performance and the individual sellers are endowed with low performance by the platform (e.g., Wu and Gereffi, 2018). That means the buyers think the reputation of the professional sellers is better than individual sellers. The products’ price of the professional sellers is higher than that of the individual sellers. In a certain period of time, the two types of sellers compete for the buyers in the platform. The platform charges the sellers with high performance subscription fee and transaction fee and provides them value-added service or not. The platform charges the sellers with low performance transaction fee only and provides basic service. To make profit optimal, the platform should balance on the two types of the sellers of the charge fee and the level of value-added service, in the paper below the authors call it investment.

Two-sided platforms have been a fertile area for operations management researchers in recent years. Many researchers have focused on the decisions in pricing and design for platforms under various backgrounds. Hagiu (2006) study pricing and commitment by a monopoly platform and competing platforms. Lin et al. (2011) examine a platform’s optimal two-sided pricing strategy while considering seller-side innovation decisions and price competition. Cheng et al. (2018) examine the optimal pricing decisions for an online video platform by considering the customer’s choice behavior. Lin et al. (2020) present service and pricing strategies for both monopolistic and duopolistic platforms and investigate the impact of platforms’ life cycle on their service and pricing strategies. Li and Zhang (2020) study highlights the pricing strategy in two-sided markets under a monopoly structure considering the reference price effect. Mahdavi et al. (2008) design a dynamic coordination mechanism in electronic markets using stochastic petri nets. Other researchers have considered the price strategies of two-sided platforms under seller classification and competition (Yanan et al. 2016) or the investment decisions of value-added services of two-sided platforms (Zhang et al. 2019). However, most of the prior literature assumes that the platform adopts the same pricing strategy for all the sellers, and has paid little attention to the problem of how the interaction between different types of sellers affects the platform’s investment and pricing decisions.

This paper investigates the investment and pricing strategies for an e-commerce platform with seller classification and competition. Specifically, the authors consider a two-sided platform that is composed of an e-commerce platform, buyers and sellers. The sellers in the market can be classified into sellers with high performance requirement (the type-A sellers) and sellers with low performance requirement (the type-B sellers). The platform generates its revenue from the two types of sellers’ transaction fee and the type-A sellers’ subscription fee. The two types of sellers are endowed with different performance and they compete for the buyers in the e-commerce platform. For each type of sellers joining the platform, the platform has to provide technical support for realizing the performance and incurs the corresponding cost. It will incur the extra investment cost when providing the type-A
sellers with value-added service (large icon display, bulk upload, do promotions and download data etc.). The authors analyze two operation situations for the platform: (1) providing the type-A sellers without value-added service (situation I); and (2) providing the type-A sellers with value-added service (situation II). The situation I is the special case of the situation II. Specifically, the authors highlight the following observations:

1. The platform will gain more profits by investing in value-added services for type-A sellers because the profits of the platform under situation II is higher than that under situation I. The optimal investment of value-added service is non-monotonic function of the price of the type-A sellers’ products and each type-A seller’s utility from per unit value-added service. The platform will provide more value-added service for the type-A sellers if each type-A seller’s utility from per unit value-added service is high or the product price of the type-A sellers is extremely low or extremely high when each type-A seller’s utility from per unit value-added service is low. However, the platform will reduce the value-added service for the type-A sellers when the product price of the type-A sellers is moderate if each type-A seller’s utility from per unit value-added service is low.

2. The optimal pricing decision for the platform is the quasi-concave function of the price of the type-A sellers’ products. The platform will charge more subscription fee for the type-A sellers when the product price of the type-B sellers increasing if the products price of the type-A sellers is low.

Main contributions of the paper are two folds. First, a model that incorporates the platform investment strategy and seller classification is firstly built. Second, the optimal investment decisions for e-commerce platforms under seller heterogeneity are analyzed, which can provide managers in reality with valuable suggestions.

The remainder of the paper is organized as follows. Section 2 provides a literature review related to pricing and investment decisions of two-sided platforms. Section 3 describes the basic model and presents the optimal decisions for the platform. Numerical results are presented in Sections 4 to further reveal the managerial insights. Section 5 concludes the paper.

LITERATURE REVIEW

The e-commerce platforms belong to the two-sided platform and the growing literature on two-sided platform has mostly focused on two-sided pricing and investment strategies. The literatures examine price and investment strategies in the context of competitive and monopolistic two-sided platforms. For example, Schiff (2007), Zhong et al. (2017) and Li et al. (2020) study the pricing and investment strategy of a monopolistic two-sided platform. Hagiu and Jullien (2014), Yue et al. (2020) and Belleflamme and Peitz (2010) study the pricing and investment strategy of two competing platforms. The extant literature can be categorized into two aspects: (1) the pricing and/or investment decisions for two-sided platforms without participant competition, and (2) the pricing and/or investment strategies for two-sided platforms with participant competition. The subsequent paragraphs successively review the related researches.

Pricing and/or Investment Decisions Without Participant Competition

Schiff (2007) illustrates the first-best and second-best prices when a single platform charges either usage fees or membership fees for users. Cope (2010) investigates dynamic pricing strategies for maximizing revenue in an e-commerce platform by actively learning customers’ demand response to price. Then they investigate several strategies for sequential pricing based on index functions that consider both the potential revenue and the information value of selecting prices. Zhong et al. (2017)
study the service pricing of a two-sided delivery platform with considering network externalities. They focus on three pricing strategies, membership-based pricing, transaction-based pricing, and cross subsidization. Gao et al. (2018) provide a general model to study new pricing strategies for a mixed two-sided platform where a consumer may appear on different sides of the platform. In particular, the platform may bundle the services it provides to two sides and offer dual-side consumers a discount (or charge them a premium) when feasible. They study when such mixed bundling is desirable, and how it affects the platform’s optimal pricing strategy. Hagiu and Jullien (2014) study how platforms trade off search diversion and consumers’ fixed access fees. Platforms do not charge consumers entry fees, and the competition among platforms is high, which leads to a lower level of search transfer equilibrium compared with monopoly platforms. A light level of competition between platforms leads to more search diversion than between monopolies. Platforms charge consumers a fixed entry fee, and regardless of the nature of competition, all equilibrium levels of search transfer under platform competition are equal to the monopoly level. The above researches mainly focus on the pricing decisions for a monopolistic platform or competing platforms under various backgrounds. However, this line of research implicitly assumes that the participants are of the same type and has paid little attention to the impact of seller interaction on the platform’s decisions.

Martin and Orlando (2007) extend the basic framework of two-sided markets by focusing on participants’ investment decisions and examine incentives for network-specific investment. They model the participants how to invest in a technology that reduces the marginal cost of using the platform. When the commitment of the platform usage fees is feasible, the high-investment equilibrium can be implemented, and vice versa. Belleflamme and Peitz (2010) compare the seller investment incentives of two competing for-profit platforms with those of two open platforms. They find that trade via for-profit platforms raises seller incentives to invest in quality, but lowers incentives to invest in price discrimination when both sides single-home; trade via for-profit platforms would lead to weaker investment incentives when sellers can multi-home and buyers single-home, that because investment decisions affect the strength of indirect network effects and access prices. Edward et al. (2014) develop the two-sided market model to analyze the investment decisions of platform performance in the presence of cross-network externalities. They show that heavily investing in the core performance of a platform does not always yield a competitive edge and have characterized the conditions under which offering a platform with lower performance but greater availability of content can be a winning strategy.

Masanell and Llanes (2015) study incentives to invest in platform quality in open and proprietary two-sided platforms that brings together users and developers of applications. They have found open platforms may benefit from limited developer access, an open platform may lead to higher investment than a proprietary platform and opening one side of a proprietary platform may lower incentives to invest in platform quality. Tan et al. (2017) develop an analytic model to explore the key tradeoffs behind investment in integration tools and how that investment interacts with pricing decisions in a two-sided market. They show that higher levels of investment by the platforms into integration become desirable when the platform has access to a large pool of content providers and consumers, is able to develop integration tools that are highly effective in reducing third-party development costs, and operates in a market in which content providers earn a high-enough profit margin creating content that is highly valued by the consumer market. These above researches mainly focus on the investment decisions for the two-sided platforms under various backgrounds. However, this line of research implicitly assumes that the sellers are of the same type and has paid little attention to the impact of seller interaction on the platform’s investment decisions.

Pricing and/or Investment Decisions With Participant Competition

The decisions for two-sided platforms with participants’ competition have rarely been investigated. Among the related researches, Goes et al. (2013) classify sellers in electronic marketplaces into new and experienced groups and compare their auction performance based on success rates, ending prices and
feedback ratings from the empirical perspective. Yanan et al. (2016) study the cooperation and pricing strategies for e-commerce platforms when classifying the sellers into individual and professional. They find the platform cooperates with both the two types of sellers when the platform attractiveness for the professional sellers is high enough and cooperates only with the professional sellers when the platform attractiveness for the professional sellers is low and the performance requirement of the individual sellers is relatively high. Chen et al. (2019) set up the game models for alternative commission pricing strategies among one online platform firm and two competing vendors. They find the exclusive pricing right that the leader vendor tries to secure results in a higher commission fee paid to the online platform. Whereas these researches has paid little attention to the investment and pricing strategies for the platform providing the type-A sellers with and without value-added service from the theoretical perspective.

In general, the extant literature mainly explores the pricing and investment decisions for the platform cooperating with all the sellers joining the platform. Though participant competition has been widely adopted by practitioners in reality to gain competitive advantages, extant literature has paid little attention to the classification of sellers when making pricing and investment decisions. The authors still know little about whether the platform should provide value-added services to Type-A sellers and the level of investment in value-added services. That is, there is still a gap between the pricing decisions and investment strategies for the platforms with seller classification and competition, which is the reason for the present paper.

MODEL

In this section, the authors consider a monopoly e-commerce platform provides a trading platform for sellers and buyers. The heterogeneities of the sellers are in the product price and performance requirement. Platform performance refers to the overall impression of a platform that is set up through the platform’s all kinds of signs, such as product features, marketing strategy, style, etc. (Edward et al. 2014). Performance is an important part of the appeal to buyers, but it requires the platform to make investments. According to the heterogeneities, the platform classified the sellers in the market into two types: the sellers with high performance requirement (type-A) and the sellers with low performance requirement (type-B). The authors assume that each buyer will choose one type of seller to buy a unit product in the platform. Then the competition exists in the two types sellers because they will use the different product prices and platform performances to attract buyers. Furthermore, the platform will provide value-added services (Houssos et al. 2002, Kuo et al. 2009 and Zhang et al. 2015) such as bulk upload, do promotions, download data reports for the sellers of type-A.

To facilitate the description, some notations are defined in Table 1 (located in the Appendix A).

Suppose that each seller of type $i$ ($i=A, B$) sells its product at a unit price $p_i$, $p_A > p_B$, and incurs an operating cost $f_i$ for making its product tradable on the platform. To characterize the heterogeneity of sellers, the authors assume that $f_i$ is uniformly distributed on the interval $[0, F]$. Additionally, each seller of type-A gains a utility that increases with the value-added service of the platform provided for the sellers of type-A, $\beta \phi$ ($\beta > 0$), where $\beta_i$ is the utility of a seller of type-A per unit value-added service.

Following Edward et al. (2014), the authors assume that each buyer will choose the type of seller immediately after entering the platform and buy a unit of product in the platform, a buyer’s utility is composed of three components: available sellers ($n_{si}$), platform performance ($v_i$) and the unit price of the purchased product ($p_j$). Specifically, the authors assume that the utility of a buyer who chooses the sellers of type-$i$ ($i=A$ or $B$) is $u_{bij} = \alpha_{i} n_{si} + \beta_i v_{ij} - p_j$, ($i=A$ or $B$), where $\alpha_i$ represents a buyer’s utility from the increase of a seller of type $i$ ($i=A, B$), which characterizes the strength of cross-network externalities. Parameter $v_i$ represents the performance requirement of the platform for the sellers of type $i$ ($i=A, B$). The performance requirement of the platform makes the buyers think the reputation
of the type A sellers is higher than the type-B sellers’, that is, \( v_A > v_B \). Parameter \( \theta \) represents the preferences of the buyers for platform performance. To characterize the heterogeneity of buyers, the authors assume that \( \theta \) is uniformly distributed on the interval \([0,1]\). Thus, the utility indifference point of a buyer choosing two types sellers is \( \tilde{\theta} = (\alpha_B n_{s_B} - \alpha_A n_{s_A} + p_A - p_B)/(v_A - v_B) \). Denote by \( n_{b_A} \) and \( n_{b_B} \) respectively the number of buyers who choose the sellers of type-A and type-B, the authors can obtain that:

\[
\begin{align*}
    n_{b_A} &= 1 - \frac{\alpha_B n_{s_B} - \alpha_A n_{s_A} + p_A - p_B}{v_A - v_B} \\
    n_{b_B} &= \frac{\alpha_B n_{s_B} - \alpha_A n_{s_A} + p_A - p_B}{v_A - v_B}
\end{align*}
\]

The platform generates its revenue by charging a subscription fee (\( s \)) from each seller of type-A since high performance and value-added services also requires large investments of the platform (e.g., more technique support) and transaction fee from two types of sellers. Following Edward et al. (2014), the authors assume that the investment costs of the platform consist of a fixed cost of platform performance given as \( cv_i^2 \) (\( c > 0 \)) and a convex increasing function of value-added service given as \( k\phi^2 \) (\( k > 0 \)). In the following sections, the authors examine the optimal strategies of the platform under two situations:

**Situation I:** the platform provides the sellers of type-A without value-added service.

**Situation II:** the platform provides the sellers of type-A with value-added service.

**The Platform Provides Type-A Sellers without Value-Added Service**

The authors first consider the situation that the platform cooperates with both the two types of sellers and provides the sellers of type-A without value-added service. The platform charges a subscription fee from each type-A seller (\( s_i \)) and transaction fee from two types of sellers (\( r \)), the sellers of type \( i \) sell the product at a unit price of \( p_i \) and the two types of sellers attract the buyers by the respective platform performance level (\( v_i \)), unit product price (\( p_i \)) and available sellers (\( n_{si} \)).

Thus, by joining the platform, a type-A seller with fixed cost \( f_A \) makes a utility of

\[
\begin{align*}
    u_{s_A}(s_i) &= (1-r)p_A n_{b_A} - s_i - f_A \\
    u_{s_B}(s_i) &= (1-r)p_B n_{b_B} - f_B
\end{align*}
\]

and a type-B seller with fixed cost \( f_B \) makes a utility of

\[
\begin{align*}
    u_{s_A}(s_i) &= (1-r)p_A n_{b_A} - s_i - f_A \\
    u_{s_B}(s_i) &= (1-r)p_B n_{b_B} - f_B
\end{align*}
\]

where \( r \) denotes the transaction fee of the two types of sellers, that is a proportion of the price and in the interval of \([0,1]\), the term \((1-r)p_A n_{b_A}\) denotes the revenue of each seller of type \( i \). Suppose that each seller will join the platform if its utility is nonnegative, the authors can obtain that:

\[
\begin{align*}
    n_{s_A}(s_i) &= \frac{(1-r)p_A n_{b_A} - s_i}{F} \\
    n_{s_B}(s_i) &= \frac{(1-r)p_B n_{b_B}}{F}
\end{align*}
\]
Combining equations (1), (2), (3) and (4) the authors can obtain that the numbers of the two types sellers and the buyers who choose the two types sellers are the function of the subscription fee. To facilitate description, denote by:

\[ L_1 = \frac{F(v_A - v_B) - (1 - r)\alpha_B p_B}{F[F(v_A - v_B) - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A]} \]

\[ L_2 = \frac{(1 - r)\alpha_A p_B}{F[F(v_A - v_B) - \alpha_B (1 - r) p_B - (1 - r)\alpha_A p_A]} \]

\[ L_3 = \frac{\alpha_A}{F[v_A - v_B] - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A} \]

\[ L_4 = \frac{F(v_A - v_B) - (1 - r)\alpha_B p_B - F(p_A - p_B)}{F[F(v_A - v_B) - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A]} \]

The platform determines the subscription fee \( s_1 \) to maximize its profit. Accordingly, the platform’s decision problem can be formulated as:

\[ \max_{s_1} \prod_i (s_1) = rp_A n_{sA1} n_{sA1} + rp_B n_{sA1} n_{sB1} + s_1 n_{sA1} - c(v_A^2 + v_B^2) \]  \hspace{1cm} (5) \]

In the optimization problem described by equation (5), the term \( rp_A n_{sA1} n_{sA1} \) represents the revenue from the type \( i \) sellers’ transaction fee (or referral fee), the term \( s_1 n_{sA1} \) represents the revenue from the type-A sellers’ subscription fee, and the term \( c(v_A^2 + v_B^2) \) represents the investment cost of the platform for the platform performance. A higher level of performance requires a larger investment of cost.

Based on the optimization problem described in equation (5) the authors have the following results.

**Theorem 1.** Suppose that \( L_1 L_3 r p_A + L_2 L_4 r p_B - L_1 < 0 \), the optimal decisions of the e-commerce platform under situation I, \( s_1^* \), satisfies:

\[ s_1^* = \frac{2r p_B (1 - F L_4) L_2 + p_A L_4 (1 - 2r F L_1)}{2(L_4 - r p_A L_1 L_3 - r p_B L_2 L_3)} \]

Proofs of Theorem 1 and the subsequent theorems are presented in the Appendix B.

Theorem 1 states \( L_1 L_3 r p_A + L_2 L_4 r p_B - L_1 < 0 \) to ensure the impact of the subscription fee on the platform’s profit is greater than the impact of the transaction fee (or referral fee). Otherwise, the platform will set the subscription fee to infinity, and that induces the sellers of the type-A will not enter the platform. Even though the platform can still obtain the optimal profit because all the buyers will make transaction with the sellers of the type-B and the impact of the transaction fee is greater. However, this view does not tally either with the actual situation or with the starting point of our research. Accordingly, the authors only study the case of the impact of the subscription fee on the platform’s profit is greater than the impact of the transaction fee (or referral fee).
The authors can observe from Theorem 1 that the optimal subscription fee \( (s_1^*) \) is greatly dependent on the transaction fee \( (r) \), the prices of the two types of sellers \( (p) \), the cross-side network effects between the sellers and the buyers \( (\alpha) \), as well as the performance requirement \( (v) \).

To further explore the properties of the platform’s optimal decisions, the authors have the following results.

**Corollary 1.** When \( r = 0, \frac{\partial^2 s_1^*}{\partial p_A^2} < 0 \).

Corollary 1 shows that when the platform charges no transaction fee \( (r = 0) \), the optimal subscription fee charged by the platform \( (s_1^*) \) is concave in the unit price of the product sold by the sellers of type-\( A \) \( (p_A) \). Intuitively, the higher the products price of the type-\( A \) sellers in the platform, the greater the attraction of the platform for the type-\( A \) sellers. That induces the platform charges a higher subscription fee. But our observation is the platform will reduce the subscription fee when the product price of the type-\( A \) sellers is extremely high (i.e., \( p_A > \frac{F(v_A - v_B) - \alpha Б p_B + Fp_B}{2F} \)) since the platform has two types of competing sellers. When the product price of the type-\( A \) sellers is low, the competitiveness of the type-\( A \) sellers is stronger and the platform will become more attractive to the type-\( A \) sellers. Hence, when \( p_A < \frac{F(v_A - v_B) - \alpha Б p_B + Fp_B}{2F} \), the optimal subscription fee will increase with \( p_A \). When the product price of the type-\( A \) sellers is extremely high, the competitiveness of the type-\( A \) sellers is weaker and the platform will become less attractive to the type-\( A \) sellers. Hence, when \( p_A > \frac{F(v_A - v_B) - \alpha Б p_B + Fp_B}{2F} \), the optimal subscription fee will decrease with \( p_A \).

**Corollary 2.** When \( r = 0, \) if \( p_A < \frac{F(v_A - v_B)}{\alpha Б} \), \( \frac{\partial s_1^*}{\partial p_B} > 0 ; \) otherwise, \( \frac{\partial s_1^*}{\partial p_B} < 0 \).

Corollary 2 shows that when the platform charges no transaction fee \( (r = 0) \), the platform can charge a higher subscription fee from the sellers of type-\( A \) \( (s_1^*) \) if the product price of the type-\( A \) sellers \( (p_A) \) is low or the strength of cross-network externalities of type-\( B \) sellers for the type-\( B \) buyers \( (\alpha Б) \) is weak. That is because the attraction of the platform to the type-\( A \) sellers is strong or the attraction of the platform to the type-\( B \) sellers is weak. When the product price of the type-\( B \) sellers increases, the platform will become more attractive to the sellers because the number of the type-\( A \) buyers increases.

**Corollary 3.** When \( r = 0, \) \( \frac{\partial s_1^*}{\partial \alpha Б} < 0 \), \( \frac{\partial s_1^*}{\partial v_B} < 0 \).

Corollary 3 shows that when the platform charges no transaction fee \( (r = 0) \), the platform will charge a lower subscription fee from the sellers of type-\( A \) \( (s_1^*) \) if the strength of cross-network
externally of the type-B sellers for type-B buyers \((\alpha_B)\) or the performance requirement of the platform for the sellers of type-B \((v_B)\) is increasing. That is because the revenue of the platform only comes from the type-A sellers in this case. The sellers of the type-B will obtain more buyers when the strength of cross-network externalities of the type-B sellers for type-B buyers \((\alpha_B)\) or the performance requirement of the platform for the sellers of type-B \((v_B)\) is increasing. That means the platform will become more attractive to the sellers of the type-B, and because there is competition between the two types of sellers therefore, the platform will reduce the subscription fee to attract the sellers of type-A to obtain optimal profit.

The Platform Provides Type-A Sellers With Value-Added Service

In this section, the authors consider the case when the platform cooperates with both the two types of sellers and provides the sellers of type-A with value-added service. The interactions among the platform are the platform charges a subscription fee from each type-A seller \((s_2)\) and transaction fee from two types of sellers \((r)\), the sellers of type \(i\) sell the product at a unit price of \(p_i\), each buyer of type \(i\) \((i=A\) or \(B)\) respectively purchases one unit of product from the sellers of type \(i\) at a unit price of \(p_i\). Specifically, the platform provides the sellers of type-A with value-added service. Due to the value-added service provided by the platform to the sellers of type-A can’t directly benefit the buyers, the two types of sellers attract the buyers by the respective platform performance level \((v_i)\), unit product price \((p_i)\) and available sellers \((n_i)\).

Thus, by joining the platform, the utility that a buyer of type \(i\) can also be formulated as functions \((1)\) and \((2)\), a type-A seller with fixed cost \(f_A\) and value-added service \(\phi\) makes a utility of 
\[
u_{sA}(\phi, s_2) = (1-r)p_A n_{sA} + \beta \phi - s_2 - f_A
\]
and a type-B seller with fixed cost \(f_B\) makes a utility of 
\[
u_{sB}(\phi, s_2) = (1-r)p_B n_{sB} - f_B
\]
where \(r\) denotes the transaction fee of the two types of sellers, that is a proportion of the price and in the interval of \([0,1]\), the term \((1-r)p_i n_{sB}\) denotes the revenue of each seller of type, and term \(\beta \phi\) denotes the utility from the value-added service. Suppose that each seller will join the platform if its utility is nonnegative. The authors can obtain that:

\[
n_{sA}(\phi, s_2) = \frac{(1-r)p_A n_{sA} + \beta \phi - s_2}{F} \tag{6}
\]

\[
n_{sB}(\phi, s_2) = \frac{(1-r)p_B n_{sB}}{F} \tag{7}
\]

Combining equations \((1)\), \((2)\) and \((6)\) and \((7)\) the authors can obtain that the numbers of the two types of the sellers and the buyers are the function of the subscription fee and the value-added service.

The platform determines the subscription fee \((s_2)\) and the value-added service \((\phi)\) to maximize its profit. Accordingly, the platform’s decision problem can be formulated as:

\[
\max_{s_2, \phi} \Pi(s_2, \phi) = r p_A n_{sA} n_{A} + r p_B n_{sB} n_{B} + s_2 n_{sA} - c(v_A^2 + v_B^2) - k \phi^2 \tag{8}
\]
s.t. $v_i > 0$, $\phi > 0$

In the optimization problem described by equation (8), the term $r p_i n_{si2} n_{si2}$ represents the revenue from the type $i$ sellers’ transaction fee (or referral fee), the term $s_2 n_{sA2}$ represents the revenue from the type-A sellers’ subscription fee, the term $c(v_A^2 + v_B^2)$ represents the investment cost of the platform for the platform performance, and the term $k\phi^2$ represents the investment cost of the platform for the value-added service.

Based on the optimization problem described in equation (8) the authors have the following results.

**Theorem 2.** Suppose that $r p_A L_A L_3 \beta^2 + r p_B L_2 L_3 \beta^2 - k < 0$, the optimal decisions of the e-commerce platform under situation II, $(s^*_2, \phi^*)$, satisfy:

$$s^*_2 = \frac{2r p_B L_3 (1 - FL_4)(2k - \beta^2 L_4) + p_A L_4 [2k(1 - 2r FL_4) - r^2 \beta^2 (2FL_4^2 - L_4)]}{4k[L_4 - r L_3 (p_A L_4 + p_B L_2)] - \beta^2 L_4^2}$$

$$\phi^* = \frac{p_A L_4 \beta L_4 [L_3 (1 - 2r + 2r FL_4) - 2\beta^2 L_3 (L_4 - \beta)] - 2r p_B L_4 L_3 \beta (1 - FL_4)}{4k[L_4 - r L_3 (p_A L_4 + p_B L_2)] - \beta^2 L_4^2}$$

Theorem 2 states $r p_A L_A L_3 \beta^2 + r p_B L_2 L_3 \beta^2 - k < 0$ to ensure that the impact of the value-added service on the platform’s revenue is weaker than the impact on the platform’s cost. Otherwise, the platform will invest the value-added service to infinity for increasing the profit. However, this view does not tally either with the actual situation or with the starting point of our research. Accordingly, the authors only study the case of the impact of the value-added service on the platform’s revenue is weaker than the impact of the value-added service on the platform’s investment cost. The authors can observe from Theorem 2 that the optimal value-added service level $(\phi^*)$ is greatly dependent on the transaction fee $(r)$, the prices of the two types of sellers $(p_i)$, the cross-side network effects between the sellers and the buyers $(\alpha_i)$, as well as the performance requirement $(v_i)$.

To further explore the properties of the platform’s optimal decisions, the authors have the following results.

**Corollary 4.** When $r = 0$, if $\beta^2 \geq \frac{(4kF - 4k\alpha_A) [F(v_A - v_B) - \alpha_B p_B] - 4kF \alpha_A p_B}{F(v_A - v_B) - \alpha_B p_B}, \frac{\partial \phi}{\partial p_A} \geq 0$;

if $\beta^2 < \frac{(4kF - 4k\alpha_A) [F(v_A - v_B) - \alpha_B p_B] - 4kF \alpha_A p_B}{F(v_A - v_B) - \alpha_B p_B}, \frac{\partial \phi}{\partial p_A} > 0$;

when $p_{A1} < p_A < p_{A2}$, $\frac{\partial \phi}{\partial p_A} < 0$; $p_{A1}$ and $p_{A2}$ ($p_{A1} < p_{A2}$) satisfy:

$4kF^2 \alpha_A p_A^2 - 2F(4kF - \beta^2) [F(v_A - v_B) - \alpha_B p_B] + (4kF - \beta^2) [F(v_A - v_B) - \alpha_B p_B]$

$[F(v_A - v_B) - \alpha_B p_B + Fp_B] = 0$
Corollary 4 shows that when the platform charges no transaction fee \((r = 0)\), if each type A seller’s utility from per unit value-added service \((\beta)\) is high, the value-added service provided by the platform \((\phi^*)\) is increasing in the unit price of the product sold by the sellers of type-A \((p_A)\); if each type A seller’s utility from per unit value-added service \((\beta)\) is low, the value-added service provided by the platform \((\phi^*)\) is increasing in the unit price of the product sold by the sellers of type-A \((p_A)\) when \(p_A\) is relatively low or relatively high, and the value-added service provided by the platform \((\phi^*)\) is decreasing in the unit price of the product sold by the sellers of type-A \((p_A)\) when \(p_A\) is moderate.

Intuitively, the higher the products price of the type-A sellers in the platform, the greater the attraction of the platform for the type-A sellers. That induces the platform provides more value-added service to charge a higher subscription fee because the revenue of the platform is only from the subscription fee. But our observation are the platform will provide more value-added service for the type-A sellers if each type A seller’s utility from per unit value-added service is high (i.e., \(\beta^2 \geq \frac{(4kF - 4k\alpha_A)[F(v_A - v_B) - \alpha_B p_B] - 4kF\alpha_A p_B}{F(v_A - v_B) - \alpha_B p_B}\)), and the platform will provide more value-added service for the type-A sellers when the product price of the type A sellers is extremely low (i.e., \(p_A < p_{A1}\)) or extremely high (i.e., \(p_A > p_{A2}\)) if each type A seller’s utility from per unit value-added service is low (i.e., \(\beta^2 < \frac{(4kF - 4k\alpha_A)[F(v_A - v_B) - \alpha_B p_B] - 4kF\alpha_A p_B}{F(v_A - v_B) - \alpha_B p_B}\)), but the platform will reduce the value-added service for the type-A sellers when the product price of the type-A sellers is moderate (i.e., \(p_{A1} < p_A < p_{A2}\)) if each type A seller’s utility from per unit value-added service is low.

That because each type-A seller’s utility from per unit value-added service is high, provided more value-added service by the platform has strong attractive to the type-A sellers. Because the platform has two types of competing sellers, the competitiveness of the type-A sellers is stronger when the product price of the type-A sellers is extremely low. The platform will become more attractive to the type-A sellers. If each type-A seller’s utility from per unit value-added service is low, provided more value-added service by the platform has not much attractive to the type-A sellers. The competitiveness of the type-A sellers is not strong when the product price of the type-A sellers is moderate and the impact of the value-added service on the platform’s cost is stronger. Hence the platform will obtain more profit by reducing the value-added service for the type-A sellers if each type-A seller’s utility from per unit value-added service is low when the product price of the type-A sellers is moderate.

**Corollary 5.** When \(r = 0\), if \(p_A < \frac{(4kF - \beta^2)F(v_A - v_B)}{\alpha_B(4kF - \beta^2) + 4k\alpha_A(F - \alpha_B)}\), \(\frac{\partial \phi^*}{\partial p_B} > 0\); otherwise, \(\frac{\partial \phi^*}{\partial p_B} < 0\).

Corollary 5 shows that when the platform charges no transaction fee \((r = 0)\), the platform will provide more value-added service for the type-A sellers \((\phi^*)\) when the product price of the type-B sellers \((p_B)\) increasing if the product price of the type-A sellers \((p_A)\) is low. Otherwise, the platform should reduce the value-added service for the type-A sellers. That is because the attraction of the platform to the type-A sellers is strong if the product price of the type-A sellers is low, and the attraction becomes stronger when the product price of the type-B sellers increases. Therefore, the platform should provide the type-A sellers with more value-added service and charges higher subscription fee.
Corollary 6. When \( r = 0 \), if \( 4kF - \beta^2 > 4k\alpha_A \cdot \frac{p_A}{p_B} > \frac{4kF - \beta^2}{4kF - \beta^2 - 4k\alpha_A} \), \( \frac{\partial \phi^*}{\partial v_A} > 0 \), \( \frac{\partial \phi^*}{\partial v_B} < 0 \); otherwise, \( \frac{\partial \phi^*}{\partial v_A} < 0 \), \( \frac{\partial \phi^*}{\partial v_B} > 0 \).

Corollary 6 shows that when the platform charges no transaction fee \( (r = 0) \), the platform will provide more value-added service for the type-A sellers \( (\phi^*) \) when the performance requirement of the platform for the sellers of type-A \( (v_A) \) is increasing if the product price of the type-A sellers \( (p_A) \) is high \( (i.e., \frac{p_A}{p_B} > \frac{4kF - \beta^2}{4kF - \beta^2 - 4k\alpha_A}) \). The higher performance requirement of the platform for the type-A seller, the more buyers may choose the type-A seller. The attraction of the platform for the type-A sellers becomes strong. Hence the type-A sellers are willing to join the platform when the platform provides more value-added service and charges higher subscription fee. The platform will take the contrary measurement when the performance requirement of the platform for the sellers of type-B \( (v_B) \) increases because the type-A sellers have a competition with the type-B sellers.

**Discussion**

Some relevant literatures have reached different conclusions from this study. Yanan et al. (2016) show that the optimal subscription fee charged by the platform for sellers with high performance is the increasing function of the cross-side network effect between the sellers and buyers with low performance, which is different from our results. Because the competition between the two types of sellers is not considered. When the cross-side network effect between the sellers and buyers of low performance is strong, the platform is more attractive to the sellers. Hence, the platform may require a higher subscription fee when the two types of sellers without competition. Dou et al. (2018) consider the influence of negative network effect within the sellers on the platform value-added service investment and finds that the optimal investment of value-added service decreasing in the negative intra-group network externality. However, they did not consider the impact of positive cross-network externalities between the sellers and buyers on platform value-added service investment.

**NUMERICAL STUDIES**

In this section, the authors conduct numerical studies to gain more insights on the optimal decisions of the platform under the general case \( (i.e., r \neq 0) \). To guarantee \( n_{si} \geq 0 \) and \( n_{bi} \geq 0 \), the base setting of the parameters is as follows: \( p_A = 1 \), \( p_B = 0.5 \), \( \alpha_A = 0.002 \), \( \alpha_B = 0.001 \), \( v_A = 0.2 \), \( v_B = 0.1 \), \( \beta = 0.1 \), \( k = 1 \), \( c = 1 \), \( F = 1 \). The transaction fee that is a proportion of the price, so the authors confine \( r \) on the interval \( r \in [0,1] \). Figure 1 (located in the Appendix A) shows that the platform’s optimal profit under situation II and the margin between the optimal profit in the two situations are the quasi-concave function of \( r \). That is, a lower transaction fee will lead the platform obtain more profit; a higher transaction fee will lead the sellers to be reluctant to participate. The platform profit under situation II is greater than situation I and the margin increases first and then decreases. That explains the platform will gain more profits by investing in value-added services for type-A sellers and it will obtain the optimal profit when the transaction fee is moderate.

Figure 2 (located in the Appendix A) shows that the platform’s optimal profit is increasing on \( p_A \) or decreasing on \( p_B \) under situation II. According to corollary 4 and corollary 5, the investment of the value-add service decreases on \( p_A \) when \( p_A \) is moderate and increases on \( p_B \) when \( p_A \) is
higher. That means the platform will pay a lower investment cost when $p_A$ is high or $p_B$ is lower, then the platform will get higher profits.

Figure 3 (located in the Appendix A) shows that the platform’s optimal profit is decreasing on $v_A$ or increasing on $v_B$ under situation II. Specifically, when the price ratio of the two types of sellers is large ($\frac{p_A}{p_B} > \frac{4kF - \beta^2}{4kF - \beta^2 - 4k\alpha}$), the investment of the value-add service increases on $v_A$ and decreases on $v_B$. That means the platform will pay a higher investment cost when $p_A$ is high or $p_B$ is lower, then the platform will get lower profits.

Figure 4 (located in the Appendix A) shows that the platform’s optimal profit is increasing on $\alpha_A$ or $\alpha_B$ under situation II. That means the platform should provide the sellers of type-A with the value-add service when the strength of cross-network externalities between the sellers and the buyers is stronger.

**CONCLUSION**

This paper investigates the investment of value-added service and pricing strategies for the e-commerce platform with two types of competing sellers. Specifically, the authors consider a two-sided platform that is composed of an e-commerce platform, buyers and sellers. The sellers in the platform are classified into two types: the sellers with high performance requirement (the type-A sellers) and the sellers with low performance requirement (the type-B sellers). The platform generates its revenue from the subscription fee of type-A sellers and transaction fee of two types of sellers. The authors examine the optimal strategies of the platform under two situations: the type-A sellers without value-added service (situation I); and the type-A sellers with value-added service (situation II). The findings can provide e-commerce managers in reality with some suggestions in making relevant decisions. The platform will make more profit if he provides value-added service for the type-A sellers. The optimal pricing decisions of the platform are the quasi-concave function of the price of the type-A sellers’ products. The platform will provide more value-added service for the type-A sellers if each type-A seller’s utility from per unit value-added service is high or the product price of the type-A sellers is extremely low or extremely high when each type A seller’s utility from per unit value-added service is low. However, the platform will reduce the value-added service for the type-A sellers when the product price of the type-A sellers is moderate if each type-A seller’s utility from per unit value-added service is low.

This paper establishes a mathematical model to study the service investment and pricing strategies of e-commerce platforms based on the actual situation. Since the model considers many factors, the analytical formula of the optimal decision is more complicated, which makes it difficult to analyze the conclusions comprehensively. In particular, there is a lack of theoretical comparative analysis of the optimal decision in the two situations.

The present research, however, considers only the monopoly platform case. It would be interesting to investigate service investment and pricing strategies when the sellers access to multi-platforms or multi-channels. The research could also be extended to the different stages of cooperation between the sellers and the platform. Additionally, if the authors consider the transaction fee as an endogenous variable, the conclusions in this paper need be re-speculated, which will be the topic of future research.

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REFERENCES


## APPENDIX A. DEFINITIONS AND FIGURES

### Table 1. Definition of notations

<table>
<thead>
<tr>
<th>Notations</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Decision variables</strong></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>Subscription fee charged by the platform from each seller of type $A$.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Value-added service of the platform provided for the sellers of type $A$.</td>
</tr>
<tr>
<td><strong>Market primitives</strong></td>
<td></td>
</tr>
<tr>
<td>$p_i$</td>
<td>Unit price of the product sold by the sellers of type $i$ ($i=A, B$).</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>Each buyer’s utility from the increase of a seller of type $i$ ($i=A, B$), which characterizes the strength of cross-network externalities.</td>
</tr>
<tr>
<td>$v_i$</td>
<td>Performance requirement of the platform for the sellers of type $i$ ($i=A, B$).</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Preference of the buyers for the performance.</td>
</tr>
<tr>
<td>$r$</td>
<td>Transaction fee of the two types of sellers.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Each type $A$ seller’s utility from per unit value-added service.</td>
</tr>
<tr>
<td>$c$</td>
<td>Investment cost of the platform per unit performance squared.</td>
</tr>
<tr>
<td>$f_i$</td>
<td>Operating cost of each seller joining the platform.</td>
</tr>
<tr>
<td>$k$</td>
<td>Investment cost of the platform per unit value-added service squared.</td>
</tr>
<tr>
<td><strong>Derived quantities</strong></td>
<td></td>
</tr>
<tr>
<td>$n_{si}$</td>
<td>Number of sellers of type $i$ ($i=A, B$).</td>
</tr>
<tr>
<td>$n_{bi}$</td>
<td>Number of buyers who choose the sellers of type $i$ ($i=A, B$).</td>
</tr>
<tr>
<td>$u_{bi}$</td>
<td>Utility of a buyer choosing the sellers of type $i$ ($i=A, B$).</td>
</tr>
<tr>
<td>$u_{si}$</td>
<td>Utility of a seller of type $i$ ($i=A, B$).</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Profit of the platform.</td>
</tr>
</tbody>
</table>
Figure 1. Impact of $r$ on the platform’s optimal profit

Figure 2. Impacts of $p_A$ and $p_B$ on the platform’s optimal profit under Strategy II
Figure 3. Impacts of $v_A$ and $v_B$ on the platform’s optimal profit under Strategy II

Figure 4. Impacts of $\alpha_A$ and $\alpha_B$ on the platform’s optimal profit under Strategy II
APPENDIX B. THEOREMS

Proof of Theorem 1. By solving equations (1), (2), (3) and (4), we obtain:

\[ n_{sA} = \frac{F(v_A - v_B) - (1 - r)\alpha_B p_B}{F[v_A - v_B] - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A} \]

\[ n_{bB} = \frac{(1 - r)p_B [F(p_A - p_B) - (1 - r)\alpha_A p_A + \alpha_A s]}{F[v_A - v_B] - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A} \]

\[ n_{bA} = \frac{F(v_A - v_B) - (1 - r)\alpha_B p_B - F(p_A - p_B) - \alpha_B s}{F[v_A - v_B] - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A} \]

\[ n_{bB} = \frac{F(p_A - p_B) - (1 - r)\alpha_A p_A + \alpha_A s}{F[v_A - v_B] - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A} \]

In order to facilitate calculation, we order:

\[ L_1 = \frac{F(v_A - v_B) - (1 - r)\alpha_B p_B}{F[v_A - v_B] - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A} \]

\[ L_2 = \frac{(1 - r)\alpha_A p_B}{F[v_A - v_B] - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A} \]

\[ L_3 = \frac{\alpha_A}{F[v_A - v_B] - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A} \]

\[ L_4 = \frac{F(v_A - v_B) - (1 - r)\alpha_B p_B - F(p_A - p_B)}{F[v_A - v_B] - (1 - r)\alpha_B p_B - (1 - r)\alpha_A p_A} \]

And obtain

\[ \Pi_p(s) = r p_A [L_4 F - L_3 s][L_4 (1 - r) p_A - L_4 s] + r p_B (1 - L_4 F + L_3 s)[(1 - L_4 F) F(1 - r) p_B + L_3 s] \]

\[ + s [L_4 (1 - r) p_A - L_4 s] - c(v_A^2 + v_B^2) \]

Since \( \frac{\partial^2 \Pi_p}{\partial s^2} = 2(L_1 L_3 r p_A + L_2 L_3 r p_B - L_1) \), if \( L_1 L_3 r p_A + L_2 L_3 r p_B - L_1 < 0 \), then the optimum decision for the platform satisfy \( \frac{\partial \Pi}{\partial s} = 0 \); if \( L_1 L_3 r p_A + L_2 L_3 r p_B - L_1 > 0 \), \( \frac{\partial^2 \Pi_p}{\partial s^2} > 0 \). Then we can obtain \( s^* \) as given by theorem 1.

Proof of Proposition 1, 2 and 3. When \( r = 0 \), the platform’s optimum decision is

\[ s^* = \frac{p_A [(v_A - v_B) - (p_A - p_B)]}{2(v_A - v_B)} \]

and satisfies:

\[ \frac{\partial^2 s^*}{\partial p_A^2} = \frac{-F_B}{2[F_B (v_A - v_B) - \alpha_B p_B]} < 0 \]
\[
\frac{\partial s^*}{\partial p_B} = \frac{p_A F_B [F_B (v_A - v_B) - \alpha_B P_A]}{2[F_B (v_A - v_B) - \alpha_B P_B]^2}
\]

\[
\frac{\partial s^*}{\partial \alpha_B} = \frac{-p_B p_A F_B (p_A - P_B)}{2[F_B (v_A - v_B) - \alpha_B P_B]^2} < 0
\]

\[
\frac{\partial s^*}{\partial v_B} = \frac{-F_B p_A F_B (p_A - P_B)}{2[F_B (v_A - v_B) - \alpha_B P_B]^2} < 0 , \text{ as given by proposition 1, 2 and 3.}
\]

**Proof of Theorem 2.** By solving equations (1), (2), (6) and (7), we obtain:

\[
n_{A} = \frac{[F(v_A - v_B) - (1-r)p_A\alpha_B][1-r)p_A + \beta\phi - s] - (1-r)p_A F(p_A - P_B)}{F[F(v_A - v_B) - (1-r)p_B\alpha_B - (1-r)p_A\alpha_A]}
\]

\[
n_{B} = \frac{(1-r)p_B[F(p_A - P_B) - (1-r)\alpha_A p_A - \alpha_A(\beta\phi - s)]}{F[F(v_A - v_B) - (1-r)p_B\alpha_B - (1-r)p_A\alpha_A]}
\]

\[
n_{A} = \frac{F(v_A - v_B) - (1-r)p_B\alpha_B - F(p_A - P_B) + \alpha_A(\beta\phi - s)}{F(v_A - v_B) - (1-r)p_B\alpha_B - (1-r)p_A\alpha_A}
\]

\[
n_{B} = \frac{F(p_A - P_B) - (1-r)\alpha_A p_A - \alpha_A(\beta\phi - s)}{F(v_A - v_B) - (1-r)p_B\alpha_B - (1-r)p_A\alpha_A}
\]

\[
\Pi_p(s) = r p_A[L_A(1-r)p_A + L_B\beta\phi - L_A s]L_A^4 + L_B^3\beta\phi - L_A^2 s] + r p_B[F_B L_B - L_A\beta\phi + L_A s]
\]

\[
\cdot [(1-r)p_B L_B - L_A\beta\phi + L_A s]^2 + s[L_A(1-r)p_A + L_B\beta\phi - L_A s] - c(v_A^2 + v_B^2) - k\phi^2
\]

Since \(\frac{\partial^2 \Pi}{\partial s^2} = 2(L_A^4 r p_A + L_A L_B - L_A) < 0\), \(\frac{\partial^2 \Pi}{\partial \phi^2} = 2(r p_A L_A^3 \beta^2 + r p_B L_B L_A \beta^2 - k) < 0\), and Det(Hessian) = \(\frac{\partial^2 \Pi}{\partial s^2} \frac{\partial^2 \Pi}{\partial \phi^2} - \frac{\partial^2 \Pi}{\partial s \partial \phi}^2 = 4k(L_A - r p_A L_A - r p_B L_B - \beta^2 L_A^2) > 0\), then the optimum decisions for the platform satisfy \(\frac{\partial \Pi}{\partial s} = \frac{\partial \Pi}{\partial \phi} = 0\). Then we can obtain \(s^*\) and \(\phi^*\) as given by theorem 2.

**Proof of Proposition 4, 5 and 6.** When \(r = 0\), the platform’s optimum decisions are \(s^* = \frac{2kp_A L_A}{L_A(4k - \beta^2 L_A)}\), \(\phi^* = \frac{\beta L_A p_A}{4k - \beta^2 L_A}\) and satisfy:

\[
\frac{\partial \phi}{\partial p_A} = \frac{\beta[F(v_A - v_B) - \alpha_B P_B - F(2p_A - P_B)][(4kF - \beta^2)[F(v_A - v_B) - \alpha_B P_B] - 4kF\alpha_A p_A]}{(4kF - \beta^2)[F(v_A - v_B) - \alpha_B P_B] - 4kF\alpha_A p_A}^2
\]

when \(4kF^2\alpha_A p_A^2 - 2F(4kF - \beta^2)[F(v_A - v_B) - \alpha_B P_B]p_A + (4kF - \beta^2)[F(v_A - v_B) - \alpha_B P_B]^2\)
\[ F(v_A - v_B) - \alpha_B p_B + Fp_B > 0, \quad \frac{\partial \phi}{\partial p_A} > 0; \]

\[ y(p_A) = \frac{4ac - b^2}{4a} = \frac{4kF\alpha_A p_B - (4kF - 4k\alpha_A - \beta^2)[F(v_A - v_B) - \alpha_B p_B]}{16kF^2\alpha_A} \]

\[ 4kF^2\alpha_A p_A^2 - 2F(4kF - \beta^2)[F(v_A - v_B) - \alpha_B p_B]p_A + (4kF - \beta^2)[F(v_A - v_B) - \alpha_B p_B] \]

\[ [F(v_A - v_B) - \alpha_B p_B + Fp_B] > 0, \quad \frac{\partial \phi}{\partial p_A} > 0; \]

\[ p_A = \frac{(4kF - \beta^2)[F(v_A - v_B) - \alpha_B p_B]}{4kF\alpha_A} \]

\[ y(p_A) = \frac{4ac - b^2}{4a} = \frac{4kF\alpha_A p_B - (4kF - 4k\alpha_A - \beta^2)[F(v_A - v_B) - \alpha_B p_B]}{16kF^2\alpha_A} \]

If \( 4kF\alpha_A p_B - (4kF - 4k\alpha_A - \beta^2)[F(v_A - v_B) - \alpha_B p_B] \geq 0, \quad \frac{\partial \phi}{\partial p_A} \geq 0; \)

If \( 4kF\alpha_A p_B - (4kF - 4k\alpha_A - \beta^2)[F(v_A - v_B) - \alpha_B p_B] < 0, \) when \( p_A < p_{A1} \) and \( p_A > p_{A2}, \) \( \frac{\partial \phi}{\partial p_A} > 0; \)

when \( p_{A1} < p_A < p_{A2}, \) \( \frac{\partial \phi}{\partial p_A} < 0; \) \( p_{A1}, p_{A2} \) satisfy \( y = 0. \)

\[ \frac{\partial \phi}{\partial p_B} = p_A \beta F \{ F(v_A - v_B)(4kF - \beta^2) - p_A [4kF(4kF - \beta^2) + 4k\alpha_A (F - \alpha_B)] \} \]

\[ \frac{\partial \phi}{\partial \alpha_A} = \frac{4kF\beta p_A^2 [F(v_A - v_B) - \alpha_B p_B - F(p_A - p_B)]}{\{(4kF - \beta^2)[F(v_A - v_B) - \alpha_B p_B] - 4kF\alpha_A p_A \}^2} \]

\[ \frac{\partial \phi}{\partial \alpha_B} = \frac{F \beta p_A p_B \{ [4k\alpha_A - (4kF - \beta^2)]p_A + (4kF - \beta^2)p_B \}}{\{(4kF - \beta^2)[F(v_A - v_B) - \alpha_B p_B] - 4kF\alpha_A p_A \}^2} \]

\[ \frac{\partial \phi}{\partial v_A} = \frac{\beta p_A^2 \{ [4kF - \beta^2 - 4k\alpha_A]p_A - (4kF - \beta^2)p_B \}}{\{(4kF - \beta^2)[F(v_A - v_B) - \alpha_B p_B] - 4kF\alpha_A p_A \}^2} \]

\[ \frac{\partial \phi}{\partial v_B} = \frac{\beta p_A^2 \{ [4k\alpha_A - (4kF - \beta^2)]p_A + (4kF - \beta^2)p_B \}}{\{(4kF - \beta^2)[F(v_A - v_B) - \alpha_B p_B] - 4kF\alpha_A p_A \}^2}, \text{ as given by proposition 4, 5 and 6.} \]

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