Who Should Open Offline Showrooms?
The Brand Owner or E-Tailer

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ABSTRACT
The consumption experience is taken seriously by consumers, and increasingly, firms open offline showrooms for this. Showrooms can help to resolve the uncertainty of consumer perceived value and brand preference in online shopping. This paper considers two kinds of showroom investment scenarios for a two-echelon supply chain consisting of a brand owner and e-tailer. The two scenarios are the brand owner opening showrooms (B) or e-tailer opening (E). The authors evaluate the impact of brand spillover, platform spillover, and offline experience spillover effects on the optimal choice to open showrooms. The results indicate that the conditions of brand owners opening showrooms for win-win are richer and more relaxed than the e-tailer. This requirement for spillovers reflects different motivations for opening showrooms. Moreover, regarding the inroad into showrooms, the brand owner and e-tailer should be cognizant of the trade-off between brand and platform spillover and willing to open showrooms in appropriate conditions.

KEYWORDS
Brand Spillover, Offline Experience, Offline Showroom, Platform Spillover, Supply Chain Management

1. INTRODUCTION
Advances in the continuous development of intelligent terminals and mobile technologies contribute to the growing prevalence of online shopping (Izogo and Jayawardhena 2018) and the change of consumer behavior and psychology. Consumers care more about quality experience and service in purchasing not only about low prices and discounts (Bachrach et al., 2016). They increasingly find it advantageous to first inspect the product at a physical store for sure whether to buy before online purchase (Balakrishnan et al. 2014; Rapp et al. 2015; Kuksov and Liao 2018). An Accenture survey on customer experience founds that 60% of consumers are willing to pay a higher price if it ensures a superior consumption experience (Jacobson et al. 2016). Therefore, increasing firms have invested significantly to open showrooms as one of the most effective ways to deliver product information and provide a better consumer experience. Besides most showrooms originally performed in stores of furniture and electronics industry, the concept of showrooms has also been recently incorporated into the electronic retailing and fashion industry such as Amazon, Warby Parker and Bonobos. With Amazon as an example, since the first offline showroom opening in Seattle in November 2015, Amazon has maintained a great customer experience and achieved a significant improvement in operational efficiency (Dustin 2015; Paul 2017). In addition, Amazon from 2019 began to open 100 showrooms in India (Merlyn 2019). Some other brand owners, such as Apple, Canon, Warby
Parker, Bonobos and Casper, have also opened brand showrooms (Du et al. 2019; Macmillan 2014; Bell et al. 2018; Chavie 2018). These firms mostly set up showrooms purely displaying products to improve the consumers’ shopping experience, and then consumers purchase products they like from the same firm’s online store.

Besides increasing the demand for their products, for brand owners and e-tailers, the main reason for opening offline showrooms is different. Specifically, brand owners can provide experiential service for consumers to enhance brand awareness and satisfaction. The practice of Bonobos and Warby Parker has shown that opening showrooms helps to expand brand influence and improve consumer brand awareness (Bell et al. 2017). E-tailers opening showrooms can drive more traffic to their platforms by offline information delivery and improve overall operational efficiency by reducing returns and observing consumers’ behavior. For example, Amazon’s showrooms always provide experiential products and corresponding barcodes at the same time. Consumers can experience the products and buy them on their mobile devices (Paul 2017). Furthermore, consumers’ uncertainty about whether products fit their anticipation is also a major issue that curbing their purchase desire when purchasing online. By improving the effectiveness of information, consumers can inspect the products and obtain professional advice from shopping guides in showrooms. Meanwhile, showrooms can also promote consumers’ visiting. However, brand owners and e-tailers opening showrooms are not as easy as bricks-and-mortar retailers adding online stores, because the cost of offline showroom investment is far higher than the cost of online store. They need to invest appropriate costs that building and serving to ensure the effectiveness of experience. Thus, for a supply chain composed of the brand owner and e-tailer, members should make careful decisions on an investor, investment cost and pricing concerning the above implications of showroom investment.

In recent years, the impact of showrooms on consumer behavior and supply chain members’ decisions and profits are attracting increasing attention in academic literature. Some literature analyzes the influence of showrooms and consumers’ showrooming behavior that is offline experience online purchase, most of which are qualitative and empirical study (Bell et al. 2017; Chopra 2016; Avery et al. 2012; Wang and Goldfarb 2017). For example, Bell et al. (2017) show that showrooms contribute to not only demand but also brand awareness, and they further explore that showrooms also increase platform operational efficiency by reducing returns (Bell et al. 2018). Another literature on quantitative models is mainly aimed at the influence of “offline experience online purchase” and showrooms on one or two parties under having set showrooms (Dzyabura and Jagabathula 2017; Gao and Su 2016; Li 2022; Jing 2018; Li et al. 2019; Basak et al. 2017; Tan et al. 2019). However, there is less literature considering the impact of showrooms from the perspective of the manufacturer or both. Only Kuksov and Liao (2018) consider the manufacturer to discuss the impact of offline experience on the retailer and manufacturer. Differently, our paper simultaneously considers the choice of showrooms for a brand owner and an e-tailer, and compares the optimal decisions and profits that influenced by offline experience under different investor scenarios.

With all the above background, we consider a supply chain comprising a single brand owner and a single e-tailer and study the following two showrooms investment scenarios. One scenario is that the brand owner opens showrooms which generating brand spillover and offline experience spillover effect (i.e., \(B\) scenario). The other scenario is the e-tailer opens showrooms that generating platform spillover and offline experience spillover effect (i.e., \(E\) scenario). This motivates the research questions addressed in the paper, which are as follows,

(1) Under different showroom investment scenarios, considering the effect of these spillovers on decisions, what are the optimal showroom investment cost and pricing strategies for the supply chain members?
(2) How do the profitability of the supply chain system and members change with different showroom investment with above spillovers?
(3) Whether it can achieve win-win for the brand owner or e-tailer to open showrooms? If so, what are conditions?
To address these questions, we present an analytical framework to capture the supply chain’s relation and decisions under different scenarios. In the \( B \) scenario, the brand owner first decides the showrooms’ investment cost and wholesale price, and then the e-tailer decides the online retail price. In the \( E \) scenario, the e-tailer first decides the cost of showrooms investment, the brand owner further decides the wholesale price, and the e-tailer decides the online retail price at last. We compare the optimal decisions and profits in the \( B \) and \( E \) scenario, and further explore the impact of brand spillover, platform spillover and offline experience spillover effect on the optimal choice for the supply chain opening showrooms.

The remainder of this paper is organized as follows. The next section reviews the related literature. Section 3 introduces the notation and formulates the decision models for the supply chain. Section 4 analyzes the optimal decisions and the change of the decisions under different showroom investment scenarios. Section 5 analyzes the effects of different scenarios on the supply chain’s performance and gives the conditions of the brand owner or e-tailer opening. Section 6 concludes this study and outlines directions for future research. All proofs of this paper are in Appendix.

2. LITERATURE REVIEW

Our work is primarily related to the research of showrooms phenomenon, and is divided into two streams of research: (1) showrooming behavior; (2) offline showrooms.

2.1. Showrooming Behavior

The literature on showrooming behavior generally includes two different implications by whether consumers finally buy from the same retailer online as which they experience or not. The behavior, that consumers experience the product at a retailer’s offline store and move to other retailers to buy it, is showrooming phenomenon in the traditional sense (Balakrishnan et al. 2014). The literature on this can be generally seen as a start point of showrooms. Previous literature mainly focused on the behavior feature of showrooming (Chiu et al., 2012; Huang et al., 2009; Neslin et al., 2014; Vanheems et al., 2013). It is also called a research shopping or free-riding phenomenon in some literature (Kucuk and Maddux, 2010; Kalyanam and Tsay, 2013; VanBaal and Dach, 2005; Verhoef et al., 2007). Some research analyzes the internal and external factors of showrooming and potential cause (Bachrach et al., 2016; Daunt and Harris, 2017). Most of these existing papers consider that showrooming behavior is negative for offline retailers due to changing the final purchase channel (Mehra et al., 2013; Teixeira and Gupta, 2015). However, some paper also verifies that showrooming can play a positive role in retail. For example, Gensler et al. (2017) find that expected average price savings from showrooming and the perceived dispersion in online prices are positively associated with showrooming.

These studies above are mainly about qualitative and empirical studies. Furthermore, some quantitative studies focus on the influence of showrooming on supply chain members’ decisions. The popular arguments usually show the threat of showrooming for traditional retailers but not e-tailers or both of them (Rapp et al. 2015; Jing 2017; Basu et al. 2017; Qi et al. 2022). For example, Balakrishnan et al. (2014) indicate showrooming is when a shopper visits an offline store to check out a product but then purchases the product at an e-tailer, which is more representative research. They considering the heterogeneity of perceived cost study the impact of channel migration on the supply chain, and find this migration behavior will increase channel competition. Jing (2017) shows that showrooming may not benefit the e-tailer considering the increasing competition by showrooming. Basu et al. (2017) indicate that high levels of showrooming are adverse to not only the physical retailer but also the online retailer, but it benefits consumers due to a reduction in retail prices. Besides, Mehra et al. (2017) focus on several strategies to fight showrooming including price-matching strategy and exclusivity of product assortments as a long-term strategy. Nevertheless, Kuksov and Liao (2018) specifically emphasize the strategic role of the manufacturer in showrooming, and, considering the manufacturer’s decisions, show that consumers’ showrooming behavior may make the profitability of physical retailers increase rather than decrease. However, the research on traditional showrooming
behavior focus on the negative influence and how to defeat it, and less attention has been paid to the benefit and how to use it on the channel operation.

2.2. Offline Showrooms
Another positive showrooming behavior that consumers experience a product at a retailer’s offline store and buy it at this retailer’s online store generally appears in the research on showrooms. Showrooms are one place proactively provided by supply chain members to support consumers’ showrooming behavior and meet their demand before the final purchase. Some research study the influence of showrooms on enterprises by qualitative and empirical studies. For example, Bell et al. (2015; 2017) study showrooms using a propensity scoring approach on quasi-experimental obtained data from Warby Parker, and find showrooms have a significantly positive impact on product information disclosure, consumer brand awareness, purchase behavior and overall operational efficiency. Moreover, in their latest research analyzing the 10-year data of Bonobos, they show that showrooms will create stronger environment feeling and brand emotional relationships, which will not only increase the rate and volume of subsequent purchases, but also benefit retailers observe customer behavior and respond positively (Bell et al. 2018). Chopra (2016) also takes Bonobos as an example to study the influence of “offline experience, online purchase” model on the channel cost. Avery et al. (2012) conduct a study on whether and when to open offline stores and discuss the short-term and long-term impact of offline stores on the online channel through the relevant data analysis. Wang and Goldfarb (2017) find that showrooms have a great influence on channel substitution and demand complementarity.

In addition to the above qualitative and empirical studies, the existing literature also adopts the quantitative lens to discuss showrooms that seeking to shed a more positive way on the channel operation. More specifically, in the presence of showrooms, it explores to inquire about consumers’ decision activities and the impact on channel selection. For example, Gao and Su (2016) establish a model starting with consumer utility maximization to depict the omnichannel retail environments. The first under product value uncertainty and availability uncertainty consider and study three information mechanisms including physical showrooms, and then inspect potential interaction effects among them to determine the optimal information structure. For physical showrooms, their result shows showrooms help to resolve product value uncertainty for consumers and reduce returns. Dzyabura and Jagabathula (2017) study the offline product assortment problem considered with channel interactions when the offline channel as a showroom posting information for consumers. They incorporate the impact of offline experience perception assessment into the consumer demand model to examine its impact on consumer preferences and online sales and further discuss the decision-making optimization of channels. Zhou et al. (2018) analyze that how to price service strategies and profits of supply chain members are affected by offline experience services provided by the traditional offline channel acting a showroom under the differential and non-differential pricing methods. Li et al. (2019b) consider a supply chain consisting of an offline showroom, an existing e-tailer and a new competing e-tailer, and study the optimal equilibrium results under asymmetric information. They further analyze the impact of competition and the optimal channel cooperation strategy of the showroom. They find that competition generates positive service effects and information asymmetry injures both the offline showroom and the e-tailer benefits in some conditions. Considering the fixed cost of showrooms and return for retailers and consumers, Li et al. (2019a) construct a model to capture consumer showrooming behaviors and the retailer’s decisions under three types of showroom deployment strategies providing different kinds of product information. They find the performance of an omnichannel retailer is influenced by the showroom feasibility and the show intensity of product information at showrooms. However, the above literature pays less attention to the choice of showroom investor and the conditions on opening showrooms for the supply chain at the same time.

Our study highlights the positive effect of showrooming behavior on the supply chain operation. The key that distinguishes our research from the above is that with different spillovers we consider showroom investment from the perspective of brand owner and e-tailer on the supply chain operation.
and analyze the change of decisions including the showroom investment and pricing under different showroom investment scenarios. The findings give the conditions and suggestions on opening showrooms.

3. MODELING FRAMEWORK

We consider a two-echelon supply chain comprising one brand owner and e-tailer. The brand owner produces a single product at a unit cost $c$ and distributes it for an independent e-tailer at a wholesale price $w$. The e-tailer will sell the product through her online channel at price $p$. To optimize the consumer shopping experience and boost demand, each supply chain member may have a motivation and option to open showrooms at investment cost $C$ determined by himself (e.g., constructing the showroom and designing the interior product arrangement layout). Consumers decide whether to purchase the product online after visiting the showroom. Therefore, we present two scenarios: (1) the brand owner opens showrooms denoted as the $B$ scenario, and (2) the e-tailer opens showrooms denoted as the $E$ scenario. In Table 1, we summarize the notations in this paper.

Figure 1 shows supply chain structures and the game sequence in different scenarios. For each scenario, we consider a Stackelberg game and the timing of the game in different scenarios is specifically described as follows. In the $B$ scenario (see Figure 1(a)): (1) the brand owner decides the investment cost of showrooms $C$ and then provides the e-tailer a wholesale price $w$; (2) the e-tailer decides on the e-retailing price $p$.

In the $E$ scenario (see Figure 1(b)): (1) the e-tailer decides the investment cost of showrooms $C$; (2) the brand owner decides the wholesale price $w$; (3) the e-tailer decides on the e-retailing price $p$.

Following the firm instance and literature (Bell et al. 2016; Huang and Swaminathan 2009), we assume that offline experience can promote the growth of consumer demand, and the market demand function for the supply chain is given by

$$d = a - p + \lambda \sqrt{C}$$

(1)

where $a$ is the potential market demand, $\lambda$ (0 < $\lambda$ < 1) is offline experience spillover effect denoting the impact of offline experience on consumers’ purchase decisions, and the greater value means the better experience effect.
According to Bell et al. (2017; 2018), we further assume that there is a spillover effect as an incentive for establishing the showrooms at the brand owner and the e-tailer. More specifically, in the \( B \) scenario, the brand owner can gain a unit brand spillover \( r_b \) (e.g., contributing to brand awareness and satisfaction); similarly, in the \( E \) scenario, the e-tailer can gain a unit platform spillover \( r_e \) (e.g., reducing returns and improving operational efficiency overall). The amount of each spillover affected by the investment cost of showrooms and offline experience spillover is \( \lambda \sqrt{C} \).

In this paper, the subscripts “\( b \)” and “\( e \)” denote the parameters corresponding to the brand owner and the e-tailer, and the superscripts “\( B \)” and “\( E \)” respectively indicate the parameters corresponding to \( B \) scenario and \( E \) scenario. In general, we assume the cost of production for the brand owner and the operation cost for e-tailer is zero. Then the profit functions can be presented as follows:

1) The brand owner opens showrooms (\( B \) scenario).

\[
\pi^B_e = (p - w)d
\]  \hspace{1cm} (2)

\[
\pi^B_b = wd + r_b \lambda \sqrt{C} - C
\]  \hspace{1cm} (3)

Table 1. Summary of basic notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>the wholesale price of the brand owner</td>
</tr>
<tr>
<td>( p )</td>
<td>the e-retailing price of the e-tailer</td>
</tr>
<tr>
<td>( C )</td>
<td>the showroom investment cost</td>
</tr>
<tr>
<td>( d )</td>
<td>the market demand</td>
</tr>
<tr>
<td>( a )</td>
<td>the potential market demand</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>the offline experience spillover effect</td>
</tr>
<tr>
<td>( r_b )</td>
<td>a unit brand spillover</td>
</tr>
<tr>
<td>( r_e )</td>
<td>a unit platform spillover</td>
</tr>
<tr>
<td>( B )</td>
<td>the scenario that the brand owner opens showrooms</td>
</tr>
<tr>
<td>( E )</td>
<td>the scenario that the e-tailer opens showrooms</td>
</tr>
<tr>
<td>( b )</td>
<td>the brand owner’s subscript</td>
</tr>
<tr>
<td>( e )</td>
<td>the e-tailer’s subscript</td>
</tr>
<tr>
<td>( s )</td>
<td>the supply chain system’s subscript</td>
</tr>
<tr>
<td>( \pi^J_i )</td>
<td>the ( i )'s profit in the ( J ) scenario, where ( i \in {b, e, s} ) and ( J \in {B, E} )</td>
</tr>
</tbody>
</table>
2) The e-tailer opens showrooms (E scenario).

\[ \pi^E_c = (p - w)d + r_\epsilon \sqrt{C} - C \]  

(4)

\[ \pi^E_b = wd \]  

(5)

4. OPENING SHOWROOMS FOR ONLINE SHOPPING

4.1. Opening Showrooms by Brand Owner (B Scenario)

In this scenario, the brand owner chooses to open showrooms. The e-tailer determines the e-retailing price to maximize profit. We obtain the optimal e-retailing price, which is given in Lemma 1.

**LEMMA 1:** Given the showroom investment cost and the wholesale price, in the B scenario, the e-tailer’s best response to the e-retailing price is:

\[ p(w, C) = \frac{a + w + \lambda \sqrt{C}}{2} \]  

(6)

Proof of lemma 1 as well as the proofs of the other propositions, are given in Appendix.

Given the e-tailer’s optimal e-retailing price as the best response, the brand owner maximizes his profit by setting the showroom investment cost and the wholesale price. We can derive the optimal C and w for the brand owner. Combining these values with the e-tailer’s best response function yields the equilibrium of the investment cost of showrooms, wholesale price, e-retailing price, and optimal profits, which are given in Proposition 1.

**PROPOSITION 1:** In the B scenario, the optimal solutions and profits are listed in Table 2.

Based on the optimal decisions and profits, we obtain the effects of parameter changes on the optimal solutions and profits in Proposition 2.

**PROPOSITION 2:** In the B scenario, the optimal showroom investment cost, wholesale price, e-retailing price and the brand owner’s and e-tailer’s profits increase with offline experience spillover effect and brand spillover.

Proposition 2 describes the impact of offline experience spillover effect and brand spillover on the optimal decisions and profits in the B scenario. When the brand owner opens showrooms, under the growing offline experience spillover effect and brand spillover, he is willing to increase the showroom investment cost, and then improves the wholesale price to gain the premium for products. The e-tailer further may take a free-riding of the brand owner’s showroom investment to charge a high e-retailing price to ensure her profit. Moreover, not only the brand owner’s profit but also the e-tailer’s profit can be improved when offline experience spillover effect and brand spillover increase.

Table 2. Optimal decisions and profits in the B scenario

<table>
<thead>
<tr>
<th>Decisions</th>
<th>( w^{**} = \frac{2(2a + \lambda^2 \tau_b)}{8 - \lambda^2} )</th>
<th>( p^{**} = \frac{3(2a + \lambda^2 \tau_b)}{8 - \lambda^2} )</th>
<th>( C^{**} = \frac{\lambda^2(a + 4\tau_b)^2}{8 - \lambda^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits</td>
<td>( \pi^*_c = \frac{(2a + \lambda^2 \tau_b)^2}{(8 - \lambda^2)^2} )</td>
<td>( \pi^*_b = \frac{a^2 + (a + 2\tau_b)\lambda^2 \tau_b}{(8 - \lambda^2)} )</td>
<td>( \pi^*_e = \frac{\lambda^2(16 - \lambda^2)\tau_b^2 + a^2(12 - \lambda^2)^2}{(8 - \lambda^2)^2} )</td>
</tr>
</tbody>
</table>
4.2. Opening Showrooms by E-tailer (E Scenario)

In this scenario, the e-tailer chooses to open showrooms and determines the e-retailing price to maximize the profit. The brand owner chooses the wholesale price after the e-tailer’s showroom investment cost to maximize her profit. The optimal e-retailing price and wholesale price are obtained, as given in Lemma 2.

**LEMMA 2:** In the E scenario, given the showroom investment cost, the e-tailer’s and brand owner’s best response are:

\[
p(w, C) = \frac{a + w + \lambda \sqrt{C}}{2} \tag{7}
\]

\[
w(C) = \frac{a + \lambda \sqrt{C}}{2} \tag{8}
\]

Based on the above response functions, the e-tailer maximizes her profit by setting the optimal \( C \). Then given the e-tailer’s and brand owner’s best responses, we obtain the optimal investment cost of showrooms. Combining it with the e-tailer’s and brand owner’s response functions yields the equilibrium the showroom investment cost, wholesale price, e-retailing price, and optimal profits, which are given in Proposition 3.

**PROPOSITION 3:** In the \( E \) scenario, the optimal decisions and profits are listed in Table 3.

<table>
<thead>
<tr>
<th>Decisions</th>
<th>( w^{*} )</th>
<th>( p^{*} )</th>
<th>( C^{*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^{*} )</td>
<td>( \pi^{*} )</td>
<td>( \pi^{*} )</td>
<td></td>
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</tbody>
</table>

**PROPOSITION 4:** In the \( E \) scenario, the optimal showroom investment cost, wholesale price, e-retailing price and the brand owner’s and e-tailer’s profits increase with offline experience spillover effect and platform spillover.

Proposition 4 describes the effect of offline experience spillover effect and platform spillover on the optimal decisions and profits in the \( E \) scenario. When the e-tailer chooses to open showrooms, under the growing offline experience spillover effect and platform spillover, she is willing to increase the showroom investment cost. Then, the brand owner may take a free-riding of the e-tailer’s showroom investment to set a high wholesale price to realize the premium for products. To achieve the maximum profit, furthermore, the e-tailer will respond accordingly by raising the e-retailing price to ensure her income. In addition, as offline experience spillover effect and platform spillover increase, both the brand owner’s and e-tailer’s profits increase.
4.3 Effects of Showroom Investment on the Supply Chain Members’ Optimal Decisions

In this subsection, we discuss how the showroom investment cost and pricing strategy change under different showroom investment scenarios.

To explore the impact of opening showrooms on the optimal decisions and profits of supply chain members, we first present equilibrium analyses under a benchmark scenario, in which neither the brand owner nor the e-tailer opens showrooms, denoted as the “N” scenario. Then, the brand owner’s and e-tailer’s profit functions are given as follows:

\[ \pi_e^N = (p - w)d \]  \hspace{1cm} (9)

\[ \pi_b^N = wd \]  \hspace{1cm} (10)

The e-tailer determines the e-retailing price to maximize her profit, and the brand owner maximizes his profit by determining the wholesale price. The optimal solutions are given in Lemma 3.

**LEMMA 3:** In the N scenario, the brand owner’s optimal wholesale price, the e-tailer’s optimal e-retailing price, and corresponding optimal profits are listed in Table 4.

**Table 4. Optimal decisions and profits in the N scenario**

<table>
<thead>
<tr>
<th>Decisions</th>
<th>( w^{N^*} = \frac{a}{2} )</th>
<th>( p^{N^*} = \frac{3a}{4} )</th>
<th>( \pi_e^{N^*} = \frac{a^2}{16} )</th>
<th>( \pi_b^{N^*} = \frac{a^2}{8} )</th>
<th>( \pi_s^{N^*} = \frac{3a^2}{16} )</th>
</tr>
</thead>
</table>

Next, we compare the optimal decisions in different scenarios to examine how the showroom investment cost and prices change. The following Proposition 5 and Figures 2 and 3 present the main outcomes.

**PROPOSITION 5:** Prices and showroom investment cost react to showroom investment scenarios as follows:

(i) If \( r_e < a / 8 \) or \( a / 8 \leq r_e < a / 7 \) and \( r_b \geq \max \{ f_0(\lambda, r_e), 0 \} \) or \( r_e \geq a / 7 \) and \( r_b \geq f_0(\lambda, r_e) \), then \( w^{B^*} \geq w^{E^*} \geq w^{N^*} \), \( p^{B^*} \geq p^{E^*} \geq p^{N^*} \) and \( C^{B^*} \geq C^{E^*} \geq C^{N^*} \);

(ii) If \( a / 8 \leq r_e < a / 7 \) and \( \lambda \leq f_0(r_e) \) or \( r_e \geq a / 7 \), when \( r_b < f_0(\lambda, r_e) \), then \( w^{E^*} \geq w^{B^*} \geq w^{N^*} \), \( p^{E^*} \geq p^{B^*} \geq p^{N^*} \) and \( C^{E^*} \geq C^{B^*} \).

where \( f_0(\lambda, r_e) = 2(8r_e - a - \lambda^2 r_e) / (16 - \lambda^2) \), \( f_0(r_e) = \sqrt{(8r_e - a) / r_e} \).

Proposition 5 shows the adjustment of showroom investment cost and the change in prices in different scenarios. Overall, it indicates that either party opening showrooms makes the brand owner and e-tailer raise the wholesale price and e-retailing price. However, the change in optimal decisions in the B and E scenario is not constant. First, if platform spillover is not high under any brand spillover or platform spillover is relatively high and brand spillover is high enough, the showroom investment...
cost in the B scenario is higher than that in the E scenario. In other words, under a lower platform spillover or a higher platform spillover and brand spillover, the brand owner will invest more than the e-tailer for showrooms. The brand owner who invests showrooms has an opportunity to set a higher wholesale price and offset the showroom investment cost, and the e-tailer follows a higher e-retailing price. Under these conditions, their prices in the B scenario are higher than those in the E scenario. Second, if platform spillover is medium and offline experience spillover effect is not too high or platform spillover is relatively high with arbitrary offline experience spillover effect, under a lower brand spillover, the showroom investment cost in the E scenario is higher than that in the B scenario. That is, at this point, the e-tailer is willing to make a higher showroom investment cost than the brand owner. The brand owner who gains a price premium from showrooms charges a higher wholesale price for the e-tailer, and the e-tailer follows a higher e-retailing price and offset the showroom investment cost.

5. EFFECT OF SHOWROOM INVESTMENT ON THE PERFORMANCE

In this section, we examine the effects of different showroom investment scenarios on the supply chain’s performance. We discuss the conditions that are beneficial for the supply chain system and one party or both parties by comparing the equilibriums in different investment scenarios.

5.1 Comparison Analysis of the Profits

Comparing the supply chain members’ and system’s profits under the B and E scenario, we obtain proposition 6 and 7 as follows.

**PROPOSITION 6:**
(i) If \( r_b \geq f_1(\lambda, r_e) \), \( \lambda \leq \min\{f_1(r_e), 1\} \) or \( \lambda > f_1(r_e) \), \( r_e \leq (15\sqrt{14} - 56)a / 28 \), then \( \pi^*_{bE} \geq \pi^*_{bB} > \pi^*_{N} \); otherwise, \( \pi^*_{bE} > \pi^*_{bB} > \pi^*_{N} \).

(ii) If \( r_b \leq f_2(\lambda, r_e) \) when \( a / 8 \leq r_e < a / 7 \), \( \lambda \leq f_3(r_e) \) or \( r_e \geq a / 7 \), then \( \pi^*_{eE} \geq \pi^*_{eB} > \pi^*_{eN} \); otherwise, \( \pi^*_{eB} \geq \pi^*_{eE} > \pi^*_{eN} \).

where
\[
f_1(\lambda, r_e) = \left(\sqrt{8 - \lambda^2}(a + 8r_e)((32 - \lambda^2)a + 8\lambda^2r_e) - a(16 - \lambda^2)\right) / 4(16 - \lambda^2),
\]
\[
\lambda = f_2(r_e) = \sqrt{(16r_e + \sqrt{m})(a + 8r_e) - m} / 4r_e, \quad m = a^2 + 48ar_e + 64r_e^2, \quad f_3(r_e) = \sqrt{8r_e - a} / r_e,
\]
\[
f_2(\lambda, r_e) = \left((8 - \lambda^2)\sqrt{(16 - \lambda^2)(a^2 + \lambda^2r_e(a + 4r_e)) - 2a(16 - \lambda^2)}\right) / (16 - \lambda^2)\lambda^2.
\]

Proposition 6 indicates the changing of the brand owner’s and e-tailer’s profit under different showrooms’ investment scenarios. First, the brand owner’s and e-tailer’s profits under either party opening showrooms are higher than without showrooms. Put differently, compared with the N scenario, the supply chain members ought to invest proactively in showrooms, and it also means the one side surely benefits from the other side’s showrooms investment. Then, for each member, it is not always constant who is better off opening showrooms. For the brand owner, we derive a condition in which he can obtain a higher profit by opening showrooms by himself (i.e., (i)). This condition means that the brand owner prefers to open showrooms under a higher brand spillover or, whatever brand spillover, higher offline experience spillover effect and lower platform spillover, due to the improvement of his total profit. Specifically, if brand spillover is high enough, the brand owner will open showrooms in arbitrary offline experience spillover effect. Even though brand spillover tends to zero, it is also beneficial to his profits in higher offline experience spillover effect. For the e-tailer, if brand spillover is relatively low, she will open showrooms under a modest platform spillover and not too high offline experience spillover effect or higher platform spillover (i.e., (ii)). Under the
above condition, the e-tailer opening showrooms can bring more profit for herself than the brand owner opening. Otherwise, she will wait for the brand owner to open showrooms. In the condition of e-tailer opening, higher brand spillover can lead to a higher wholesale price, so higher brand spillover suppresses the e-tailer’s desire to open showrooms.

Furthermore, we find that when other factors satisfy certain conditions, even low brand spillover tending to zero, the brand owner opening showrooms still makes the brand owner himself obtain more profit than waiting for the e-tailer to open. But for the e-tailer, the same logic does not apply to the situation that platform spillover is low even tending to zero. This indicates that the brand owner cares more about the brand spillover from showroom than the increase in market demand. The e-tailer mainly obtains more profit by the increasing demand and improving e-reselling price to offset showroom investment cost, and has a higher requirement for platform spillover. Besides, proposition 6 has a certain significance to answer the question about whether the two parties get a free ride or create more profits according to the conditions. Being a freeloader may easily lead to back to square one as a result of factors such as equity. Hence, supply chain members should actively open showrooms in some appropriate conditions and not always be a free rider.

**PROPOSITION 7:**

(i) If \( r_e < f_e(\lambda) \) or \( r_b \geq f_b(\lambda, r_e) \), then \( \pi_s^{B} \geq \pi_s^{E} > \pi_s^{N} \);

(ii) \( r_e \geq f_e(\lambda), r_b < f_b(\lambda, r_e) \), then \( \pi_s^{E} \geq \pi_s^{B} > \pi_s^{N} \).

where

\[
f_e(\lambda) = a((16 - \lambda^2)\sqrt{\lambda^4 - 80\lambda^2 + 768 - (48 - \lambda^2)(8 - \lambda^2)} \bigg/ 8(16 + \lambda^2)(8 - \lambda^2))
\]

\[
f_b(\lambda, r_e) = ((8 - \lambda^2)\sqrt{(16 - \lambda^2)(4\lambda^2 r_e(a + 4r_e) + a^2(20 - \lambda^2) + 4m) - a(16 - \lambda^2)(12 - \lambda^2)} \bigg/ 2(16 - \lambda^2)^2),
\]

\[
m = a^2 + 48ar_e + 64r_e^2.
\]

Proposition 7 reveals the change of supply chain system’s profit under different showroom investment scenarios. Opening showrooms first will benefit to the supply chain system’s profit. If platform spillover is low whatever brand spillover or platform spillover and brand spillover are relatively high, the supply chain system will derive a higher profit in the \( B \) scenario. And if platform spillover is relatively high and brand spillover is relatively low, the supply chain system will derive a higher profit in the \( E \) scenario. Figure 3 further illustrates the change of the supply chain’s profit under the \( B \) and \( E \) scenario.

**Figure 2.**
In Figure 2, it is obvious that with factors changing the chance of the $B$ scenario is always more than the $E$ scenario by observing the area of region $B$ and $E$ (i.e., $S_B > S_E$). The e-tailer opening is beneficial to the supply chain system only under a higher platform spillover (i.e., $r_c \geq 0.6026$) and lower brand spillover (i.e., $r_b < f_d(0.5, r_c)$). Moreover, the brand owner opening is beneficial to the supply chain system under a lower platform spillover (i.e., $r_c < 0.6026$) or a higher platform spillover and brand spillover (i.e., $r_c \geq 0.6026$, $r_b \geq f_d(0.5, r_c)$). It is noticeable that the offline experience spillover effect is a weak influence factor for the supply chain to select who should open showrooms. Because, for the whole supply chain, the effect of showrooms’ offline experience spillover effect on consumer purchase is faintly affected by changing investor.

5.2 Win-win Conditions of Showroom Investment under Different Scenarios

To examine when the brand owner and e-tailer can achieve win-win under different showroom investment scenarios, we present the following Proposition 8 and 9 and further illustrate the main outcomes in Figure 3 by a set of numerical simulations.

**PROPOSITION 8:** If $r_j \geq \max\{f_f(\lambda, r_e), f_f(\lambda, r_e), 0\}, \text{ the brand owner and the e-tailer can realize win-win through opening showrooms by the brand owner.}$

Proposition 8 shows the condition that the brand owner opening showrooms makes supply chain members realize win-win in the $B$ scenario. On the whole, it indicates that if brand spillover is relatively high, the brand owner and e-tailer will obtain more profits through opening offline showrooms by the brand owner. It should specifically be emphasized in two situations in this condition. First, the situation that brand spillover is low and tends to zero is similar to Proposition 6(i). It means that, under the win-win, the brand owner will still open showrooms when other factors satisfy certain conditions, even low brand spillover tending to zero. The main motivation of the brand owner’s showroom investment is to obtain brand spillover. Second, the lower limit of brand spillover under win-win is higher than that just considering the brand owner’s profit (i.e., Proposition 6(i)) when platform spillover is high. In this case, high platform spillover makes the e-tailer prefer to open showrooms, so the brand owner still opens and realizes win-win only if brand spillover is higher than the value determined by himself.

**PROPOSITION 9:** If $r_j \geq a / 8, \lambda \leq \min\{f_f(\lambda, r_e), f_f(\lambda, r_e)\}$ and $r_j < \min\{f_f(\lambda, r_e), f_f(\lambda, r_e)\}, \text{ then the brand owner and the e-tailer can realize win-win through opening the showrooms by the e-tailer.}$

Proposition 9 shows the condition that the e-tailer opening showrooms makes supply chain members realize win-win in the $E$ scenario. It indicates that if platform spillover is relatively high and brand spillover is low, the brand owner and e-tailer obtain higher profits through the e-tailer opening showrooms. Under the win-win, the e-tailer still does not open showrooms when platform spillover is low even tending to zero. It means that the e-tailer’s motivation to open showrooms is more profits from the increasing demand and improving e-retailing price, and a higher level of platform spillover. Besides, it differs from the condition that just considering the e-tailer’s profit (i.e., Proposition 6(ii)) in the ceiling of brand spillover. When offline experience spillover effect is higher, the lower of the two brand spillover thresholds can make them realize win-win through opening showrooms by the e-tailer.

We further illustrate the above results in Figure 3 based on a set of numerical simulations. According to Proposition 6, we have got that without opening showrooms is a strictly dominated strategy in this paper. Thus, we only consider comparing the $B$ scenario and $E$ scenario in the following analysis and discuss the impact of offline experience spillover effect, brand spillover and platform spillover on the optimal choice of supply chain members’ showroom investment. In Table 5, we indicate the naming way and definition of each region in Figure 3 as follows.

Figure 3(a) describes the case in which platform spillover is high enough such as $r_c = 4 > 2.3386$. We find that if platform spillover is high enough, the brand owner’s showroom investment can make the supply chain members achieve win-win under a higher brand spillover, but the e-tailer opening can make members achieve win-win under a lower brand spillover. Under a modest brand spillover
Table 5. Region name and Definition

<table>
<thead>
<tr>
<th>Region name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Win-Win (B)</td>
<td>The brand owner and e-tailer win more profits in the $B$ scenario. The brand owner will open showrooms.</td>
</tr>
<tr>
<td>Win-Win (E)</td>
<td>The brand owner and e-tailer win more profits in the $E$ scenario. The e-tailer will open showrooms.</td>
</tr>
<tr>
<td>Win-Lose (B)</td>
<td>The brand owner wins more profit and the e-tailer loses more profit in the $B$ than $E$ scenario, and the brand owner loses more profit and the e-tailer wins more profit in the $E$ than $B$ scenario. Both members want to open showrooms.</td>
</tr>
<tr>
<td>Lose-Win (B)</td>
<td>The brand owner loses more profit and the e-tailer wins more profit in the $B$ than $E$ scenario, and the brand owner wins more profit and the e-tailer loses more profit in the $E$ than $B$ scenario. Neither member wants to open the showroom.</td>
</tr>
</tbody>
</table>

Figure 3.
and high enough platform spillover, the brand owner and e-tailer opening by himself or herself are better than waiting for the other opening.

Figure 3(b) corresponds to the case in which platform spillover is high (but not too high) such as \( r_e = 2 > 1.4286 \). We can see that if offline experience spillover effect is low and brand spillover is higher, opening showrooms by the brand owner is the optimal choice strategy for both the brand owner and e-tailer in this setting. Furthermore, compared with the \( B \) scenario, we can find that the supply chain members in the \( E \) scenario exist in Pareto improvement when the brand spillover is not high. It means that the brand owner desires the e-tailer to open showrooms and the e-tailer also is willing to do. However, with a high offline experience spillover effect, if the brand owner (e-tailer) opens showrooms under a not high enough brand spillover, it will lead to loss of the brand owner’s (e-tailer’s) profit but boost the e-tailer’s (brand owner’s) profit. Namely, neither the supply chain member at this moment wants to open showrooms. Each member as a free rider waits for the other to open and sinks into prisoner dilemma. Besides, with low offline experience spillover effect, if the brand owner (e-tailer) opens showrooms under a not too low brand spillover, it will achieve profitable growth of the brand owner (e-tailer) but take away the e-tailer’s (brand owner’s) chance to increase the profit. In other words, both supply chain members at this moment are willing to open showrooms.

Figure 3(c) describes the regions for optimal showrooms choice strategy for the case with modest platform spillover such as \( 1.25 < r_e = 1.4 < 1.4286 \). From Figure 3(c), we can see that if the brand spillover is not low, both the brand owner and e-tailer in the \( B \) scenario will get higher profits than profits in the \( E \) scenario for arbitrary offline experience spillover effect. Figure 3(c) suggests that the brand owner always should open showrooms due to the improvement of their profits in that sitting. In addition, we can find that if offline experience spillover effect is not high and brand spillover is low, the supply chain members in the \( E \) scenario have higher profits than them in the \( B \) scenario. Figure 3(c) also suggests that the brand owner wishes the e-tailer to open showrooms and the e-tailer will do. However, even if offline experience spillover effect is higher but brand spillover is low, supply chain members in the \( B \) scenario may not achieve win-win.

Figure 3(d) depicts the regions for optimal showrooms choice strategy for the case with lower platform spillover such as \( r_e = 0.04 < 1.25 \). From Figure 3(d), we have an intuitive knowledge of which opening showrooms by the brand owner is a strictly dominant strategy and the \( E \) scenario never happens. Specifically, even the brand spillover is low to tend zero, it is fine for supply chain members to open showrooms by the brand owner under certain conditions. On the contrary, when the offline experience spillover effect is not very high and brand spillover is low enough, the brand owner does not open showrooms because of the loss of profit, i.e., neither of supply chain members open showrooms.

Summarizing the above analysis of Figure 3, from Figure 3(a) to (d), the region of win-win in the \( B \) scenario tends to go up as platform spillover decrease and the region of win-win in the \( E \) scenario drops with the decrease of platform spillover. The region of win-win in the \( B \) scenario is more than the region of win-win in the \( E \) scenario. It further verifies that the motivations of the brand owner and e-tailer that we describe in the above propositions. Besides, Figure 3 also indicates that the brand spillover and platform spillover play critical roles in whether the brand owner and e-tailer open showrooms. At the same time, while offline experience spillover effect also influences optimal choice strategy but not significantly by comparing with brand spillover or platform spillover. Put differently, whatever the value of offline experience spillover effect is, the brand owner opens showrooms under high brand spillover and low platform spillover. Comparatively, if platform spillover is high and brand spillover is low, the e-tailer ought to open showrooms.
6. CONCLUSIONS

6.1. Summary

This paper investigates the choice of showrooms for a supply chain that comprises a brand owner and an e-tailer, and provides new insights for supply chain members facing consumption experience. Following industrial practice and the previous literature, we assume that the brand owner and e-tailer make showroom investment that provides a wonderful consumption experience for customers and stimulates demand within online shopping. Thus we study two scenarios the brand owner opens showrooms (i.e., B scenario) and the e-tailer opens (i.e., E scenario). We develop a framework model to analyze the strategic roles of the showrooms in different scenarios and use the multi-stage optimization technique and comparison analysis to derive and analyze the optimal decisions and choice for showroom investment.

We first provide insights into the supply chain members’ showroom investment cost and prices in different showroom investment scenarios. With the showroom investment, how to adjust investment cost, the wholesale price and e-retailing price and determine the changing intensity of decisions is an open question. Our findings suggest brand spillover, platform spillover and offline experience spillover effect are significant in the members’ decision-making. Regarding the tendency of changing, we have demonstrated that the brand owner and e-tailer will increase the wholesale price and e-retailing price when opening showrooms and have a not constant tendency in the B and E scenario. Specifically, under a lower platform spillover or a higher platform spillover and brand spillover, if the investor of showrooms changes from the e-tailer to the brand owner, the brand owner will invest more than the e-tailer and both of members will increase prices. But if the investor changes from the brand owner to the e-tailer, the e-tailer will invest less than the brand owner and both of the members will cut prices. And the contrary changing trend applies to the situation that platform spillover is not too low and brand spillover is relatively low.

In the presence of showroom investment, we examine the supply chain’s performance under different scenarios. We have identified the choice of showroom investment scenario is measured by brand spillover, platform spillover and offline experience spillover effect. This study figures out the conditions of showroom investment on which one or two parts of members and the supply chain system can derive a higher profit when investing showrooms. In total, higher brand spillover can motivate the brand owner to open showrooms but suppress the e-tailer to open, and higher platform spillover can motivate the e-tailer to open showrooms but suppress the brand owner to open.

Furthermore, we find the impact of brand spillover on the brand owner is different from the impact of platform spillover on the e-tailer. In other words, their requirement for respective spillover is not the same when opening showrooms. Specifically, whether opening showrooms is beneficial to one or two members, when other factors satisfy certain conditions, even low brand spillover tending to zero, the brand owner will still open showrooms to obtain more profit. But for the e-tailer, the same logic does not apply to the situation that platform spillover is low even tending to zero.

6.2. Managerial Implications

Due to the continuous emergence of the offline experience activities and the transformation of consumer consumption behavior, our research on the choice of showroom investment mode is valuable to the brand owner’s and e-tailer’s business mode decisions in the experience economy. Specifically, we can provide insights for the brand owner and e-tailer on whether to invest showrooms and who should open (the brand owner or e-tailer open). The results may depend on the factors such as brand spillover, platform spillover and offline experience spillover effect. It is noticeable that comparing with brand spillover or platform spillover, offline experience spillover effect is an inconspicuous factor influencing the investment scenario choice, as the effect of showrooms’ offline experience spillover effect on consumer purchase is faintly affected by changing investor.
Moreover, it is interesting that the impacts of brand spillover on the brand owner and platform spillover on the e-tailer present different trends. This indicates two members have differences in the main motivation to open showrooms, the brand owner’s is to obtain brand spillover but the e-tailer is to obtain more profit and a higher level platform spillover by the increasing demand.

Finally, the conditions of showroom investment for a single member or win-win are not unique and more complex because of the joint effect of brand spillover, platform spillover and offline experience spillover effect. The conditions of the brand owner opening are richer and more relaxed than that of the e-tailer. And the threshold value of respective spillover is required more stringently under the win-win than that just considering the single member’s profit. Regarding the inroad into showroom investment, the brand owner and e-tailer should be cognizant of the trade-off between brand spillover and platform spillover, and be active to open showrooms in some appropriate conditions and not always be a free rider. Our study reveals the optimal scenarios for the supply chain to open showrooms considering the role of showrooms, as well as the investment suggestions for the supply chain members to achieve better performance in e-tailing.

6.3. Future Research

Our study also might have some limitations. We focus on two cases—opening showrooms by the brand owner or e-tailer. In doing so, we leave out a case that both the brand owner and e-tailer open showrooms such as Apple and Amazon. This issue would be interesting to combine the two cases and discuss the choice of the showroom’s investment strategy. Moreover, the information on brand spillover and platform spillover could be asymmetric, so we may need to extend the game equilibrium under asymmetric information settings.

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APPENDIX A - DERIVATION AND PROOF OF EQUILIBRIUM SOLUTIONS UNDER DIFFERENT SCENARIOS

Proof of Lemma 1
The e-tailer maximizes his profit by determining the e-retailing price \( p \), which leads to the following first-order condition:

\[
\frac{\partial \pi^e}{\partial p} = a - 2p + \lambda \sqrt{C} + w = 0
\]  \( \text{(A.1)} \)

We have \( \frac{\partial^2 \pi^e}{\partial p^2} = -2 < 0 \), therefore, the e-tailer’s profit is concave in e-retailing price. Solving the above equation leads to Equation (6).

Proof of Proposition 1
Substituting the e-tailer’s best responses obtained from Lemma 1 in the brand owner’s profit function, we have the first-order condition of \( w \):

\[
\frac{\partial \pi^b}{\partial w} = \frac{a + \lambda \sqrt{C} - 2w}{2} = 0
\]  \( \text{(A.2)} \)

It can be shown that \( \frac{\partial^2 \pi^b}{\partial w^2} = -2 < 0 \), hence, the brand owner’s profit is concave in wholesale price. Solving the above equation leads to \( w(C) = \frac{a + \lambda \sqrt{C}}{2} \), and substituting Equation (6) and \( w(C) \) in the brand owner’s profit function, we have the first-order condition of \( C \):

\[
\frac{\partial \pi^b}{\partial C} = \frac{\sqrt{C} \lambda^2 + (a + 4\lambda) \lambda - 8\sqrt{C}}{8\sqrt{C}}
\]  \( \text{(A.3)} \)

We have \( \frac{\partial^2 \pi^b}{\partial C^2} = -\frac{(a + 4\lambda) \lambda}{16\sqrt{C^3}} < 0 \), therefore, the brand owner’s profit is concave in the showroom investment cost. Solving the above first-order condition leads to the optimal showroom investment cost \( C^B^* \), where:

\[
C^B^* = \frac{\lambda^2 (a + 4\lambda)^2}{(8 - \lambda^2)^2}
\]  \( \text{(A.4)} \)

Substituting \( C^B^* \) into \( w(C) \), we get the optimal wholesale price \( w^B^* \):
\[ w^{B^*} = \frac{2(2a + \lambda^2 r_b)}{8 - \lambda^2} \]  
(A.5)

Finally, substituting \( C^{B^*} \) and \( w^{B^*} \) into Equations (6), (2) and (3), we can gain \( p^{B^*} \), \( \pi_e^{B^*} \) and \( \pi_b^{B^*} \) as shown in Table 2. Therefore, this Proposition is proved.

**Proof of Proposition 2**

From \( w^{B^*} \), \( p^{B^*} \) and \( C^{B^*} \), we have:

\[ \frac{\partial w^{B^*}}{\partial \lambda} = \frac{8\lambda(a + 4r_b)}{(8 - \lambda^2)^2} > 0 , \quad \frac{\partial p^{B^*}}{\partial \lambda} = \frac{12\lambda(a + 4r_b)}{(8 - \lambda^2)^2} > 0 , \]
\[ \frac{\partial C^{B^*}}{\partial r_b} = \frac{8\lambda^2(a + 4r_b)}{(8 - \lambda^2)^2} > 0 . \]

Furthermore, from \( \pi_e^{B^*} \) and \( \pi_b^{B^*} \), we get:

\[ \frac{\partial \pi_e^{B^*}}{\partial \lambda} = \frac{8\lambda(a + 4r_b)(2a + \lambda^2 r_b)}{(8 - \lambda^2)^2} > 0 , \]
\[ \frac{\partial \pi_b^{B^*}}{\partial \lambda} = \frac{2\lambda(2a + \lambda^2 r_b)}{(8 - \lambda^2)^2} > 0 , \quad \frac{\partial \pi_b^{B^*}}{\partial r_b} = \frac{\lambda^2(a + 4r_b)}{8 - \lambda^2} > 0 . \] Therefore, this Proposition is proved.

**Proof of Lemma 2**

The e-tailer maximizes his profit by determining the e-retailing price \( p^* \), which leads to the following first-order condition:

\[ \frac{\partial \pi_e^E}{\partial p} = a - 2p + \lambda \sqrt{C} + w = 0 \]  
(A.6)

It is obvious that the e-tailer’s profit is concave in e-retailing price. Solving the above equation leads to Equation (7). Then, substituting (7) in the brand owner’s profit function, the brand owner chooses the wholesale price \( w^* \) to maximize his profit, which leads to the following first-order condition of \( w^* \):

\[ \frac{\partial \pi_b^E}{\partial w} = a + \lambda \sqrt{C} - 2w = 0 \]  
(A.7)

It is obvious that the brand owner’s profit is concave in wholesale price. Solving the above equation leads to Equation (13).

**Proof of Proposition 3**

Substituting (12) and (13) in the e-tailer’s profit function, we have the first-order condition of \( C^* \):
It can be shown that $\frac{\partial^2 \pi_e}{\partial C^2} = -\frac{(a + 8r_e)\lambda}{32\sqrt{C^3}} < 0$, hence, the e-tailer's profit is concave in the showroom investment cost. Solving the above first-order condition leads to the optimal showroom investment cost $C_{E^*}$, where:

$$C_{E^*} = \frac{\lambda^2(a + 8r_e)^2}{(16 - \lambda^2)^2}$$  \hspace{1cm} (A.9)

Substituting $C_{E^*}$ into (13), we have $w_{E^*} = \frac{4(a + \lambda^2r_e)}{16 - \lambda^2}$. Then, substituting $w_{E^*}$ and $C_{E^*}$, respectively, lead to $p_{E^*}$, $\pi_{eE^*}$ and $\pi_{bE^*}$ as shown in Table 3. Therefore, this Proposition is proved.

**Proof of Proposition 4**

It is easy to prove Proposition 4 by the similar to Proposition 2, so we omit it.

**APPENDIX B - PROOF OF COMPARATIVE ANALYSIS OF OPTIMAL DECISIONS AND PROFITS**

**Proof of Proposition 5**

Comparing the wholesale price, e-retailing price and showroom cost under different scenarios, we get

$$w_{B^*} - w_{N^*} = \frac{\lambda^2(a + 4r_b)}{2v} > 0$$  \hspace{1cm} (B.1)

$$w_{E^*} - w_{N^*} = \frac{\lambda^2(a + 8r_e)}{2U} > 0$$  \hspace{1cm} (B.2)

$$w_{B^*} - w_{E^*} = \frac{2\lambda^2M}{vU}$$  \hspace{1cm} (B.3)

$$p_{B^*} - p_{N^*} = \frac{3\lambda^2(a + 4r_b)}{4v} > 0$$  \hspace{1cm} (B.4)
\[ p^{E^*} - p^{N^*} = \frac{3\lambda^2(a + 8r_e)}{4U} > 0 \] (B.5)

\[ p^{B^*} - p^{E^*} = \frac{3\lambda^2 M}{vU} \] (B.6)

\[ C^{B^*} - C^{E^*} = \frac{8\lambda^2 M(2Ur_b - (a + 4r_e)\lambda^2 + 4(3a + 8r_e))}{v^2U^2} \] (B.7)

where \( M = (8 + v)r_e + 2(\lambda^2 r_e + a - 8r_e) \), \( v = 8 - \lambda^2 \), \( U = 16 - \lambda^2 \). It is obvious that if \( M \geq 0 \), then \( w^{B^*} - w^{E^*} \geq 0 \), \( p^{B^*} - p^{E^*} \geq 0 \) and \( C^{B^*} - C^{E^*} \geq 0 \). We get further that if \( r_e \geq f_0(\lambda, r_e) \) when \( \frac{a}{8} \leq r_e < \frac{a}{7} \) and \( \lambda < f_0(r_e) \) or \( r_e \geq \frac{a}{7} \) or if \( r_e < \frac{a}{8} \) or \( \frac{a}{8} \leq r_e < \frac{a}{7} \) and \( \lambda > f_0(r_e) \), then \( M \geq 0 \); if \( r_e < f_0(\lambda, r_e) \) when \( \frac{a}{8} \leq r_e < \frac{a}{7} \) and \( \lambda < f_0(r_e) \) or \( r_e \geq \frac{a}{7} \), then \( M < 0 \), where \( f_0(\lambda, r_e) = \frac{(8r_e - a - 8\lambda r_e)}{U} \). To sum it up, it is generalized that if \( r_e \geq \max\{f_0(\lambda, r_e), 0\} \), then \( w^{B^*} \geq w^{E^*} \geq w^{N^*} \), \( p^{B^*} \geq p^{E^*} \geq p^{N^*} \) and \( C^{B^*} \geq C^{E^*} \); if \( r_e < f_0(\lambda, r_e) \) when \( \lambda \leq \min\{f_0(r_e), 1\} \) and \( r_e \geq \frac{a}{8} \), then \( w^{E^*} \geq w^{B^*} \geq w^{N^*} \), \( p^{E^*} \geq p^{B^*} \geq p^{N^*} \) and \( C^{E^*} \geq C^{B^*} \). Therefore, this proposition is proved.

**Proof of Proposition 6**

The superscripts “BN”, “EN” and “BE” respectively denote the supply chain members’ profit margins between the “B” scenario and “N” scenario, “E” scenario and “N” scenario and “B” scenario and “E” scenario. The supply chain members’ profit margin is denoted as \( \Delta \pi \).

1. The brand owner’s profit margins are respectively given by

\[ \Delta \pi^{BN}_b = \pi^{B^*}_b - \pi^{N^*}_b = \frac{\lambda^2(a + 4r_e)^2}{8v} > 0 \] (B.11)

\[ \Delta \pi^{EN}_b = \pi^{E^*}_b - \pi^{N^*}_b = \frac{\lambda^2(a + 8r_e)((8r_e - a)(2\lambda^2 + 32a) + 2\lambda^2 a)}{8U^2} > 0 \] (B.12)

\[ \Delta \pi^{BE}_b = \pi^{B^*}_b - \pi^{E^*}_b = \frac{\lambda^2(2U^2r_b^2 + aU^2r_b + 8r_e^2\lambda^4 + (a^2 + 32ar_e - 64r_e^2)\lambda^2 - 256ar_e)}{vU^2} \] (B.13)
In (B.13), let \( f_1(r_b) = 2U^2r_b^2 + aU^2r_b + 8r^2_e\lambda^4 + (a^2 + 32ar_e - 64r^2_e)\lambda^2 - 256ar_e \), and we get if \( f_1(r_b) > 0 \), \( \pi^{\text{r\_um}}_b - \pi^{\text{r\_um\_r}}_b > 0 \). Then, it is obvious that if \( r_b \geq f_1(\lambda, r_e) \) when \( \lambda \leq f_1(r_e) \) and \( r_e \leq \frac{15\sqrt{14} - 56}{28} a \) or \( r_e > \frac{15\sqrt{14} - 56}{28} a \), or if \( r_e \leq \frac{15\sqrt{14} - 56}{28} a \) and \( \lambda > f_1(r_e) \), we obtain that \( \pi^{\text{r\_um}}_b - \pi^{\text{r\_um\_r}}_b > 0 \); if \( r_b < f_1(\lambda, r_e) \) when \( \lambda \leq f_1(r_e) \) and \( r_e \leq \frac{15\sqrt{14} - 56}{28} a \) or \( r_e > \frac{15\sqrt{14} - 56}{28} a \), we get \( \pi^{\text{r\_um}}_b - \pi^{\text{r\_um\_r}}_b < 0 \), where

\[
f_1(\lambda, r_e) = \sqrt{v(a + 8r_e)((16 + U)a + 8\lambda^2r_e)} - aU
\]

(B.14)

\[
f_1(r_e) = \frac{\sqrt{(16r_e + \sqrt{m})(a + 8r_e) - m}}{4r_e}
\]

(B.15)

\[m = a^2 + 48ar_e + 64r^2_e
\]

(B.16)

(2) The e-tailer’s profit margins are respectively given by

\[
\Delta\pi^{\text{BN}}_e = \pi^{\text{r\_um\_r}}_e - \pi^{\text{N\_r}}_e = \frac{\lambda^2(a + 4r_e)(16a + (4r_e - a)\lambda^2)}{8v^2} > 0
\]

(B.17)

\[
\Delta\pi^{\text{EN}}_e = \pi^{\text{r\_um\_r}}_e - \pi^{\text{N\_r}}_e = \frac{\lambda^2(8r_e)^2}{16U} > 0
\]

(B.18)

\[
\Delta\pi^{\text{BE}}_e = \pi^{\text{r\_um\_r}}_e - \pi^{\text{N\_r}}_e = \frac{\lambda^2(\lambda^2Ur_b^2 + 4aUUr_b - ((a + 4r_e)\lambda^2 - 4(3a + 8r_e)))(\lambda^2r_e + a - 8r_e)}{vU^2}
\]

(B.19)

In (B.19), let \( f_2(r_b) = \lambda^2Ur_b^2 + 4aUUr_b - ((a + 4r_e)\lambda^2 - 4(3a + 8r_e))(\lambda^2r_e + a - 8r_e) \), and we get if \( f_2(r_b) > 0 \), \( \pi^{\text{r\_um\_r}}_e - \pi^{\text{N\_r}}_e > 0 \); if \( f_2(r_b) < 0 \), \( \pi^{\text{r\_um\_r}}_e - \pi^{\text{N\_r}}_e < 0 \). Then, we can get the conditions that \( \pi^{\text{r\_um\_r}}_e - \pi^{\text{N\_r}}_e > 0 \) are given by

(i) if \( r_e < \frac{a}{8} \) or \( \frac{a}{8} \leq r_e < \frac{a}{7} \) and \( \lambda > f_0(r_e) \), \( \pi^{\text{r\_um\_r}}_e - \pi^{\text{N\_r}}_e > 0 \);

(ii) if \( r_b > f_2(\lambda, r_e) \), when \( \frac{a}{8} \leq r_e < \frac{a}{7} \) and \( \lambda \leq f_0(r_e) \) or \( r_e \geq \frac{a}{7} \), \( \pi^{\text{r\_um\_r}}_e - \pi^{\text{N\_r}}_e > 0 \);
It is obvious that if \( r_b \leq f_2(\lambda, r_e) \) when \( \lambda \leq f_0(r_e) \) and \( \frac{a}{8} \leq r_e < \frac{a}{r} \) or \( r_e \geq \frac{a}{r} \), we obtain that \( \pi^B_s - \pi^E_s < 0 \).

Therefore, Proposition 6 is proved.

**Proof of Proposition 7**

The supply chain’s profit margins are respectively given by

\[
\Delta \pi^s_{BN} = \pi^B_s - \pi^E_s = \frac{\lambda^2(a + 4r_e)(4(16 - \lambda^2)r_b^2 + (32 - 3\lambda^2)a)}{16v^2} > 0
\]  

(B.8)

\[
\Delta \pi^s_{EN} = \pi^B_s - \pi^E_s = \frac{\lambda^2(a + 8r_e)((8\lambda^2 + 128)r_e + a(80 - 3\lambda^2))}{16U^2} > 0
\]  

(B.9)

\[
\Delta \pi^s_{BE} = \pi^B_s - \pi^E_s = \frac{\lambda^2(U^2r_b^2 + a(12 - \lambda^2)U^2r_b^2 - 4Uv^2r_e^2 - 4v^2(48 - \lambda^2)r_e + 4a^2(48 - \lambda^2))}{v^2U^2}
\]  

(B.10)

In (B.10), let \( f_3(r_b) = U^2r_b^2 + a(12 - \lambda^2)U^2r_b^2 - 4Uv^2r_e^2 - 4v^2(48 - \lambda^2)r_e + 4a^2(48 - \lambda^2) \), and we get if \( f_3(r_b) > 0 \), \( \pi^B_s - \pi^E_s > 0 \). Then, it is obvious that if \( r_e < f_2(\lambda) \) we obtain that \( \pi^B_s - \pi^E_s > 0 \); if \( r_e \geq f_2(\lambda) \) when \( r_b \geq f_3(\lambda, r_e) \), we get \( \pi^B_s - \pi^E_s > 0 \), when \( r_b \geq f_3(\lambda, r_e) \), we get \( \pi^B_s - \pi^E_s < 0 \).

**Proof of Proposition 8**

To give a comprehensive discussion of conditions of \( \pi^B_s - \pi^E_s \geq 0 \) and \( \pi^B_e - \pi^E_e \geq 0 \) given in Proposition 6, we get that if \( r_e < \frac{a}{8} \), \( \lambda \leq \min\{f_i(r_e), 1\} \), \( r_b \geq \max\{f_i(\lambda, r_e), 0\} \) or \( \frac{a}{8} \leq r_e < \frac{a}{7} \), \( \lambda \leq \min\{f_i(r_e), f_i(\lambda, r_e), 0\} \) or \( \frac{a}{7} \leq r_e \), \( \lambda \leq f_i(r_e) \), \( r_b \geq \max\{f_i(\lambda, r_e), f_i(\lambda, r_e), 0\} \), then \( \pi^B_s \geq \pi^E_s \) and \( \pi^B_e \geq \pi^E_e \). This is simplified to if \( r_b \geq \max\{f_i(\lambda, r_e), f_i(\lambda, r_e), 0\} \), \( \pi^B_s \geq \pi^E_s \) and \( \pi^B_e \geq \pi^E_e \). Thus, this Proposition is proved.

**Proof of Proposition 9**

To give a comprehensive discussion of conditions of \( \pi^B_s - \pi^E_s < 0 \) and \( \pi^B_e - \pi^E_e < 0 \) given in Proposition 6, we obtain that if \( r_b < \min\{f_i(\lambda, r_e), f_i(\lambda, r_e)\} \) when \( \lambda \leq f_0(r_e) \) and \( \frac{a}{8} \leq r_e < \frac{a}{7} \) or \( r_e \geq \frac{a}{7} \), then \( \pi^B_s \geq \pi^E_s \) and \( \pi^B_e \geq \pi^E_e \). This is simplified to if \( r_b < \min\{f_i(\lambda, r_e), f_i(\lambda, r_e)\} \) when \( \lambda \leq \min\{f_i(r_e), 1\} \) and \( r_e \geq \frac{a}{8} \), \( \pi^E_s \geq \pi^B_s \) and \( \pi^E_e \geq \pi^B_e \). Thus, this Proposition is proved.