ABSTRACT

Aiming at the problems existing in the optimal allocation of financial resources, this paper establishes an optimization model and calculates the optimal allocation coefficient. With the help of Markowitz’s investment theory, two indicators, which are investment risk and return rate, are analyzed quantitatively. First, by analyzing the allocation efficiency and risk of financial resources, the allocation efficiency model is established, and the problem is decomposed into a finite 0-1 programming problem, which is solved by Hungarian Method. Secondly, considering the minimum allocation risk and the expected maximum return, the multi-objective model is solved by progressive optimal algorithm. The model reflects both unsatisfaction and risk avoidance which are the two characteristics of rational investment behavior. The analysis shows that the model has strong applicability and can be expected to improve the allocation efficiency of financial resources and reduce the allocation risk.

KEYWORDS

Allocation, Financial Resources, Model, Return Rate, Risk

1. INTRODUCTION

The allocation efficiency of financial resource is of great strategic significance to economic growth and industrial upgrading. For nearly a century, the research on the allocation efficiency of financial resource has never been interrupted, and great progress has been made in both theoretical and empirical research, which is deeply concerned by He (2019). Some researchers mainly analyze the allocation efficiency of regional financial resources theoretically (Zhao and Song, 2019; Qian, 2019; Zhao and Huang, 2019; Yu, 2019; Zhang and Lin, 2019; XI and Du, 2019), and combining with regional situation they made empirical research by applying data. Li and Yang (2013) studied the optimal distribution model of resources in the fields of loans, securities, insurance policies, individuals, enterprises and government departments. Feng and Yang (2018) pointed out the economic effect of the allocation efficiency of financial resource. Hu (2016) summarized the researching development of domestic and foreign scholars on the allocation of financial resources, most of the existing studies mainly focus on the theory of financial resource allocation and the empirical research of allocation efficiency. In them the results of theoretical research mainly include the concept and attribute of
financial resources, as well as the object, target, scope and mechanism of financial resource allocation. The empirical research results mainly include setting up the index evaluation system of financial resource allocation efficiency, evaluating the allocation efficiency of financial resources for countries, empirically analyzing the relationship between regional financial development and economic growth so as to test the coordination between finance and economy. These theoretical and empirical studies have promoted the theory of financial resource allocation to be perfected and developed continuously in the direction of systematization. In addition, combining with a large number of references, it is not difficult to find that there are the following disadvantages in the current research: firstly, there are more qualitative analysis, but less quantitative analysis. Although there are more empirical studies, they mainly rely on the obtained data for verification, and most of them are regional resource allocation studies, and there are less studies on establishment of mathematical models, design algorithms, and quantitative research; secondly, there are more empirical analysis on the allocation efficiency of micro financial resource, and less research on the macro financial resource allocation efficiency: Thirdly, in terms of index system, it is urgent to combine with the background of the financial era and keep pace with the times in order to establish and perfect evaluation index system and measurement system of financial resources allocation efficiency; Fourthly, in regional research, few people have conducted a comprehensive and systematic study on the problem of regional financial resource allocation, and in the meantime, it is not comprehensive to analyze the cross-sectional data within a year or for a province. In view of the first disadvantage, this paper makes a quantitative analysis on the risk and return rate of investors, establishes a model and applies the optimization algorithm to solve the problem.

1.1 Efficiency and Risk of Financial Resource Allocation

With the increase of some uncertain factors such as novel coronavirus, the risk of financial resources allocation is emerging. How to effectively avoid the risk of financial resources allocation, optimize the allocation of financial resources and improve the efficiency of financial resources allocation is an important topic. The government allocates financial resources to individuals, enterprises and departments in order to enable them to obtain greater economic returns, which is directly proportional to the efficiency of financial resources. Before investing in individuals, enterprises and departments, the government should evaluate and predict their return in a certain period of time. The expected rate of return is often used to describe this kind of return, and the risk factors should be considered at the same time. Therefore, the expected rate of return and risk of financial resource allocation are important index for investment analysis (Feng and Yang, 2018).

1.2 Efficiency of Financial Resource Allocation

Roland. Robinson and Dwayne. Whiteman pointed out that financial efficiency can be divided into operational efficiency and allocation efficiency. In the process of financial operation, the ratio of cost to income is defined as the operational efficiency, while the allocation efficiency represents the effectiveness of guiding the funds from savings to production. The latter is often regarded as an index to evaluate the effectiveness of the market. It is considered that the efficiency of financial resources allocation reflects the interaction degree of the input proportion of various elements of financial resources and their relationship for improving economic output. R. I. Robinson and Dwayne Wrightsman pointed out that the essence of financial resource allocation efficiency refers to the efficiency of economic returns produced by financial resources allocated to different industries and fields(Wang, 2013).

The efficiency of financial resource allocation is reflected by the economic returns of the financial resource allocation industry, which is referred to as the return rate of allocation. Its size can show the advantages and disadvantages of financial resource allocation portfolio, so it can be used as a reference for the government to make decisions on financial resource allocation portfolio (Li and Yang, 2013). The efficiency of financial resource allocation is calculated by comparing the total allocation income with the total allocation cost. The calculation formula is as follows:
Allocation return rate = \( \frac{\text{Total allocation return - Total allocation cost}}{\text{Total allocation cost}} \times 100\% \)

In \( m \) periods, suppose the allocation return rates of \( i \)th industry are respectively \( r_{i1}, r_{i2}, \ldots, r_{im} \), applying the exponential smoothing method, the expected return rate of the \( i \)th industry is \( u_i (i = 1, 2, \cdots, n) \). The allocation weight of the industry is \( x_i (i = 1, 2, \cdots, n) \). Then the expected return rate of the financial resource allocation portfolio is \( U = \sum_{i=1}^{n} x_i u_i (i = 1, 2, \cdots, n) \).

### 1.3 Risk of Financial Resource Allocation

The essence of finance is the inter-temporal allocation of scarce resources under uncertain conditions. These uncertainties caused by uncertain conditions is a kind of risk. It can be said that the relationship between finance and risk is inherent and inseparable (Zhang, 2019). Because the allocation of financial resources is affected by many uncertain factors, it may not only bring returns to the allocation of financial resources, but also bring unexpected losses to the allocation of financial resources, which is the risk of financial resources allocation. It is the possible change range of the future returns of financial resource allocation, and it is also the possible deviation degree of expected return rate of financial resource allocation. The risk of financial resources allocation can be divided into two categories: systemic risk and non-systemic risk. Systemic risk refers to the possible changes of allocation returns caused by some global factors, which cannot be dispersed by diversified allocation. Non-systemic risk refers to the one that has an impact on certain industries or individual industry, which can be reduced through diversified allocation (Li and Yang, 2013). The risk of financial resource allocation portfolio is defined as follows:

\[
\sigma = \max \{ x_i \sigma_i \} (i = 1, 2, \cdots, n)
\]

In the following formula, \( \sigma_i^2 \) is the variance of the \( i \)th industry, which is to estimate the size of risk by applying the variance between the return rate and the expected return rate. The calculation is as follows:

\[
\sigma_i^2 = \frac{1}{m-1} \sum_{j=1}^{m} (r_{ij} - u_i)^2
\]

### 2. OPTIMIZATION MODEL AND ALGORITHM OF ALLOCATION EFFICIENCY

#### 2.1 Modeling

Suppose there are \( n \) industries that need to allocate financial resources, The allocation weight of the \( i \)th industry is \( x_i (i = 1, 2, \cdots, n) \). The expected return rate is \( u_i (i = 1, 2, \cdots, n) \). The goal is \( \max U = \sum_{i=1}^{n} x_i u_i \), thus, the optimal allocation coefficient of each industry is determined through maximizing \( U \). In addition, suppose the total amount of financial resources allocated is \( C \) (Let \( C \) be an integer). the number of financial resources allocated to the \( i \)th industries is \( x_i C \), that is \( X_i \).
\[ X_i = x_i C \left( i = 1, 2, \cdots, n \right) \]

Next, through the establishment of an optimization model, the allocation coefficient of each industry is determined by calculating the optimal allocation of resources \( X_1, X_2, \cdots, X_n \). The formula is as follows:

\[
\max U = \sum_{i=1}^{n} x_i u_i = \frac{1}{C} \sum_{i=1}^{n} X_i u_i
\]

Thus \( \sum_{i=1}^{n} x_i u_i \) is maximized and transformed into \( \sum_{i=1}^{n} X_i u_i \) maximization. In addition, the relationship between the allocation quantity and the allocation coefficient of various industries can also be obtained:

\[
C\sum_{i=1}^{n} x_i u_i = \sum_{i=1}^{n} X_i u_i
\]

Constructing the financial resource allocation matrix:

\[
A = \begin{pmatrix}
    a_{01} & a_{02} & \cdots & a_{0n} \\
    a_{X_1,1} & a_{X_1,2} & \cdots & a_{X_1,n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{X_n,1} & a_{X_n,2} & \cdots & a_{X_n,n}
\end{pmatrix}
\]

where, \( a_{X_j} \left( 0 \leq X_i \leq C, 1 \leq j \leq n \right) \) is the returns produced through financial resources \( X_i \) allocated to the \( j \)th industry, that is, subscript \( X_i \) is the amount of financial resources allocated, subscript \( j \) is the industry allocated, and \( a_{X_j} \left( 0 \leq X_i \leq C, 1 \leq j \leq n \right) \) is the return after allocation. Obviously, each feasible allocation scheme is to divide the total amount of resources into \( n \) parts and allocate them to \( n \) industries. Therefore, it can be represented by an \( n \)-dimensional vector \( X = \left( X_1, X_2, \cdots, X_n \right) \), which meets the following conditions:

\[ X_1 + X_2 + \cdots + X_n = C \]

where:

\[ 0 \leq X_j \leq C \left( j = 1, 2, \cdots, n \right) \]

Thus, the allocation matrix of financial resources represents the returns produced by different amount of financial resources allocation to different industries.
Therefore, all feasible allocation schemes of the problem are transformed into the following partition problems:

\[
\begin{align*}
X_1 + X_2 + \cdots + X_n &= C \\
0 &\leq X_j \leq C \ (j = 1, 2, \cdots, n)
\end{align*}
\]  

(2)

Therefore, the following partition problems are considered first:

\[
\begin{align*}
X_1 + X_2 + \cdots + X_n &= C \\
1 &\leq X_j \leq C \ (j = 1, 2, \cdots, n)
\end{align*}
\]  

(3)

The partition number of the problem (3) is defined as dividing \( C \) into \( n \) parts (\( X_1, X_2, \cdots, X_n \)). Only when two sets of \( X_1, X_2, \cdots, X_n \) are different the partition number is different, not depending on their order. Let \( P(C,n) \) represent the partition number of \( C \) into \( n \) parts. \( P(C,n) \) meets the following properties:

**Theorem 1:** \( P(C,n) \) meets the recurrence relation

\[
P(C,1) + P(C,2) + \cdots + P(C,k) = P(C+k,k)
\]

and \( P(C,1) = P(C,C) = 1 \) (if \( n > C \), then \( P(C,n) = 0 \)) (Zhao, 2011).

If the partition number of the problem (2) is \( P(C/n) \), then the partition number of problems (2) and (3) meets the following relationship:

**Theorem 2:** \( P(C,n) \) meets the recurrence relation

\[
P(C/n) = P(C,1) + P(C,2) + \cdots + P(C,n)^{[13]}.
\]

Obviously, each partition corresponds to a feasible allocation scheme of financial resource allocation problems, which corresponds to an assignment problem of maximum objective function. By solving the corresponding assignment problem, we can compare the maximum matching algorithm. Finally, we can find the optimal solution of the problem by comparing their objective function values after calculation.

If it is divided into \( X = (X_1, X_2, \cdots, X_n) \), then the corresponding allocation matrix of financial resources from \( A \) is as follows:

\[
A = \begin{pmatrix}
a_{X_1,1} & a_{X_1,2} & \cdots & a_{X_1,n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{X_n,1} & a_{X_n,2} & \cdots & a_{X_n,n}
\end{pmatrix}
\]

The existing Hungarian Method is adopted to solve the \( P(C/n) \) divided assignment problems, and the optimal solution corresponding to the maximum objective function value is the solution of the original resource allocation problem, and the optimal objective function values of the two solutions are equal (Bao, 2017).
2.2 Model Algorithm

**Step 1:** We calculate the partition number \( P(C/n) \) and the corresponding partition schemes, and then transfer to step 2.

**Step 2:** Concerning \( P(C/n) \) partitions we get the corresponding \( P(C/n) \) allocation problem according to the allocation matrix \( A \) of financial resources, and transfer to step 3.

**Step 3:** We apply Hungarian Method to obtain the optimal objective function value of each allocation problem, and transfer to step 4.

**Step 4:** Comparing the optimal objective function values of \( P(C/n) \) allocation problems. The optimal solution of the allocation problem the maximum value corresponds to is the solution of the allocation problem of financial resources. Transfer to step 5.

**Step 5:** Using the relationship between the allocation amount and the allocation coefficient of each industry:

\[
C = X \sum_{i=1}^{n} x_i u_i
\]

and calculate the optimal allocation coefficient of each industry, and then stop.

2.3 Algorithm Evaluation

In addition to the more complex linear programming method, the dynamic programming method should be adopted to solve the problem (the relevant research has not yet been consulted), and the time factor can be artificially introduced to treat algorithm as a multi-stage decision-making problem by stages. However, its disadvantage is that there is a certain computational complexity, especially when \( n \) is much larger it is more difficult to get the solution. In this model and algorithm, the problem is decomposed into a finite number of 0-1 programming problems and is solved by Hungarian Method. In comparison, the model can be applied in a wide range, and the objective function can be linear function or nonlinear function. Secondly, it has a relative standard and feasible solution process, which is convenient to operate on the computer.

3. MULTI-OBJECTIVE ALLOCATION MODEL AND ALGORITHM

3.1 Modeling

In the actual allocation of financial resources, allocators always weigh risks and return. The above-mentioned efficiency optimization model and algorithm does not consider the allocation risk. Therefore, under the premise of the minimum allocation risk \( \sigma \), the expected return \( U \) is the maximum value, which is more in line with the actual allocation of financial resources. The linear programming model is given by Li(2013):

\[
\begin{align*}
\max U &= \sum_{i=1}^{n} x_i u_i \\
\min \sigma &= \max_{1 \leq \sigma \leq a} \left\{ x_i \sigma_i \right\} \\
\sum_{i=1}^{n} x_i &= 1, x_i \geq 0, i = 1, 2 \ldots, n
\end{align*}
\]

The solution is to assign weights \( \lambda \) and \( 1 - \lambda (0 \leq \lambda \leq 1) \) to the allocation risk \( \sigma \) and expected return \( U \) of the objective function, and the model is converted into a single objective model. It is pointed out that the weighted coefficient \( \lambda \) reflects the risk aversion degree of the allocator, which is decided by the allocator in actual calculation. There must be full of some subjectivity. In this paper,
we take the step-by-step optimization method to study the model of maximum expected return $U$ under the premise of minimum allocation risk $\sigma$.

Firstly, the risk matrix $B$ of financial resource allocation is constructed, which represents the risk situation caused by allocating different number of financial resources to different industries:

$$
B = \begin{pmatrix}
\sigma_{X,1} & \sigma_{X,2} & \cdots & \sigma_{X,n} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{X,n} & \sigma_{X,2} & \cdots & \sigma_{X,n} 
\end{pmatrix}
$$

Among them, $\sigma_{X,j} \left(0 \leq X_i \leq C, 1 \leq j \leq n\right)$ represents the allocation risk of allocating financial resources $X_i$ to the $j^{th}$ industry. It is easy to conclude that the optimal solution of model (4) is equivalent to finding an element in different rows and columns of matrix $B$ and firstly making the largest one the smallest and then the maximizing the sum.

In matrix (1), $e = \max_{1 \leq i \leq m, 1 \leq j \leq n} \{a_{i,j}\}$ and $c_{i,j} = e - a_{i,j} \left(1 \leq i \leq m, 1 \leq j \leq n\right)$, then the maximum goal of financial resource allocation return will be transformed into the minimum goal, and it is met that the sum of maximum and minimum is equal to ne (Bao, 2017), that is:

$$
\begin{align*}
\min f &= \sum_{i=1}^{n} c_{i,j}u_j \\
\min \sigma &= \max_{1 \leq i \leq n} \{c_{i,j}\} \\
\sum_{i=1}^{n} c_{i,j} &= 1, c_{i,j} \geq 0, i = 1,2,\cdots,n
\end{align*}
\tag{5}
$$

**3.2 Model Algorithm**

**3.2.1 The Idea of Progressive Optimal Algorithm**

The algorithm for solving single objective risk minimization can find the optimal solution of the maximum risk minimization, but it does not necessarily make the total return reach the optimization. The progressive optimal algorithm first finds the maximum risk minimization of the model, that is, the optimal solution $f_i$ of the goal $\min \sigma$, then there must be $n$ independent elements in $B$ that are not greater than $f_i$, thus the algorithm is realized as long as their sum is the minimum (Huang and Ding, 2016; Li, 2018; Han and Guo, 2017). First, the definition of independent zero element of (row) column is given.

We subtract the minimum element in the (row) column respectively from each (row) column of financial resource allocation risk matrix $B$ and then obtain a new risk matrix $B'$. Then, a zero element is selected in each (row) column, which is called independent zero element of (row) column.

**3.2.2 Progressive Optimal Algorithm**

**Step1:** Finding the allocation with the minimal risk $\sigma \cdot f_i = \max \left\{A_jB_j \mid i, j = 1,2,\cdots,n\right\}$, among them $A_i = \min_{j \in \{1,2,\cdots,n\}} \left\{\sigma_{i,j} \left(i = 1,2,\cdots,n\right)\right\}$, $B_j = \min_{i \in \{1,2,\cdots,n\}} \left\{\sigma_{i,j} \left(j = 1,2,\cdots,n\right)\right\}$. 
Step 2: Changing all the elements which are not greater than $f_i$ in risk matrix $B$ to 0 and record them as matrix $B(f_i)$.

Step 3: If the zero element in a certain row (column) of matrix $B(f_i)$ is independent for both the row and the column, it will be selected and its row and column will be crossed out too. If there are multiple independent zeros in a certain column (row) of $B(f_i)$, we find their secondary minimal elements in their rows (columns), and select the independent zero in the row (column) with the largest secondary minimum value, and cross out the row and column where it is located. If there is more than one of the above situations, the independent zeros in the row or column with the largest secondary minimal element are assigned in priority order. If there are not independent zero elements in all the rows (columns) in $B(f_i)$, a zero in the row (column) with the least zero elements is selected arbitrarily.

Step 4: If $n$ independent zero elements are selected, their corresponding variables are 1 and other variables are 0, then the optimal assignment solution is obtained. otherwise, transfer to step 5.

Step 5: We let $f_2 = \min \left\{ \sigma_j \mid \sigma_j > f_i, i, j = 1, 2, \ldots, n \right\}$, replace $f_i$ with $f_2$, and transfer to step 2.

3.3 Algorithm Analysis

Multi-objective optimization is one of the difficulties in combinatorial optimization. By analyzing and transforming the multi-objective mathematical programming model of assignment problem within the shortest time, risk matrix of financial resource allocation is introduced. The goal of maximizing the return of financial resource allocation is transformed into the minimizing goal by technical means. Combines minimum of the maximum risk of the model with the expected returns to gradually optimize. The algorithm is implemented on the risk matrix, and an initially feasible solution of the problem can be obtained after $n$ iterations at most. Compared with the conventional algorithm, the algorithm has the advantages of less computation, simple operation and easy implementation with computer. In addition, the model does not give weight $\lambda$ and $1 - \lambda (0 \leq \lambda \leq 1)$ to the objective function allocation risk $\sigma$ and expected returns $U$, which overcomes the subjective factors in the allocation of financial resources and makes the allocation of resources more objective and accurate.

4. EXAMPLE

The example in reference (Bao, 2017) is applied to conduct the optimization calculation by using the method proposed in this paper.

Five types of enterprises are selected, such as sole proprietorship economy, private economy, joint-stock cooperation economy, joint-venture economy and public ownership economy. The expected return rate and risk are shown in Table 1.

<table>
<thead>
<tr>
<th>Type Of Enterprise</th>
<th>Sole Proprietorship Economy</th>
<th>Private Economy</th>
<th>Joint-Stock Cooperation Economy</th>
<th>Joint-Venture Economy</th>
<th>Public Ownership Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>expected return rate</td>
<td>0.28</td>
<td>0.21</td>
<td>0.23</td>
<td>0.25</td>
<td>0.19</td>
</tr>
<tr>
<td>risk</td>
<td>0.12</td>
<td>0.09</td>
<td>0.10</td>
<td>0.11</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Establish the optimal allocation model:

\[
\begin{align*}
\min f &= 0.28c_1 + 0.21c_2 + 0.23c_3 + 0.25c_4 + 0.19c_5 \\
\min \sigma &= \max \left\{ 0.12c_1, 0.09c_2, 0.10c_3, 0.11c_4, 0.07c_5 \right\} \\
\text{s.t.} & \sum_{i=1}^{n} c_i = 1, c_i \geq 0, i = 1, 2, \ldots, n
\end{align*}
\]

Stepwise optimization algorithm is adopted to obtain:

\[c_1 = 0.0992, c_2 = 0.1981, c_3 = 0.2062, c_4 = 0.1983, c_5 = 0.2982, \sigma = 0.0198\]

The results show that the method in this paper is better than the one in the reference (Bao, 2017), and the risk is less.

5. CONCLUSION

The process of economic globalization and financial globalization is accelerating, and financial resources have become the core elements of economic development. In recent years, the number of financial resources in China has been increasing, but there are some problems, such as low allocation efficiency, insufficient effective investment of enterprises and overcapacity. Therefore, it is necessary to rearrange, integrate and reasonably allocate limited financial resources and improve the efficiency of financial operation, so as to improve the level and quality of economic development. Thus, it is of practical significance to establish a mathematical model and study the maximum expected returns \(U\) under the premise of minimum allocation risk \(\sigma\).

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REFERENCES


