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The Edgeworth Cube:
An Economic Model for Social Peace

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ABSTRACT

Social peace is an asset to every society. Its absence endangers the well-being and the safety of the population and the stability of states. In order to better understand the interdependencies of poverty, social peace and migration pressure the authors introduce the model of the "Edgeworth-Cube" which is an extension of the classical Edgeworth Box model by one dimension. This new dimension can either be interpreted as "aggression" (which reduces "social peace" for others) or as "migration pressure" (which results from a worldwide heterogeneous distribution of wealth), and this new dimension is modelled as a non-budget-constrained unilateral immaterial good. The "Edgeworth-Cube" also differentiates vital (essential) goods from normal (non-essential) goods. By focusing on extremely imbalanced endowments and by formal mathematical modeling the authors show in their approach that applying behavioral pressure (i.e. aggression or migration pressure) has an existential economic value for the poor on the one hand. On the other hand, the authors show that transfer payments have a systemically limited potential to keep aggression and migration pressure at bay.

KEYWORDS

Aggression, Conflict Economics, JEL Classification Codes: D51, D61, D62, H53, H55, K37, K42, Pareto Efficient Allocations, Social Peace, Transfers

1. INTRODUCTION

1.1. Motivation

Social peace is a desirable state of a social community[1]. In a peaceful environment people can enjoy their lives, societies can prosper, and economies can grow. The absence of peace endangers lives of individuals as well as democratic systems of social co-existence and decision making.

Still different levels of social peace can be observed in different socio-economic environments. Why are some societies able to maintain a desirable level of social peace, whereas in others social peace is less developed? In other words – which mechanisms secure and endanger social peace? Which mechanisms lead to aggressive behavior of parts of a society? And which mechanisms lead to migration pressure and thus actuate aggressive behavior of other parts of a society?

An understanding of these mechanisms is important for solving intra-societal problems (e.g. strikes, riots of the underprivileged, crime level etc.), as well as for inter-societal problems (especially migration issues).

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Leaving out all psychological aspects (such as e.g. pleasure in aggression or envy) and sociological aspects (such as e.g. orchestrated manipulation, purposefully misconducted religious beliefs or mass hysteria) the authors want to introduce an extension of the well-known Edgeworth Box model which solely uses economic mechanisms to explain social peace (and aggressive behavior which endangers a peaceful intra societal co-existence). The authors are aware that their model is a significant simplification of the real world, but they also consider that this reduction helps to understand hidden mechanisms more precisely.

1.2. The Problem

The different allocation of goods to different players[2] constitutes the basis for economic interactions. If one wants to understand why and to which extend economic transactions happen, the "Edgeworth Box" (see e.g. Pfingsten (1989), pp.120f) is an established basic model for a two-player-two-goods economy. It mathematically explains why "the basis of arbitration between contractors is the greatest possible utility of all concerned" (Edgeworth 1881, p vi). But what happens, if the original endowments of the players are significantly unbalanced and threaten the existence of one player? I.e. what happens, if one player owns almost all the goods and the other player owns almost none?

In other words, the research shall help to understand the nature of the micro-economic mechanisms that lead to aggression by the poor and how aggression-counterbalancing options work from a sheer economic point of view.

1.3. Structure of this Publication

Section two is a brief overview on the state of science with focus on the Edgeworth Box. In section three the authors develop the extension of the Edgeworth Box towards the Edgeworth Cube and discuss the relevant details. Section four translates the mathematical findings into real world observations. In section 5 the authors sum up the results, discuss them critically and show the need for further research.

2. STATE OF SCIENCE

The Edgeworth Box model assumes rationality[3] in the choice of goods and competitive behavior of players. Let's briefly summarize the model of the Edgeworth Box in order to introduce a model notation which we'll use and expand in section 3.

2.1. The Edgeworth Box With Two Goods

The model focusses on an economy of two players, A and B, which consume two goods, 1 and 2. Let's also take two essential assumptions as mentioned by Breyer (2008, p. 195): Both players behave as utility maximizing individuals as well as price takers and quantity adjusters. The first assumption also implies rationality. According to Bisin (2011, p. 61) a player will behave rationally, if he takes the best bundle of goods which is affordable for him. Best bundle means a choice based on the consumer's preferences. Affordable bundle means a choice determined by the consumer's budget constraint.

Let's use the following notation[4]: Player A's demand of good 1 and 2 is called x_1^A, x_2^A, and player B's x_1^B, x_2^B. The amounts of goods are infinitely divisible and have to be non-negative, i.e. $x_i^j \geq 0$ for $i \in \{1,2\}$ and $j \in \{A,B\}$. Moreover, the players own an initial endowment $\omega_i^j \geq 0$ for $i \in \{1,2\}$ and $j \in \{A,B\}$ which can be aggregated to the total supply of goods ω_1 and ω_2. The distribution of the initial endowment is shown in (1):

$$\omega_1 = \omega_1^A + \omega_1^B \text{ and } \omega_2 = \omega_2^A + \omega_2^B \tag{1}$$

The initial endowment is valued by the prices $p_1, p_2 \geq 0$. Moreover, the model only consists of one period. The preferences of the consumers are given by utility functions in (2):

$$\boldsymbol{u}^{\boldsymbol{A}}\left(\boldsymbol{x}_1^{\boldsymbol{A}}, \boldsymbol{x}_2^{\boldsymbol{A}}\right) \text{ and } \boldsymbol{u}^{\boldsymbol{B}}\left(\boldsymbol{x}_1^{\boldsymbol{B}}, \boldsymbol{x}_2^{\boldsymbol{B}}\right) \tag{2}$$

In general, the authors assume a decreasing marginal utility (3). That means with increasing consumption of a good the additional utility of each further unit of this good is decreasing:

$$\frac{\partial \boldsymbol{u}^{\boldsymbol{j}}}{\partial \boldsymbol{x}_{\boldsymbol{i}}^{\boldsymbol{j}}} > 0 \text{ and } \frac{\partial^2 \boldsymbol{u}^{\boldsymbol{j}}}{\partial \boldsymbol{x}_{\boldsymbol{i}}^{\boldsymbol{j}2}} < 0 \text{ for } \boldsymbol{i} \in \{1,2\}, \boldsymbol{j} \in \{\boldsymbol{A}, \boldsymbol{B}\} \tag{3}$$

2.2. Optimization Problem and Results

According to (4) every player maximizes his utility function by choosing amounts of good 1 and 2. At the same time consumption is restricted by his budget constraint (5). That means that the value of demand (or expenditures for consumption) on the right side of (5) is limited to the value of initial endowment (or income m^j for $j \in \{A, B\}$) on the left side of (5). By trading goods the two players can improve their utility:

$$\max_{x_1^j, x_2^j} \boldsymbol{u}^{\boldsymbol{j}}\left(\boldsymbol{x}_1^{\boldsymbol{j}}, \boldsymbol{x}_2^{\boldsymbol{j}}\right) \text{for } \boldsymbol{j} \in \{\boldsymbol{A}, \boldsymbol{B}\} \tag{4}$$

$$s.t.\ p_1\omega_1^j + p_2\omega_2^j \geq p_1x_1^j + p_2x_2^j\ ;\ x_1^j, x_2^j \geq 0 \text{ for } j \in \{A, B\} \tag{5}$$

An interior solution of this optimization problem is given, if the utility function is strictly concave, the budget constraint is binding and the consumed amounts of good 1 and 2 are not negative.

Using the first order conditions one gets a central result in (6): Every player fits his marginal rate of substitution[5] (MRS) to the relative price relationship on competitive markets. According to Varian (2011, p. 24), the price quotient signals the opportunity costs of additional consumption of good 1 measured in units of good 2:d social peace for player B due to aggre

$$\underbrace{\frac{\partial u^j\left(x_1^j, x_2^j\right) / \partial x_1^j}{\partial u^j\left(x_1^j, x_2^j\right) / \partial x_2^j}}_{=\left|MRS_{1,2}^j\right|} = \frac{p_1}{p_2} \text{ for } \boldsymbol{j} \in \{\boldsymbol{A}, \boldsymbol{B}\} \tag{6}$$

From (6) one gets the utility maximizing consumption quotient of good 1 and 2. This is inserted into the budget constraint, so that one gets the utility maximizing demand functions of player $j \in \{A, B\}$ for good $i \in \{1,2\}$.

2.3. Market Equilibrium

A market is cleared if demand and supply are equal. The supply side is determined by the initial endowments according to (1) and the demand side is determined by the utility maximizing demand functions $x_i^{j\_d}$ for $i \in \{1,2\}$ and $j \in \{A,B\}$. So, consumption is restricted to available resources of goods as you can see in (7):

$$\underbrace{\omega_1^A + \omega_1^B}_{=\omega_1} = x_1^{A\_d} + x_1^{B\_d} \text{ and } \underbrace{\omega_2^A + \omega_2^B}_{=\omega_2} = x_2^{A\_d} + x_2^{B\_d} \tag{7}$$

According to the Walras' Law, if there are n markets and n–1 markets are in equilibrium, the last market will also be in equilibrium.[6] If one takes good 1 as numeraire[7], one can calculate the price of good 2. Then the market allocation results from the players' utility maximizing demands with regard to the market prices.

2.4. Set of Pareto Efficient Allocations

A criterion to evaluate an allocation with regard to economic efficiency is Pareto efficiency. This concept is based on the unanimity rule.[8] Varian (2011, p. 652) shows that a Pareto efficient allocation can be defined by a situation where no player can increase his utility without decreasing the utility of the others.

Equation (6) shows that every player fits his marginal rate of substitution to the price relationship. Consequently, in competitive markets the marginal rates of substitution of all players have to be equal. The condition of Pareto efficiency is shown in (8). This central result is derived and explained by Edgeworth (1881, pp. 20-22). After him the graphical representation of an economy of two players and two goods is called, the Edgeworth box[9]:

$$\underbrace{\frac{\partial \boldsymbol{u}^{A}\left(\boldsymbol{x}_1^A, \boldsymbol{x}_2^A\right) / \partial \boldsymbol{x}_1^A}{\partial \boldsymbol{u}^{A}\left(\boldsymbol{x}_1^A, \boldsymbol{x}_2^A\right) / \partial \boldsymbol{x}_2^A}}_{=\left|\boldsymbol{MRS}_{1,2}^A\right|} = \frac{\boldsymbol{p}_1}{\boldsymbol{p}_2} = \underbrace{\frac{\partial \boldsymbol{u}^{B}\left(\boldsymbol{x}_1^B, \boldsymbol{x}_2^B\right) / \partial \boldsymbol{x}_1^B}{\partial \boldsymbol{u}^{B}\left(\boldsymbol{x}_1^B, \boldsymbol{x}_2^B\right) / \partial \boldsymbol{x}_2^B}}_{=\left|\boldsymbol{MRS}_{1,2}^B\right|} \Rightarrow \left|MRS_{1,2}^A\right| = \left|MRS_{1,2}^B\right| \tag{8}$$

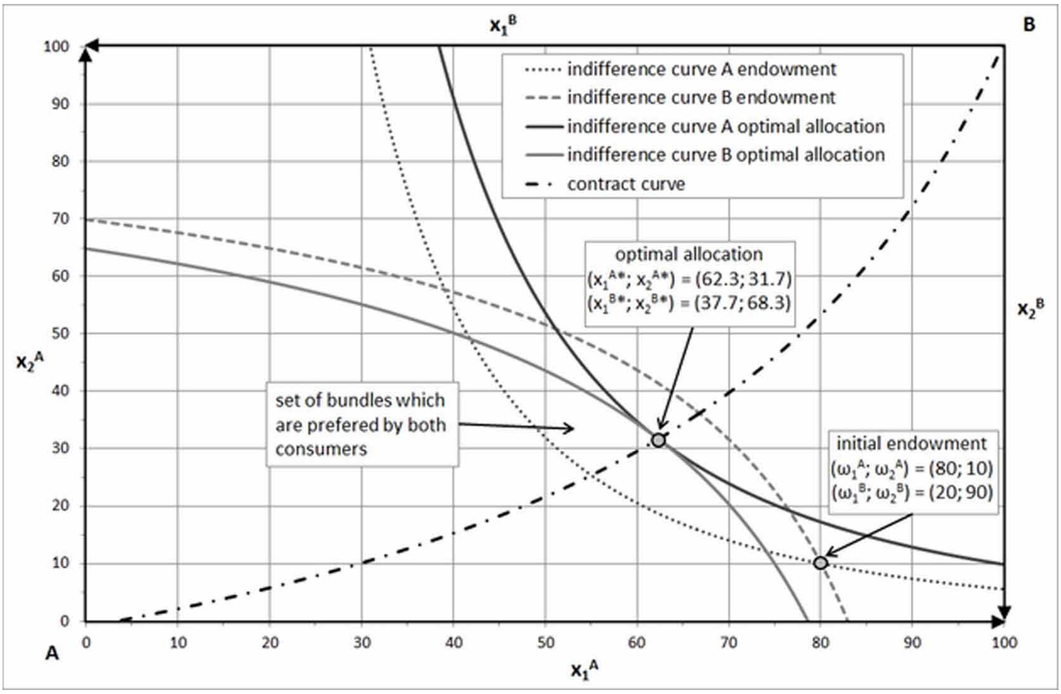
If the marginal rates of substitution of both players are equal, then their indifference curves[10] will also have one common tangential point. This can be seen in Figure 1, an Edgeworth box drawn similar to Pfingsten (1989, pp. 122, 124), with an initial endowment and an optimal allocation.

Looking at Figure 1 one recognizes that the initial endowment is no optimal allocation. Starting from this point both players can increase their utility by choosing an allocation in the area enclosed by both indifference curves, the exchange lens. Furthermore, from the condition of Pareto efficiency (8) and the resource constraints of (7) one can derive the contract curve[12] which represents all Pareto efficient allocations. It only depends on the initial endowment which efficient allocation will result. However, a Pareto efficient allocation can also be an extremely unequal distribution of goods. So, the Pareto Efficiency Criterion does not say anything about "fair" distribution.

2.5. Extensions of the Model

In contrast to Pareto efficiency the criterion of no-envy focusses on distribution. Therefore, let's consider a set of allocations which are envy free for both players. That means no player prefers the bundle of goods of the other to his own one. For example, Diamantaras and Thomson (1990) formalize the no-envy concept and Tadenuma (2002) looks at the order of Pareto optimality and the no-envy condition. Feldman and Kirman (1974) define fairness in the sense of no-envy and Baumol (1982) looks at envy, fairness and strict superfairness.

Figure 1. From initial endowment to market equilibrium[11]



A further approach deals with avarice and altruism. Scott (1972) uses preference functions with interdependencies. That means for example, that the utility of player A also depends on the amount of goods consumed by player B. Daly and Giertz (1972) show benevolent behavior of a player by modelling utility interdependencies either as utility externality or as goods externality. That means that either the utility of player B or the amount of goods consumed by player B have influence on the utility of player A.

Moreover, one can consider that players may fight for goods. For example, Anderton and Carter (2008) and Anderton and Carter (2009) integrate conflict and appropriation of goods in an economic model of exchange by means of a probabilistic conflict success function. Players have initial resources to invest in military goods and dispute about a common resource. In accordance with Hirshleifer (1995) they suggest using a conflict success function (either in a ratio form or in a logistic form). The conflict success function determines the fighting efforts of the players. Also, the players can invest a part of their initial resources into a production technology to produce two consumer goods. Anderton and Carter (2008, 2009) want to look at diversion of resources to military goods as well as destruction of a part of fixed resource and trade disruption as a consequence of fight. A similar approach was chosen by Garfinkel and Skaperdas (2007) with two stages: In the first stage players have to decide about choice of guns and in the second stage they negotiate about division of the contested goods. They can agree on the division or they fight and the winner takes all.

Note that neither Anderton and Carter, Hirshleifer nor Garfinkel and Skaperdas model aggression as an additional immaterial good, but use a functional approach (i.e. a conflict/contest success function) instead.

3. EXPANDING THE EDGEWORTH BOX TO THE EDGEWORTH CUBE

The Edgeworth Box explains the economies of trade very well within a minimalistic setting, as long as the initial allocations of resources are distributed in a sufficiently fair manner. But what happens, if one player owns all the goods and the other player owns none (see chapter 1.2)?

Thus, the research question is: "What happens, if the original endowments of the players are significantly unbalanced and threaten the existence of one player?"

The authors attempt to answer this research question model-wise by extending the Edgeworth box by one dimension.

3.1. The Edgeworth Cube

Imagine you are one of the two players in an economy which consists of two goods (good 1 = bread and good 2 = wine) only. Whereas the other player (B) owns everything in this economy, you (A) own nothing. Let's also assume that there is a free supply of water in this economy, too, which renders bread to be a vital (essential) good, whereas wine can be considered as a normal (non-essential) good.

This initial allocation of goods will pose a mortal threat to you, as your life depends on the possession of bread.

How would you react? Probably you would first politely beg for bread, but if your begging remains unanswered you most probably would threat to take away a certain amount of bread by force in the attempt of self-preservation, so that you don't starve.

Interpreting this behavior in the context of the Edgeworth Cube, you will have introduced a new unilateral and immaterial "good 3" which is your behavioral pressure level towards B (or your aggression[13] level towards B) which is only created by you (see equation (9)). Now the utility function of the other player (B) will be expanded automatically by one term which represents the utility of social peace for player B (x_3^B denotes the level of social peace for player B which lies between 0 and 100):

$$\boldsymbol{u}^{\boldsymbol{A}}\left(\boldsymbol{x}_1^{\boldsymbol{A}}, \boldsymbol{x}_2^{\boldsymbol{A}}, \boldsymbol{x}_3^{\boldsymbol{A}}\right) \text{and } \boldsymbol{u}^{\boldsymbol{B}}\left(\boldsymbol{x}_1^{\boldsymbol{B}}, \boldsymbol{x}_2^{\boldsymbol{B}}, \boldsymbol{x}_3^{\boldsymbol{B}}\right) \text{with } \boldsymbol{x}_3^{\boldsymbol{B}} = 100 - \boldsymbol{x}_3^{\boldsymbol{A}} \tag{9}$$

In an initial step (and for simplicity reasons) the authors suggest that aggression is created by player (A), the poorer of the two players[14], only (=unilateral determination of social peace level). Thus, let social peace and aggression be defined as follows:

- $x_3^B = 100$:= maximal social peace for player B represented by the absence of any aggression executed by player A (0% aggression ↔ $x_3^A = 0$);
- $x_3^B < 100$:= reduced social peace for player B due to aggression level x_3^A of player A;
- $x_3^B = 0$:= no social peace for player B represented by the maximum aggression level applied by player A (100% aggression ↔ $x_3^A = 100$).

3.2. Vital Goods, Normal Goods and Aggression

In the given example, the authors now want to differentiate two types of physical goods – vital goods (i.e. bread or good 1) and normal goods (i.e. wine or good 2). Whereas the lack of a minimum possession of vital goods is life threatening, the lack of a normal good is just non-desirable. The authors suggest to differentiate existential goods from normal goods, because they assume that the likeliness to make a threat of aggression in case of lack of a vital good is significantly higher (because the motivation of aggression is self-preservation) than the threat of aggression in case of lack of a normal good (e.g. motivated by violent envy).

The authors also suggest to introduce an immaterial good which they call ‘aggression’ (derived from the Latin word “aggredi” – going towards, approach, attack). The authors explicitly want to exclude any moral or ethical implications from this word and only focus on its meaning in terms of “behaving in a way that pressure is applied to a third party” – no matter if the reason for this pressure is legitimate or not, morally justified or not, good or bad.

Thus ‘aggression’ may appear in different instances and at different levels as e.g.:

- Polite begging, aggressive begging, pick-pocketing, theft, robbery, ransom, murder (individuals vs. individuals in a context of unevenly distributed wealth);
- Demonstrations, warning strikes, short strikes, long strikes, general strikes;
- (Individuals vs. companies in a context of unevenly distributed wage levels);
- No, little, medium, large, unlimited migration pressure (individuals vs. nations in a context of unevenly distributed wealth) laissez faire, political threats, economic warfare, physical warfare (nations vs. nations in a context of unevenly distributed wealth).

Note that this introduction of aggression as a new good in a sheer economic model together with the concept of vital vs. normal goods helps to differentiate the concept of ***envy***[15].Envy might be a cause for “aggression” if it is based on a certain level of lacking normal goods. But if “aggression” results from the lacking of vital goods, it cannot be interpreted as envy, but it must be seen as a mean of last resort for self-preservation.

Also note, that by introducing the “egoistic strive for peace & safety” as a term within the utility function of the “rich” player (B) one may argue that the model also encompasses ***altruism*** (if one interprets altruism as a special form of egoism which seeks less aggression in exchange for giving possessions away).

3.3. Unilateral Determination of Social Peace Level by Player A

Let’s now preliminarily[16] assume the following:

Assumption I: All three goods are “independent”,[17];
Assumption II: All three goods are characterized by a decreasing positive marginal utility;
Assumption III: There was a budget constraint on good 3 (as well as for goods 1 & 2).[18]

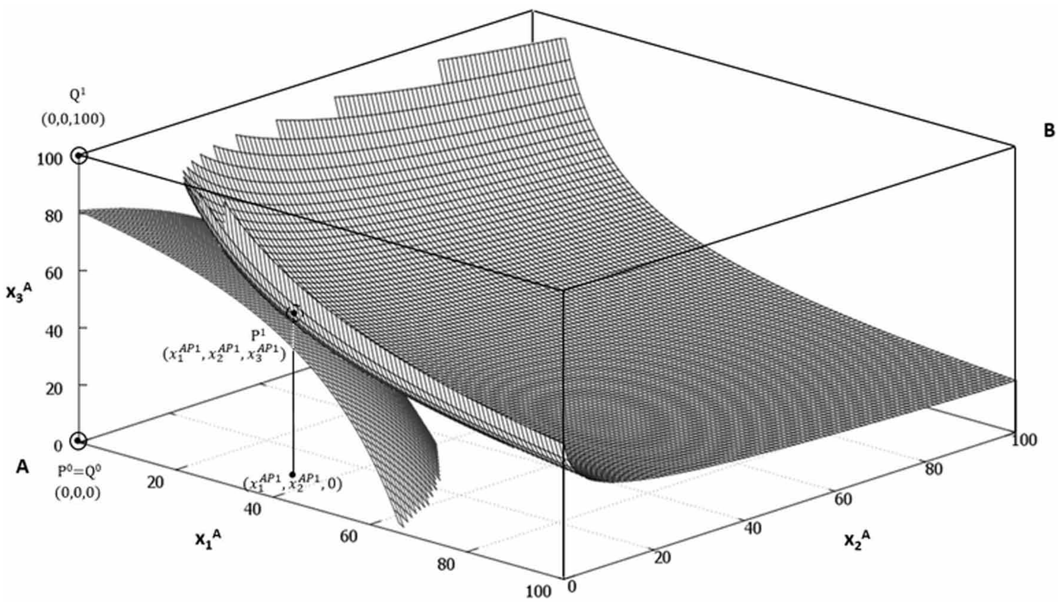
As one can see in Figure 2, there is a difference between the final allocation of the goods in the classical Edgeworth Box and the final allocation of the goods in the Edgeworth Cube.

The equilibrium-allocation P^0 within an initially aggression free 2-dimensional world of the classical Edgeworth Box (represented by the bottom plane of Figure 2) lies at the coordinates $(0,0)$ and is identical to the initial endowment Q^0, because:

- There is no possibility to improve utility of all players;
- No player can increase his utility without decreasing the utility of the others;
- All gains of exchange are exhausted;
- No further mutual profitable exchanges can be realized.[20]

Let’s now take a look at a world with existence of aggression (3-dimensional world of the Edgeworth Cube). Assume that player A applies a maximum aggression level of $\omega_3^A = 100$ at the

Figure 2. Initial endowments and Pareto optima without a sanction system for aggression[19]



point Q[1] at the coordinates $(x_1^{AQ1}, x_2^{AQ1}, x_3^{AQ1}) = \left(0, 0, 100\right)$. The utility for each player $j \in \left\{A, B\right\}$ in a three-dimensional diagram can be represented by a convex surface:

$$x_3^j = f\left(x_1^j, x_2^j, u^j\left(x_1^{jP1}, x_2^{jP1}, x_3^{jP1}\right)\right)$$

The new Pareto optimal solution in P[1] is located, where the two convex iso-utility surfaces of player A and B touch each other. At P[1] player A has reduced his aggression compared to Q[1] in exchange for a new bundle of physical goods $\left(x_1^{AP1}, x_2^{AP1}\right)$. So introducing aggression has resulted in a better endowment of the physical goods for player A. And for player B the payment of $\left(x_1^{BQ1} - x_1^{BP1}, x_2^{BQ1} - x_2^{BP1}\right)$ has "bought" a reduction of aggression by $(x_3^{AQ1} - x_3^{AP1})$.

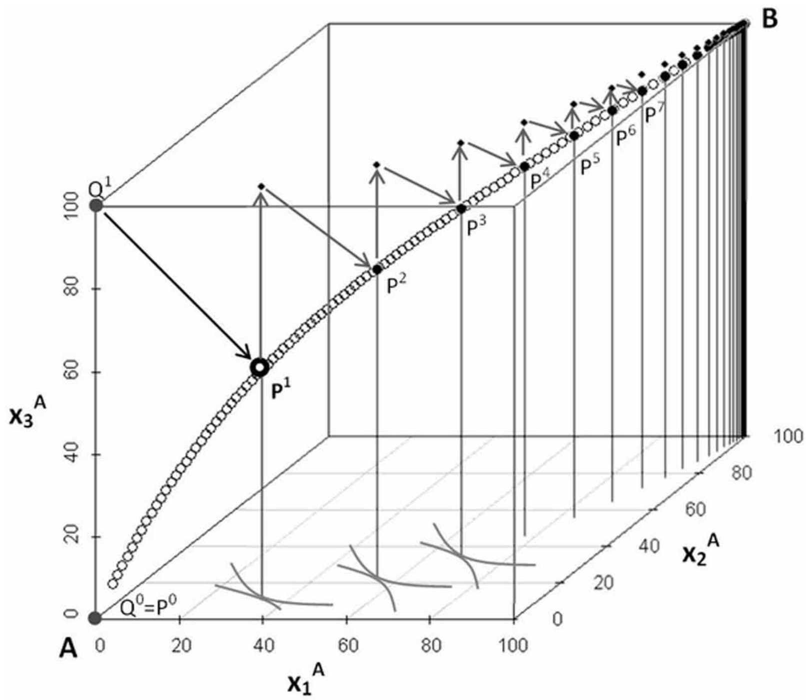
3.4. Budget Constraints

Let's now discuss assumption III) the budget constraint of good 3 (aggression). Whereas goods 1 & 2 have a budget constraint due to their nature as limited physical goods, good 3 can unilaterally be created by player A – i.e. there is no natural budget constraint for good 3.[21]

Which impact does this lacking budget constraint have on the behavior of player A? Let's assume the two players have reached a temporary allocation steady state at P[1] (see Figure 3).

Player A has learned, that applying aggression leads to a better endowment for him regarding goods 1 & 2 - i.e. the aggression level x_3^{AP1} has increased his endowment of goods 1 & 2 from $\left(x_1^{AQ1}; x_2^{AQ1}\right)$ to $\left(x_1^{AP1}; x_2^{AP1}\right)$. So if player A has no moral incentives for his behavior, but is just an utility optimizer, and as due to assumption II the marginal utility of aggression is always positive, player A could increase the aggression level again to 100% and thus re-negotiate a further increase of his endowment of goods

Figure 3. Iterative exchange of goods for peace and subsequent re-raise of aggression by Player A without budget constraint for aggression and without penalty mechanism for aggression[22]



1 & 2 because there is no budget constraint to player A's aggression. This results in the next Pareto optimal solution P^2. Thus player A can increase his possession of goods 1 and 2 over several rounds of exchange (see Figure 3) since in this model player B does not possess any sanction system which will punish A for being aggressive. In the chosen model, the lack of a budget constraint for good 3 would lead to a final allocation where all of goods 1 & 2 are allocated to player A at an aggression level A vs. B of 100%. At this point let's discuss the options of player B.

3.5. Counterbalancing Options for Player B

To avoid this dooming new equilibrium, where player B is completely bereft of all goods 1 & 2, player B has the following options while still in the original situation at P^0:

1. Exchange goods for peace;
2. Apply aggression himself; or
3. Hire someone to reduce the aggression of player A (as long as player B still has possessions to pay for such a service).

Option A: Alone does not work in the model if player A has neither a moral limit nor a budget constraint to applying aggression.

Option B: Would lead to a "fist fighting" economy, where the law of the strongest rules.

Option C: Is something that can be observed in modern societies, where police forces impose an impact on the utility function of Player A regarding good 3 at a cost for player B. So let's discuss the impacts of option c) on the utility functions of both players.

3.6. Utility Functions

Let's now discuss the utility functions of both players in more detail and add some new aspects which are not yet considered.

As the lack of a vital good poses a threat to life, assume, that the utility function of vital goods may take negative values, if the endowment is lower than the existence minimum whereas the utility of a normal good is always non-negative. This holds for both players $j \in \{A, B\}$ (see Figure 4 a) and b)).

At this point, one needs to question assumption I (independence of goods). At least as long as the existence minimum has not been reached ($x_1^A < x_{1,min}$) player A will and should apply aggression in order to survive.

3.6.1. Specifics of Aggression in the Utility Function of Player A

The utility of "aggression applied by player A" for player A is more complex. One can see three different aspects of this utility:

- First the value of exchanging/substitution aggression for vital or normal goods $u_{3s}^A\left(x_3^A\right)$ - see Figure 5 a);
- Second "fear of punishment" $u_{3p}^A\left(x_3^A\right)$ – see Figure 5 b) [the combination of a) and b) is depicted in Figure 5 c)]; and
- Third a psychological utility - i.e. feeling good or bad about applying aggression.

Let's assume that player A has no psychological inclination to apply or avoid aggression – then one can neglect the third aspect.

Now let's assume that the total utility of Player A can be computed as follows:

$$
\begin{aligned}
u^A\left(x_1^A, x_2^A, x_3^A\right) = & \\
& = u_1^A\left(x_1^A\right) + u_2^A\left(x_2^A\right) + u_3^A\left(x_3^A\right) \\
& = u_1^A\left(x_1^A\right) + u_2^A\left(x_2^A\right) + u_{3s}^A\left(x_3^A\right) + u_{3p}^A\left(x_3^A\right) \qquad (10)
\end{aligned}
$$

Figure 4. Shape of unidimensional utilities for players A and B

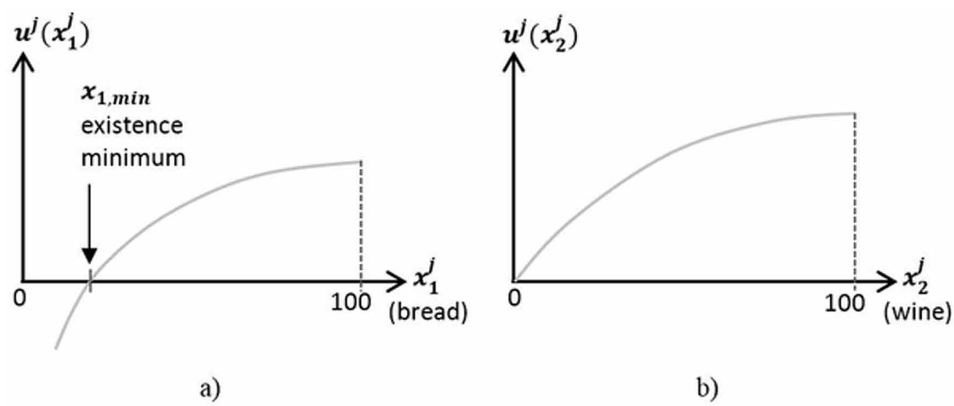
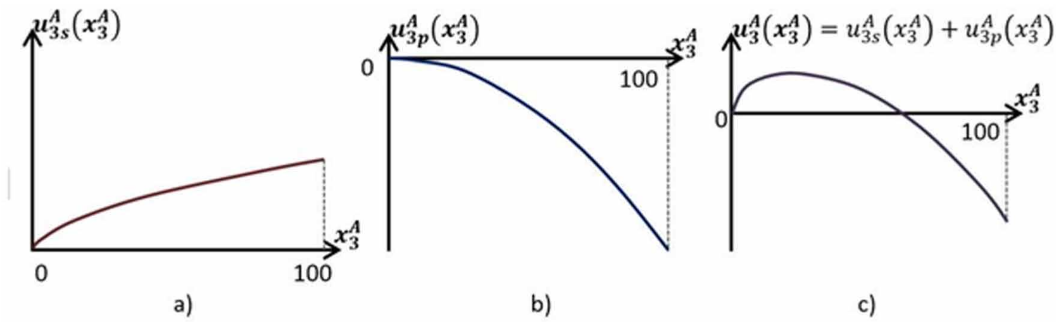


Figure 5. Aggregression related utilities for player A



with:

- $u_1^A\left(x_1^A\right)$: Utility of bread for player A;
- $u_2^A\left(x_2^A\right)$: Utility of wine for player A;
- $u_3^A\left(x_3^A\right)$: Utility of aggression for player A;
- $u_{3s}^A\left(x_3^A\right)$: Substitution utility of aggression vs. bread and/or wine for player A;
- $u_{3p}^A\left(x_3^A\right)$: Utility of aggression in terms of estimated penalty for applying aggression.

Let's now have a closer look at the first aspect– the utility of substituting aggression for goods 1 or 2. As Figure 5 a) shows, assume that a very small level of aggression will not have a significant "substitution" value for player A, but a "significant level of aggression" will probably cause B to trade some of his bread (or his wine) in exchange for a lower level of aggression.

Let's now look at the second aspect – the "utility" of expected punishment. The estimated punishment for aggression depends on two factors: the probability of being held responsible for aggression and the relevant penalty level for aggression. (Whereas petty aggression may result in no prosecution or very little punishment, violent rioting may lead to severe punishment, and murder to life-long imprisonment.) Thus, assume that A's utility of applying aggression with respect to estimated punishment is concave as depicted in Figure 5 b).

The cumulative utility of aggressive behavior for Player A is shown in Figure 5 c). Here you see that player A has an optimal level of aggression which lies above 0% and below 100%, in the range where $u_3^A\left(x_3^A\right)$ is positive. Thus A's chosen level of aggression also depends on the level of punishment which is determined by the shape of $u_{3p}^A\left(x_3^A\right)$.

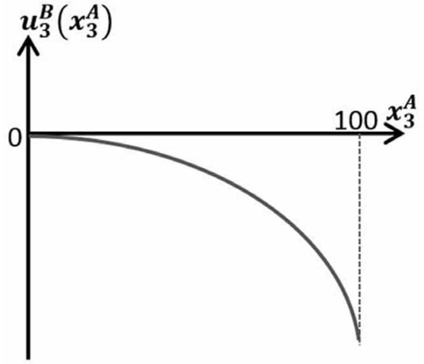
Therefore, the above assumption II does not hold in real life, if punishment mechanisms for aggression are in place.

Now let's discuss the utility function of player B in greater detail.

3.6.2. Specifics of Aggression in the Utility Function of Player B

Assume that aggression applied by player A always has a negative utility for player B, and that this negative utility grows disproportionally with the aggression level (see Figure 6). Also assume, that player B does not apply aggression himself, but hires someone to cause the utility effects of punishment

Figure 6. B's utility of suffering aggression from A



$u_{3\mathrm{p}}^{A}\left(x_3^A\right)$ for player A (i.e. player B pays wages for a security force) one can model the utility function of player B as follows:

$$\begin{aligned} u^B\left(x_1^B, x_2^B, x_3^A\right) &= \\ &= u_1^B\left(x_1^B\right) + u_2^B\left(x_2^B\right) + u_3^B\left(x_3^A\right) \\ &= u_1^B\left(x_1^B - x_{1w}^B\right) + u_2^B\left(x_2^B - x_{2w}^B\right) + u_3^B\left(x_{3c}^A\right) \end{aligned} \tag{11}$$

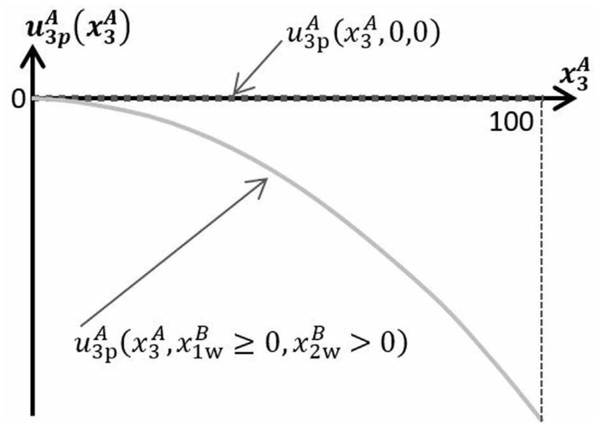
with:

- $u_1^B\left(x_1^B\right)$: Utility of bread for player B;
- $u_2^B\left(x_2^B\right)$: Utility of wine for player B;
- $u_3^B\left(x_3^A\right)$: Utility of aggression applied by player A and endured by player B without help of a security force;
- $u_3^B\left(x_{3\mathrm{c}}^A\right)$: Utility of aggression applied by player A and counterbalanced by security force of B;
- $x_{1\mathrm{w}}^B$: Consumption abstinence of good 1 used as wage for police or security force;
- $x_{2\mathrm{w}}^B$: Consumption abstinence of good 2 used as wage for police or security force.

In other words, player B has an economic choice to invest parts of his original endowment of physical goods $\left(\omega_1^B, \omega_2^B\right)$ into direct transfer payments (as e.g. social aid or foreign development aid) to A and/or into wages for security forces (as e.g. private security forces, public police forces or military forces in a transnational context) in order to reduce the aggression level of A. From an economic point of view B will apply a combination of policies which renders the highest utility to B.

Note that consequently the "utility of aggression in terms of estimated penalty for applying aggression by player A" $u_{3\mathrm{p}}^A\left(x_3^A\right)$ can be re-formulated more precisely by $u_{3\mathrm{p}}^A\left(x_3^A, x_{1\mathrm{w}}^B, x_{2\mathrm{w}}^B\right)$ as shown in Figure 7.

Figure 7. Aggression related utilities for player A with penalties



Summarizing the findings from section 3.6 the *lack of a budget constraint* for good 3 (aggression) requires the utility of player A with regard to good 3 to *show a maximum* (see Figure 5 c) if a steady state in the Edgeworth Cube shall be achieved.

4. INTERPRETATION OF POSSIBLE PARAMETERIZATION SCENARIOS

The formal model introduced above has a number of possible parameterizations, which can be interpreted from a socio-economical point of view.

4.1. Anarchy

If $x_{1w}^B = x_{2w}^B = 0$, i.e. the player B does not or cannot invest into wages for security forces, there is no penalty for aggression for player A. Thus, A might increase his aggression level, till he holds all of the goods 1 & 2. In this case now B would be facing a threat to his very existence (i.e. B is now in the same position in which player A started off before administering aggression). As now player B would have to apply aggression, too, one would be in an anarchistic world, were "the rule of the physically strong" would dominate every commercial transaction.

4.2. Non-Social Police State

In an extreme non-social police state, there would be no transfers paid by B to A, and at the same time the expenses for security forces $\left(x_{1w}^B, x_{2w}^B\right)$ would be such, that the aggression level of A is kept at bay. In this scenario player A would either be able to live (in case his aggression is tolerated on a level where he is able to carve a living on aggression) or else he'd be imprisoned or die.

4.3. Social House

In a social house, B would use part of his endowment as a transfer to A (i.e. help A to make a living above the existence minimum) whereas another part of the money is invested into a security force to counterbalance the lacking budget roof on A's aggression, and thus keep criminality at bay.

5. SUMMARY, CRITICAL DISCUSSION AND FURTHER RESEARCH

5.1. Summary

The authors' key idea, i.e. introducing either aggression or migration pressure as a unilateral non-physical good without a budget constraint within an expanded Edgeworth cube was sufficient to explain three different types of social co-existence (anarchy, non-social police-state and social house) on a sheer economic basis without the need to refer to any social or moral mechanisms.

The authors were able to show, that:

1. An extremely imbalanced allocation of goods results in aggression (or migration pressure) by the poor;
2. The lack of a budget constraint for aggression (or migration pressure) requires a local maximum within the utility function of the aggressor in order to reach a steady state in the system; and
3. In order to induce this maximum in the aggressors (or the migrants) utility function the aggressor's opponent (or the migration opponent) needs to establish an increasing threat of punishment.

These results derived from the model can be observed in different forms of societies but can also be observed in global trans-societal migration scenarios, where the industrial world can allocate funds to development aid on the one hand and/or to border protection forces on the other hand.

The second idea, i.e. differentiating vital goods from normal goods is helpful, if one wants to overcome the notion that aggression is a sheer result of psychological factors (as e.g. pleasure of applying aggression) or the absence of moral factors (e.g. aggression as a result of envy). The authors reason that applying aggression is the "ultima ratio" option in case the existence minimum of a player is jeopardized by the other player's unwillingness to share vital goods.

5.2. Critical Discussion

One key aspect of the modelling lies in the asymmetry of players A and B. Because the authors wanted to help understand social peace mechanisms within scenarios where wealth is extremely heterogeneously distributed (i.e. co-existence of very poor and very rich as represented by the corners of the Edgeworth box), they established two different types of players (poor player A and rich player B). As long as one operates with significant differences in wealth, this asymmetry is intended and reflected by the reality where the poor and the rich co-exist, and each rely on different options to act (the poor struggle for live within their very limited options and the rich try to defend their possessions - be it by means of transfer payments (social welfare aid or foreign development aid) or by means of investments into security forces (private security and domestic police or border patrol and army forces)). Thus, the authors think this asymmetric approach is acceptable. As soon as one looks into more balanced scenarios, this asymmetric modelling should be replaced by a symmetric modelling, where both players have symmetric options to act.

Another key aspect of the chosen modelling lies in the sheer possession perspective (poor vs. rich) and does not consider the option to change possession levels by means of work. This simplification was made due to model complexity reasons[23].

Last but not least one may argue that a human does not behave as a "homo economicus" (or a mindless automaton based on utility functions), only. Of cause, there are other aspects which influence aggression as e.g. moral convictions, fear of getting caught and other aspects which influence migration pressure as e.g. hopes, strive for happiness - see e.g. Di Tella and MacCulloch (2006) - or fear to leave ones' home country which are not sufficiently considered in the chosen mathematical model. Still looking "ceteris paribus" at the modeled entities (i.e. utility functions), only, helps to better understand a part of human behavior in this context.

5.3. Need for Further Research

As the presented model for now is a conceptual model, only (i.e. neither the *shape of the utility functions* has been specified in more detail yet, nor have any *system parameters* be determined on the basis of empirical data) the authors see significant room for further conceptual and empirical research e.g. can the dimensions of aggression (see section 3.2) be measured on better ordinary scales or even on a cardinal scale, and if yes, how. The authors assume that this empirical research could lead to quantitative economic models which help to better understand political concepts to deal with imbalanced resource allocations from a sheer economic and non-idealistic point of view. They also see room for further research in the already discussed model specifics ("asymmetry of aggression", "steady state computational details"[24] and "work as a missing model element").

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ENDNOTES

1 The authors understand a social community as e.g. a family, a large company, a city or a state.

2 As some authors refer to “players”, some to “consumers” and others to “contractors”, the authors uniformly use the term “player” to avoid term-confusion.

[3] This assumption can be discussed critically with respect to whether human beings really evaluate the utility of their consumption in a rational way and whether this evaluation can be modelled sufficiently by means of mathematical functions. An alternative approach is behavioral economics which also deals with irrational behavior and animal spirits. This behavior economics approach will not be taken into consideration here.

[4] The notation in this paper is inspired by Bisin's (2011, pp. 61-79) way of notation.

[5] See also Varian (2011), p. 83.

[6] See also Varian (2011), pp. 658-660.

[7] That means the authors set the price of good 1 to one.

[8] See also Breyer (2008), p. 199.

[9] Varian (2011, p. 649) explains the Edgeworth box as a two-dimensional coordinate system whose length and width represents the total initial endowments of good 1 and 2. So the remaining amounts of good 1 and 2 which do not belong to player A are owned by player B. A's amounts of goods are measured from the left bottom corner of the box while B's amounts of goods are measured from the right upper corner.

[10] According to Pindyck and Rubinfeld (2013, p. 110) an indifference curve represents a set of allocations between which the player is indifferent because each allocation of this set gives him the same utility.

[11] Self-drawn. Let's assume a total initial endowment of $\omega_1 = 100$ and $\omega_2 = 100$. As distribution: A has $\omega_1^A = 80$ and $\omega_2^A = 10$ while B has $\omega_1^B = 20$ and $\omega_2^B = 90$. The utility function of A is $u^A\left(x_1^A, x_2^A\right) = \left(x_1^A\right)^{0,7} \cdot \left(1 + x_2^A\right)^{0,3}$ and of B $u^B\left(x_1^B, x_2^B\right) = \left(x_1^B\right)^{0,4} \cdot \left(1 + x_2^B\right)^{0,6}$. The type of utility functions shows that for both players good 1 is essential and good 2 is non-essential.

[12] Breyer (2008, p. 203) emphasizes that every Pareto efficient allocation lies on the contract curve.

[13] See discussion of the term "aggression" in section 3.2

[14] Of course the other player may also apply force – but the authors will discuss this aspect later.

[15] For other interpretations of envy please refer to Thomson (2007), or Schneider and Zimmermann (2010).

[16] Note that the authors will discuss the impact of these assumptions and revise some of them later on.

[17] "Independent" especially means here that aggression is not modelled as a function of the endowment of vital and normal goods.

[18] The reader will see later that this assumption does not hold in the real world.

[19] For this illustration the authors assumed a total initial endowment of $\omega_1 = 100$, $\omega_2 = 100$ and a maximal aggression level of 100. As initial goods distribution the authors assume: Player A has $\omega_1^A = \omega_2^A = 0$ and $\omega_3^A = 100$ while player B has $\omega_1^B = \omega_2^B = 100$ and $\omega_3^B = 0$. For this illustration the utility function of player A is $u^A\left(x_1^A, x_2^A, x_3^A\right) = \left(x_1^A\right)^{1/3} \cdot \left(1 + x_2^A\right)^{1/9} \cdot \left(1 + x_3^A\right)^{5/9}$ and of player B is $u^B\left(x_1^B, x_2^B, x_3^B\right) = \left(x_1^B\right)^{1/3} \cdot \left(1 + x_2^B\right)^{4/9} \cdot \left(x_3^B\right)^{2/9}$. This choice of utility functions means that social peace is an essential immaterial good for player B but aggression is not an essential immaterial good for player A.

[20] See section 2.4

[21] Note that while there is a maximum level of aggression which can be applied by player A (which in this case is ($x_3^A = 100$) this maximum level of aggression is not the same as a "budget constraint", because player A can always decide to apply 100% of aggression unilaterally.

[22] Let's started again with an initial goods distribution: Player A has $\omega_1^A = \omega_2^A = 0$ and $\omega_3^A = 100$ while player B has $\omega_1^B = \omega_2^B = 100$ and $\omega_3^B = 0$. The utility function of player A is $u^A\left(x_1^A, x_2^A, x_3^A\right) = \left(x_1^A\right)^{1/3} \cdot \left(1 + x_2^A\right)^{1/9} \cdot \left(1 + x_3^A\right)^{5/9}$ and of player B is $u^B\left(x_1^B, x_2^B, x_3^B\right) = \left(x_1^B\right)^{1/3} \cdot \left(1 + x_2^B\right)^{4/9} \cdot \left(x_3^B\right)^{2/9}$.

[23] One might also argue that in general it is difficult for the poor to significantly change their possession level by means of work as can be seen in the existence of precarian and proletarian social classes. But the authors leave this discussion to sociology experts.

[24] Depending on the shape of the utility functions it may be impossible to derive a general analytical solution for $x_3 = f(x_1, x_2)$. A numerical approach might be required here, but this aspect would be outside the scope of this paper.