

Two-step Procedure Based on the Least Squares and Instrumental Variable Methods for Simultaneous Estimation of von Bertalanffy Growth Parameters

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ABSTRACT

Advanced in the present article is a Two-step procedure designed on the methods of the least squares (LS) and instrumental variable (IV) techniques for simultaneous estimation of the three unknown parameters L_{∞} , K and t_0 , which represent the individual growth of fish in the von Bertalanffy growth equation. For the purposes of the present analysis, specific MATLAB-based software has been developed through simulated data sets to test the operational workability of the proposed procedure and pinpoint areas of improvement. The resulting parameter estimates have been analyzed on the basis of consecutive comparison (the initial conditions being the same) between the results delivered by the two-step procedure for simultaneous estimation of L_{∞} , K and t_0 and the results obtained via the most commonly employed methods for estimating growth parameters; first, use has been made of the Gulland-and-Holt method for estimating the asymptotic length L_{∞} and the curvature parameter K , followed by the von Bertalanffy method for estimation of t_0 .

KEYWORDS

Asymptotic Length, Biological Objects, Fish, Gulland-Holt Plot, Individual Growth, MATLAB, Modeling, Parameter estimates, System Identification

INTRODUCTION

Determining the fish growth parameters plays a fundamental role not only in the correct detection of the fish stock age and size structure, but also in the process of stock assessment procedure, studies of the population dynamics and sustainable stock management in terms of fish stock exploitation restrictions or specific recommendations for minimum limits on the size (length) of species to be caught legally, selectivity of the fishing gears and equipment, etc. Historic data on growth parameters estimates has been successfully used as a basis for analysis of the stock biological development over time in addition to the analysis of the impact of environmental factors – food availability, variability in specific environmental conditions, such as water temperature, the concentration of dissolved oxygen, the types of species interactions and others. Several functions have been developed to model the growth of fish (Gompertz growth model, Schnute-Richards, logistic, etc.), however the von Bertalanffy model is the most popular and is in the scope of the present material.

The von Bertalanffy mathematical model of individual-based approach to predicting fish growth expresses the length (L) as a function of the age of the fish (t):

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$$L(t) = L_{\infty} \left[1 - \exp(-K(t - t_0)) \right] \quad (1)$$

where: t is the age of the respective fish, L_{∞} - is the asymptotic length (or mean length of the cohort at age equal to infinity when the study refers to population dynamics), K is a curvature parameter which shows how fast a given individual approaches its asymptotic (finite) length, t_0 - is the initial condition parameter (determining the point in which the length of the fish is =0) (Cadima, 2003, Sparre & Venema, 1998).

Equation (1) has three unknown coefficients K , L_{∞} and t_0 , to be determined analytically on the basis of given experimental data (i.e. the length and age measurements of the species under analysis). A great number of methods for estimating von Bertalanffy growth parameters have been successfully implemented in practice, with their main disadvantage being the requirement for regularity of the corresponding measurements. Failure to collect and process the data on a regular basis distorts the results of the methods discussed so far except for the Gulland-and-Holt plot, and what is more, they usually provide estimates for only 2 of the 3 unknown parameters (Sparre & Venema, 1998), with the exception of (Melnikova, 2009).

Norbert Winner and Ludwig von Bertalanffy conclusively proved in their works the existence of close similarities in the operation and control of the machines and the living organisms. It follows, therefore, that the common principles, methods and approaches developed for the analysis and control of large technical systems can be successfully modified or adapted in the research and modeling of biological objects and systems. The System theory and more specifically the development of the System identification scientific area appear to be very suitable theoretical basis for further improvement and elaboration of the object/system modeling methods and principles (Eykhoff, 1974; Genov, 2004; Hoffman & Frankel, 2001; Soderstrom & Stoica, 1989).

The Least Squares Method (LSM) techniques are well-known and widely applied in various scientific areas for experimental data processing and its subsequent fitting to a model (Stigler, 1981; Walter & Pronzato, 1997). The LSM is a standard approach in regression analysis to approximate the solution of overdetermined systems, i.e., sets of equations in which the number of equations is much higher than the parameters unknown. "Least squares" means that the overall solution minimizes the sum of the squares of the residuals made in the results of every single equation. It is widely practiced in engineering for solving regulator synthesis problems and system modeling, where a variety of LSM modifications have been developed aiming at obtaining general improvement of the accuracy in parameter estimates. The LSM also takes place in modeling and parameter estimation of biological objects. Its wide applicability is generally assumed to be associated with normal distribution of input data used for analysis provided that the number of data should exceed the number of the unknown (estimated) parameters. The present article proposes a two-step procedure designed on the methods of least squares and instrumental variables for simultaneous estimation of the three unknown parameters in von Bertalanffy growth model equation - L_{∞} , K and t_0 .

BACKGROUND

For the purposes of the present analysis and development of the method proposed and the corresponding algorithm, the von Bertalanffy growth equation will be re-stated as follows:

$$L(t) = L_{\infty} - L_{\infty} e^{-K(t-t_0)} \quad (2)$$

In terms of the General Theory of Automatic Control and System Identification, equation (2) is considered transition (step) function of the object/system under consideration.

Numerous methods have been developed to determine the system parameters by its step response, which prove to be ineffective in offering a solution to the von Bertalanffy growth equation. Thus, despite having the same model, the methods for determining the parameters of the transition (step) function based on experimental data valid for the technical systems are not applicable for biological objects. Hence, a new approach to solving the Growth equation was considered necessary.

In the case of fish, the growth rate measured in length is represented as:

$$\frac{dL(t)}{dt} = L_{\infty} e^{-K(t-t_0)} K \quad (3)$$

Equation (2) can be then rewritten:

$$L_{\infty} e^{-k(t-t_0)} = L_{\infty} - L(t) \quad (4)$$

If (4) is substituted in (3), the following shall be obtained:

$$\frac{dL(t)}{dt} = k.L_{\infty} - k.L(t) \quad (5)$$

Since L_{∞} is constant, the relationship between the growth rate $\frac{dL(t)}{dt}$ and the length $L(t)$ is a linear function. Accordingly, it makes it possible for the unknown parameters L_{∞} and K to be easily estimated by a linear regression analysis (provided that length-at-age data (measurements) is available). It should be emphasized, however, that the accuracy of the parameter estimates is strongly dependent on the input data quality. Great accuracy of estimates can only be expected if the length and age are determined precisely. Experimental data collection and sampling are normally accompanied by added noise of different types and levels, for example as a result of inaccurate measurements. The present analysis takes into consideration random error occurrence, rather than systematic measurement error, and thereupon it is reasonable to assume that those random errors have zero mean or expectation $M = 0$, dispersion σ and are normally distributed. The a priori analysis of the problem is of major significance for further selection of mathematical (instruments) solution (procedure) to the growth equation. Given the above listed considerations and assumptions, the present paper introduces Two-step LS and IV- method based procedure for simultaneous estimation of the individual growth parameters L_{∞} , K and t_0 .

Growth Parameters Estimation

Length-at-age samples in general present discrete data in relation to the parameters measured and the values $L(i\Delta t)$ are accordingly discrete, which is to mean that the continuous derivative (5) can

be duly expressed as the finite difference $\frac{\Delta L}{\Delta t}$. The theory of finite difference approximations of derivatives is covered in most numerical analysis textbooks and has been widely used in scientific and engineering computations. It is generally recognized, however, that the quality of the approximation greatly depends on the regularity of the function. If the initial data is sampled from a sufficiently

smooth function, or if the data is ‘dirty’ because of measurement errors, simple one-sided finite difference appears the most appropriate solution. Interpolation of data is usually related to selection of sampling time $\Delta t = const$. Moreover, in numerical differentiation even small variations of the function can lead to major changes in its derivative. Provisions are made in the analysis under discussion for non-regular measurements of the parameters ($L(t)$ and t) and respectively $\Delta t \neq const$. In accordance with the mean values theorem (finite increments) (Bradistilov, 1961; Bretscher, 1995), which is to be applied hereinafter for derivation of the procedure proposed:

$$L(x + t_{i+1}) - L(x + t_i) = \Delta t \cdot L'(x + \xi), \text{ where: } \xi \text{ is numerical value in the interval: } t_i < \xi < t_{i+1}. \text{ here: } \xi = \xi(x, \Delta t).$$

The calculations further are based on the following assumption:

$$t_{si} = t_i + \xi = t_i + 0.5(t_{i+1} - t_i), \quad L'(t_{si}), \quad i = \overline{1, N-1}.$$

Step 1: Estimation of the Asymptotic Length L_∞ and the Parameter K

It is assumed that $N = \text{number of length measurements of fish}$ – i.e. $L(t_1), L(t_2), L(t_3), \dots, L(t_N)$, which correspond to ages: t_1, t_2, \dots, t_N , the linear equation (5) then can be expressed by the following system of equations:

$$L'(t_{si}) = k \cdot L_\infty - k \cdot L(t_{si}), \quad i = \overline{1, N-1} \tag{6}$$

or in matrix-vector form as:

$$L' = k \cdot L_\infty \cdot 1 - k \cdot Ls \tag{7}$$

where:

$$L' = [L'(t_{s1}), L'(t_{s2}), L'(t_{s3}), \dots, L'(t_{s, N-1})]^T;$$

$$Ls = [L(t_{s1}), L(t_{s2}), L(t_{s3}), \dots, L(t_{s, N-1})]^T;$$

$$1 - \text{is a vector of ones with size } N - 1. \tag{8}$$

Equation (7) can also be conveyed as:

$$L' = [1 - Ls] \cdot \begin{bmatrix} kL_\infty \\ k \end{bmatrix} = X \cdot b \tag{9}$$

Where the elements of the vector b are the unknown von Bertalanffy growth parameters K and L_{∞} .

Supposing that \hat{b} is the estimation of vector b , the error vector e should be defined and respectively taken into account when \hat{b} is additionally substituted in (9) to obtain the following equation:

$$e = L' - X\hat{b} \quad (10)$$

The elements of the error vector e can also contain measurement errors.

The unknown vector \hat{b} will be determined in accordance with the minimum norm least squares solution requirement, i.e.:

$$J = e^T e = \underbrace{\min}_b \quad (11)$$

In compliance with the requirement for minimization of the norm (11), it follows that:

$$\hat{b} = [X^T \cdot X]^{-1} X^T L' = C \cdot X^T L' \quad (12)$$

As a result, the elements of vector \hat{b} shall determine L_{∞} and K :

$$L_{\infty} = \frac{b(1)}{b(2)}, \quad K = b(2) \quad (13)$$

Step 2: Estimation of t_0

Once the 2 growth parameters L_{∞} and K have been calculated in the first step of the procedure being described - equation (2) can be re-stated as follows:

$$e^{-K(t-t_0)} = 1 - \frac{L(t)}{L_{\infty}} = 1 - l(t) \quad (14)$$

After taking a logarithm on both sides and carrying out some simple transformations we have:

$$K \cdot t_0 = K \cdot t + \ln |1 - l(t)| \quad (15)$$

If $y(t) = (\ln |1 - l(t)|) / K$ is substituted in (15): (16)

It proceeds from (16) that:

$$y(t) = t_0 - t \tag{17}$$

(17) is also a linear equation and for $t_i, i = \overline{1, N}$ a system of linear algebraic equations will take place and may be hereby presented in a matrix-vector form as:

$$Y = t_0 \cdot 1 - T \tag{18}$$

where:

$$Y = [y(t_1), y(t_2), \dots, y(t_N)]^T, T = [t_1, t_2, t_3, \dots, t_N]^T, \text{ and } 1 - \text{ is a vector of ones with size } N.$$

(18) is an overdetermined system of linear equations. The unknown parameter t_0 will be established in accordance with the minimum norm least squares solution requirement, analogously to (11), once the estimation \hat{t}_0 is substituted in (18).

$$\mu = (T + Y) - \hat{t}_0 \cdot 1 = Z - \hat{t}_0 \cdot 1 \tag{19}$$

The unknown parameter \hat{t}_0 is to be determined through compliance with the requirement for minimization of the norm:

$$G = \varepsilon^T \varepsilon = \min_{\hat{t}_0} \tag{20}$$

$$\Rightarrow \hat{t}_0 = [1^T \cdot 1]^{-1} 1^T \cdot Z = \frac{\sum_{i=1}^N Z_i}{N} \tag{21}$$

Implementation of the Instrumental Variable (IV)

IV is used when an explanatory variable of interest is correlated with the error term, in which case ordinary least squares are likely to produce biased results. A valid instrument induces changes in the explanatory variable but has no independent effect on the dependent variable, allowing a researcher to uncover the causal effect of the explanatory variable on the dependent variable. Instrumental variable methods allow for consistent estimation when the explanatory variables (covariates) are correlated with the error terms in a regression model. Such correlation may occur when changes in the dependent variable change the value of at least one of the covariates, which is subject to measurement error. Explanatory variables affected severely by one or more of these issues in a regression-based context are commonly referred to as endogenous. In such a situation, ordinary least squares produces biased and inconsistent estimates. However, if an instrument is available, consistent estimates may still be obtained. An instrument is a variable that does not itself belong to the explanatory equation but is correlated with the endogenous explanatory variables, contingent upon the value of the other covariates. Using IVs in linear models relates to two main requirements:

The instrument must be correlated with the endogenous explanatory variables, conditional on the other covariates. If this correlation is strong, then the instrument is said to have a strong first step. A weak correlation may provide misleading inferences about parameter estimates and standard errors (Bound, Jaeger & Baker, 1995).

The instrument cannot be correlated with the error term in the explanatory equation, conditional on the other covariates. In other words, the instrument cannot run into the same problem as the original predicting variable. If this condition is met, then the instrument is said to satisfy the exclusion restriction (Angrist & Krueger, 2001; Bowden & Turkington, 1984; Stock & Trebbi, 2003).

The primary aim of IV implementation (instrumental matrix V) in the procedure brought forward is to ensure improvement in the quality of the estimates (vector \hat{b}), more specifically in the presence of added noise (measurement error). IV has been selected to have a form identical to the structure of and in close correlation with the matrix X rather than with the added noise (the measurement error). It should be regarded as a special supplement to the LSM, completing and enhancing it. In general, IV derivation methodology follows the basic algorithm for the LSM.

1. Implementation of the IV for calculation of the vector \hat{b}_{ip}

In equation (12) used in place of \hat{b} is the following expression:

$$\hat{b}_{ip} = [V^T . X]^{-1} V^T L' \tag{22}$$

where the instrumental matrix V^T is in a form identical to the matrix X^T , and yet its elements are not the measured values $L(t_{si})$ but the ones forecasted by the model calculated with the (previously) estimated parameters L_{∞}, K and t_o (as determined in Step 1. and Step 2.)

The IV is implemented in line with the following algorithm (Genov, 2004):

1. Estimate vectors \hat{b} and \hat{t}_0 on the basis of the LSM;
2. Use the estimates \hat{b} and \hat{t}_0 to forecast the model values L_{model} ;
3. Use the model values L_{model} to form the instrumental matrices V and V_1 and obtain the new estimates by applying the equations (21) and (22);
4. Go back to step (2);
5. The procedure is iterative and can be executed as many times as needed (usually 2 to 5) to secure stabilization of the estimated parameter values.

2. Implementation of the IV for calculation of the parameter \hat{t}_{0ip}

The instrumental matrix V_1 should be selected in a form identical to the matrix Z , but in this case y will be formed by implementing the model forecasted values L_{model} , that have been calculated using the already estimated values \hat{b} and \hat{t}_0 ;

Upon substitution of the vector of ones in (21) with V_1^T , the parameter \hat{t}_{0ip} is to be calculated through the following equation:

$$\hat{t}_{0ip} = [V_1^T \cdot 1]^{-1} V_1^T \cdot Z = \frac{\sum_{i=1}^N v_{1i} \cdot z_i}{\sum_{i=1}^N v_{1i}} \quad (23)$$

3. Analysis of the model accuracy L_{model}

The unknown parameters of the model (2) having been estimated, its accuracy should be evaluated, i.e. at what extent the model data comes close to the experimental (the measured) data. To that effect, it is necessary for the calculated model values to be compared to the experimental data. The smaller the difference is, the greater the model accuracy. Higher deviations of the model estimates L_{model} from the measurement data $L(t)$ are indications for incorrect model structure and respectively, low quality of the parameter estimates or inaccurately defined measurement error impact (incorrect measurement in the length or age of fish). As a measure of the model accuracy it is appropriate to

adopt the error variance (or the standard deviation): $S_{emodel}^2 = \frac{1}{N-1} \sum_{i=1}^N (L_i - L_{model,i})^2$, or the model

relative error: $S_{otn_model} = S_{emodel} / S_L$.

MATLAB Program for the Realization of the Proposed Two-step LS and IV- Method Based Procedure for Simultaneous Estimation of the Individual Growth Parameters L_{∞} , K and t_0

For the purposes of the practical implementation of the procedure described and its satisfactory performance a MATLAB-environment script has been developed – provided for reference in Appendix A.

MAIN FOCUS OF THE ARTICLE

Performance and Operational Examination of the Two-step Procedure for Simultaneous Estimation of the von Bertalanffy Growth Parameters L_{∞} , K and t_0

For the objectives of the present research, the subsequent in-depth analysis and concrete proof of the workability of the improved Two-step LS and IV- method based procedure for simultaneous estimation of the individual growth parameters L_{∞} , K and t_0 put forward by the paper, the results obtained will be compared with the results produced by other widely used methods for estimating von Bertalanffy growth parameters. Simultaneously with the execution of the procedure under discussion and with the provision of equal conditions applied consecutively are two methods for estimating the above-mentioned parameters L_{∞} , K and t_0 . K and L_{∞} are determined first by applying the Gulland-and-Holt method and then the calculated asymptotic size value was used to estimate the t_0 by the von Bertalanffy method.

Once the goal was set, a specific MATLAB-based program was developed for consecutive execution of Gulland-and-Holt method for estimation of K and L_{∞} and the von Bertalanffy method for estimation of t_0 , which is fed along with the asymptotic length value L_{∞} previously estimated using the former method. The content of the program script is presented for reference in Appendix B.

Simulation Data

Input data used for the simulation series:

$t_0 = 0.087$; $K = 0.78$; $L_\infty = 50.00$ cm, the data is simulated to present non-regular measurements ($\Delta t \neq const$) in the interval 0.1-4.35 years, by using the von Bertalanffy model:
 $L(t) = L_\infty \left[1 - \exp(-K(t - t_0)) \right]$ in MATLAB program environment with the following program script:

```
clc
clear
K=0.78
t0=0.087
Lasimpt=50
load('t_sim.mat')
L=Lasimpt*(1-exp(-K*(t_sim-t0)))
```

Along with the study into the accuracy of the growth parameter estimates obtained by the methods described so far, another study was carried out into the probable way the measurements might affect the estimates and the level of resistance to the added noise of the methods themselves (Fuller, W. A. (1987)). Provisions were made for simulated cases of added measurement error through implementation of random normally distributed error, which is considered either incorrect measurement of L or incorrect determination of the corresponding ages. The values $L = L_sim$ and $t = t_sim$ simulated by the von Bertalanffy growth model were completed with a normally distributed error, generated by using the following script:

```
R = normrnd(mu, sigma, 1, 50)
mu=0
sigma=0.01 (*0.05, 0.1, 0.15, 0.2, 0.3, 0.5)
R = normrnd(mu, sigma, 1, 50)
R'
e01=R'
Se01=std(e01)
S_L=std(L)=14.7815
S_t=std(t)= 1.2132
sotnL=(Se01/S_L)*100
sotnt=(Se01/S_t)*100
```

$R = \text{normrnd}(\mu, \sigma)$ generates random numbers from the normal distribution with a mean parameter μ and standard deviation parameter σ . μ and σ can be vectors, matrices, or multidimensional arrays that have the same size, which is also the size of R . A scalar input for μ or σ is expanded to a constant array with the same dimensions as the other input.

In order to address the noise levels to a certain percentage, added measurement error is calculated:

- The standard deviation of the input data is to be determined by:

$$S_L = Std(L) = 14.7815 \text{ and: } S_t = Std(t) = 1.2132 ;$$

- Signal-to-noise ratio: $S_{otnL} = \frac{S_e}{S_L} 100(\%)$ and $S_{otnt} = \frac{S_e}{S_t} 100(\%)$;

The generated noise (added measurement errors) has the following characteristics:

- e_{01} - standard deviation $S_{e_{01}} = 0.0103$; signal-to-noise ratio $S_{otnL} = 0.0730\%$ or measurement error $\approx 0.07\%$ and $S_{otnt} = 0.8897$ or measurement error $\approx 1\%$
- e_{05} - standard deviation $S_{e_{05}} = 0.0557$; signal-to-noise ratio $S_{otnL} = 0.3769$ or measurement error $\approx 0.4\%$ and $S_{otnt} = 4.5918$ or measurement error $\approx 5\%$
- e_{1} - standard deviation $S_{e_1} = 0.1036$; signal-to-noise ratio $S_{otnL} = 0.7006$ or measurement error $\approx 0.7\%$ and $S_{otnt} = 8.5356$ or measurement error $\approx 10\%$
- e_{15} - standard deviation $S_{e_{15}} = 0.1526$; signal-to-noise ratio $S_{otnL} = 1.0326$ or measurement error $\approx 1\%$
- e_{2} - standard deviation $S_{e_2} = 0.1946$; signal-to-noise ratio $S_{otnL} = 1.3164$.or measurement error $\approx 1,5\%$
- e_{3} - standard deviation $S_{e_3} = 0.3364$; signal-to-noise ratio $S_{otnL} = 2.2758$ or measurement error $\approx 2\%$
- e_{5} - standard deviation $S_{e_5} = 0.5141$; signal-to-noise ratio: $S_{otnL} = 3.4782$ or measurement error $\approx 3.5 \%$
- e_{6} - standard deviation $S_{e_6} = 0.9149$; signal-to-noise ratio: $S_{otnL} = 6.1897$ or measurement error $\approx 6 \%$

Histograms of the generated measurement errors (noise) are presented in Figure 1, Figure 2, Figure 3, Figure 4, Figure 5, Figure 6, Figure 7 and Figure 8.

The input data used for obtaining estimates of the growth parameters by the methods listed below:

- The Two-step LS and IV method-based procedure for simultaneous estimation of the individual growth parameters L_{∞} , K and t_0 advanced by the present paper
- The Gulland-and-Holt method for estimation of L_{∞} and K
- The von Bertalanffy method for estimation of t_0 as presented in Table 1

SIMULATION RESULTS

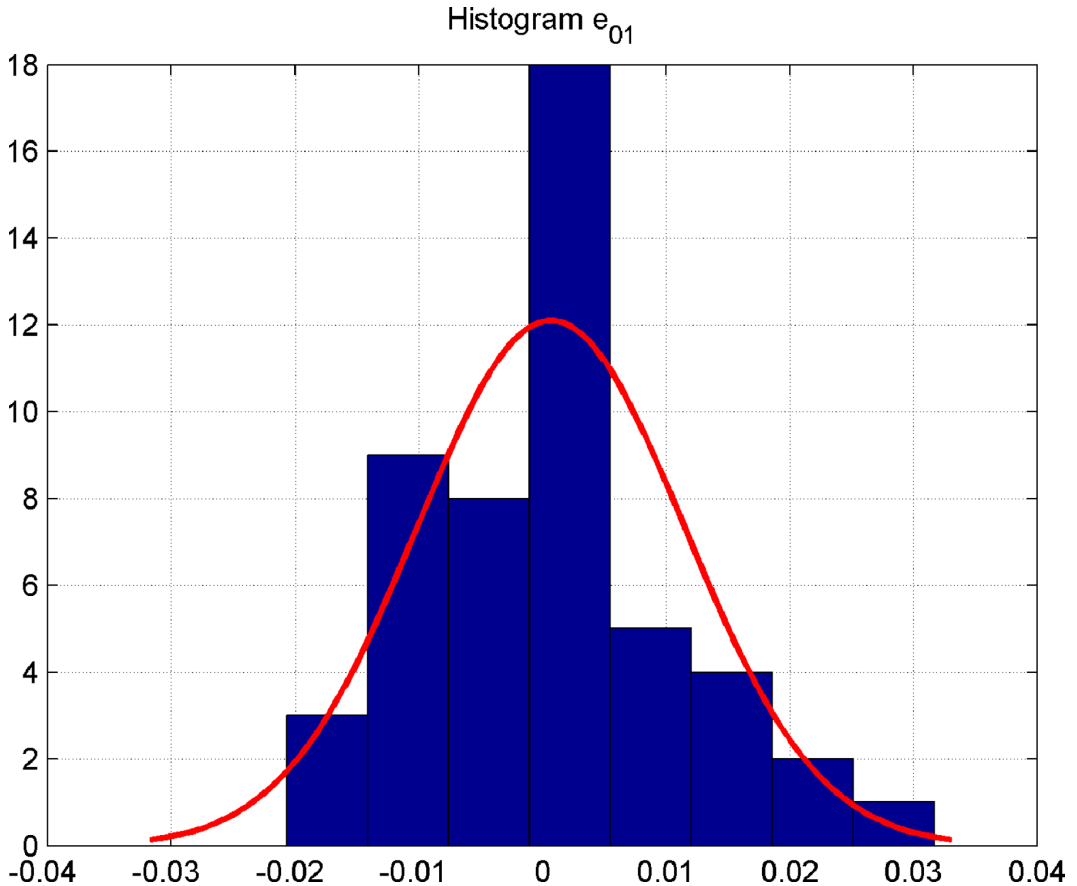
The results rendered by the execution of the above listed procedure and methods are presented in Table 2 and figures 9 to 11:

CONCLUSION

Advanced in the article is a two-step procedure (algorithm), based on the LSM and IV implementation for simultaneous estimation of the three unknown parameters in von Bertalanffy growth equation. Specific software in MATLAB programming environment has additionally been developed for the purposes of the present analysis.

To assess the forecasting quality and accuracy of the approximating model use is made of the variance (standard deviation) of the model error – i.e. the difference between the measured (simulated) values and those forecasted (estimated) by the model.

Figure 1. Histogram (distribution shape) of the simulated measurement error e_{01} (left)



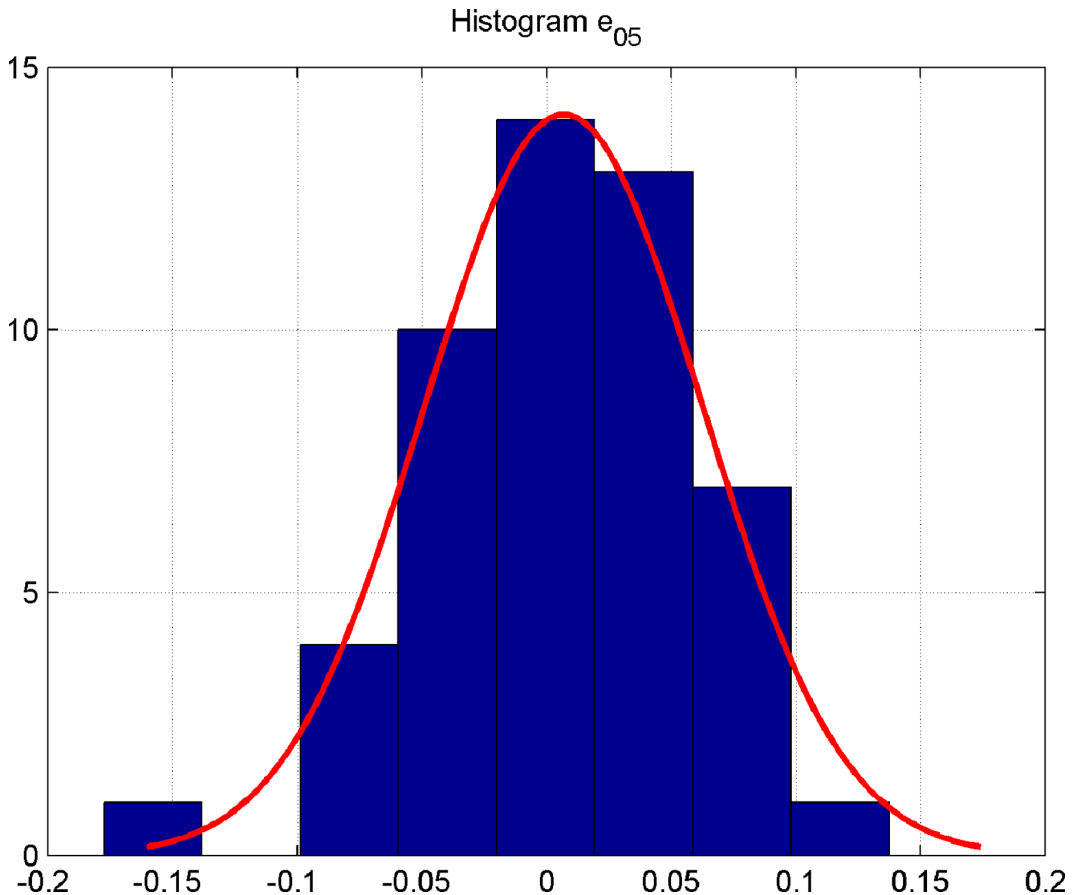
Research studies have been carried out with extra levels of added noise (input of a measurement error in the length or age determination of the given biological object) with the noise burden being normally distributed and generated by the normrnd function with a mathematical expectation of $M = 0$ and a standard deviation $\tilde{A} = S$ from 0.01 to 0.5. The simulated measurement error is 0.07% to 3.5% for L and 1% and 5% for t .

The results of the conducted experiments as to the two-step LS and IV method-based procedure for simultaneous estimation of von Bertalanffy growth parameters: L_{∞} , K and t_0 have been compared to the results obtained by the Gulland-and-Holt method for estimation of L_{∞} and K and subsequent execution of von Bertalanffy method for estimation of t_0 . To achieve the stated objective, developed has been a MATLAB-based program for consecutive implementation of the selected two methods.

The estimates of the growth parameters produced by the proposed procedure for simultaneous estimation of L_{∞} , K and t_0 in the case of non-dirty (non-noisy) data were characterized with a very high degree of accuracy and the two methods were found to be comparatively noise-resistant to added measurement error of L and with values of the added error being up to 3.5%, they tend to yield estimates with a relative accuracy of 1 to 6%, thus, identifying t_0 as the most noise-sensitive parameter.

The results show conclusively that in the presence of higher levels of added noise the estimates of the growth parameters improved significantly due to the applied instrumental variable, which makes the hitherto described two-step LS and IV method-based procedure for simultaneous estimation

Figure 2. Histogram (distribution shape) of the simulated measurement error e_{05} (right)



of von Bertalanffy growth parameters: L_{∞} , K and t_0 extremely appropriate with lower quality of input data (if there are length measurement errors of around 3-4%).

The two surveyed approaches to estimating the growth parameters proved to be particularly sensitive in terms of accuracy upon application of a measurement error in estimating t_0 , and made it necessary for the issue of age determination to be addressed with utmost attention.

The analyzed two-step LS and IV method-based procedure for simultaneous estimation of von Bertalanffy growth parameters: L_{∞} , K and t_0 is applicable not only to data measurements of parameters L and t ($\Delta t = const$) at regular intervals, but also to non-regular measurements of these parameters ($\Delta t \neq const$), provided that the number of data is greater than that of the parameters under estimation. It should be borne in mind that the qualities of the instrumental variable as regards the noise resistance of the estimates are most pronounced with substantial amount of input data and the number of iterations (klu) could be increased until the right stabilization of the estimates has been secured.

Figure 3. Histogram (distribution shape) of the simulated measurement error e_1 (left)

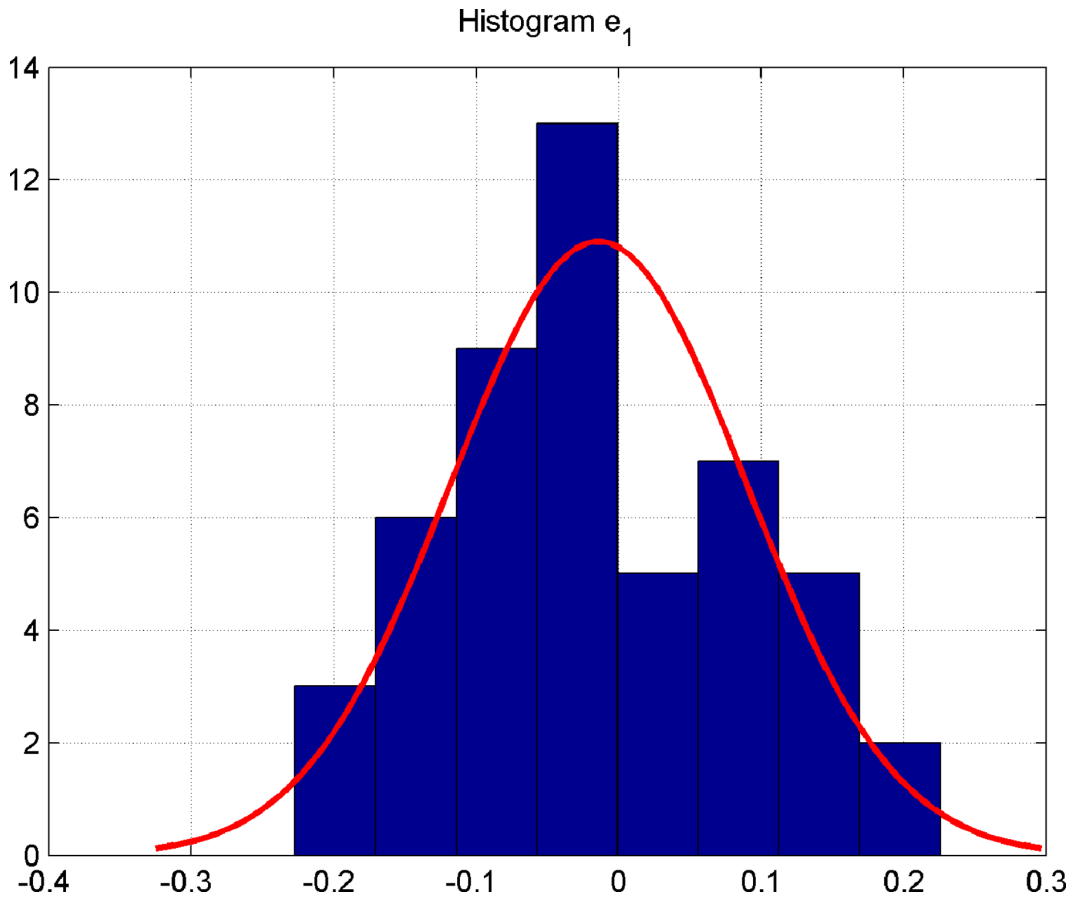


Figure 4. Histogram (distribution shape) of the simulated measurement error e_{15} (right)

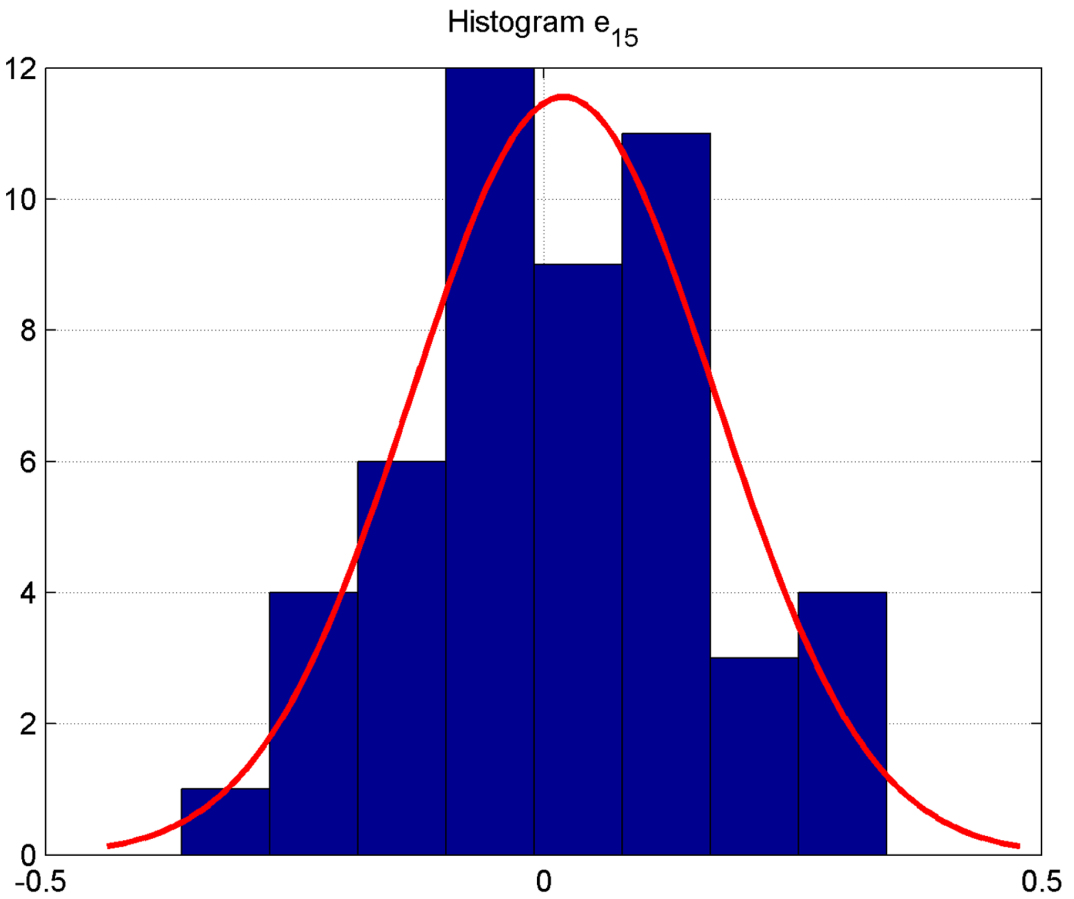


Figure 5. Histogram (distribution shape) of the simulated measurement error e_2 (left)

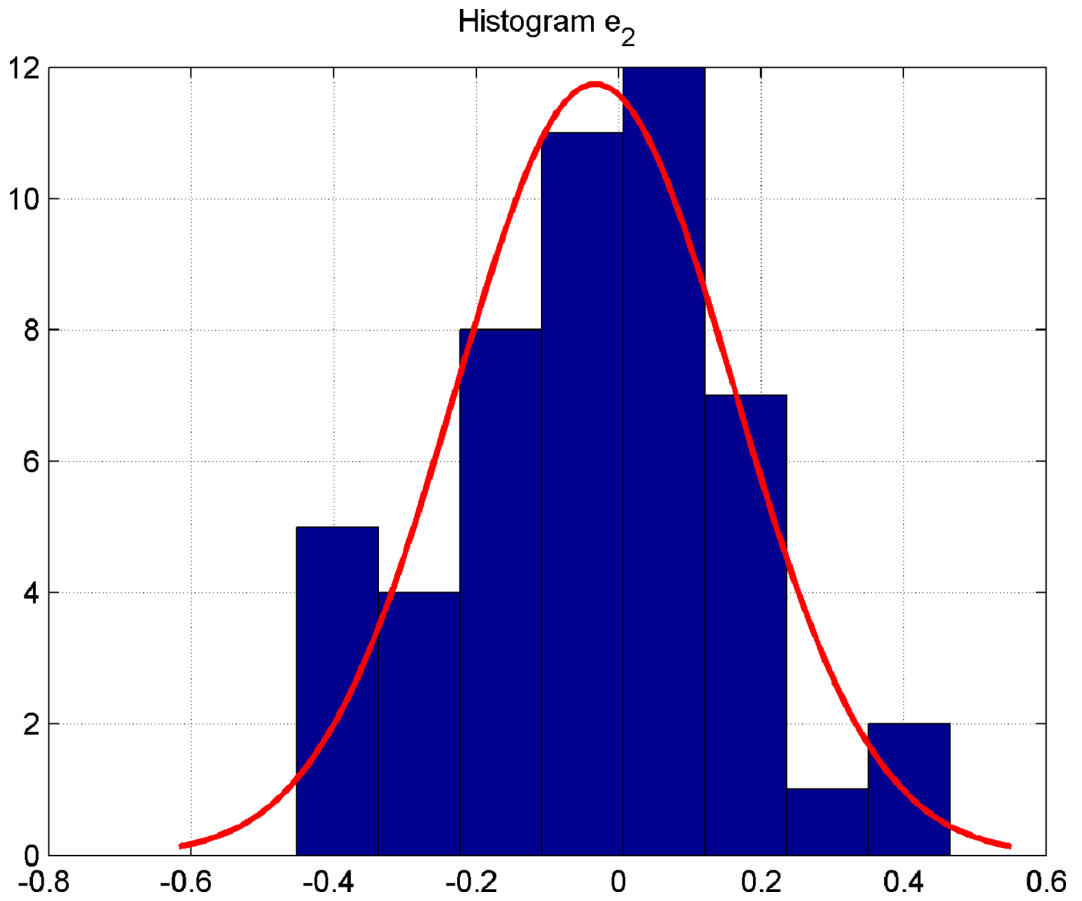


Figure 6. Histogram (distribution shape) of the simulated measurement error e_3 (right)

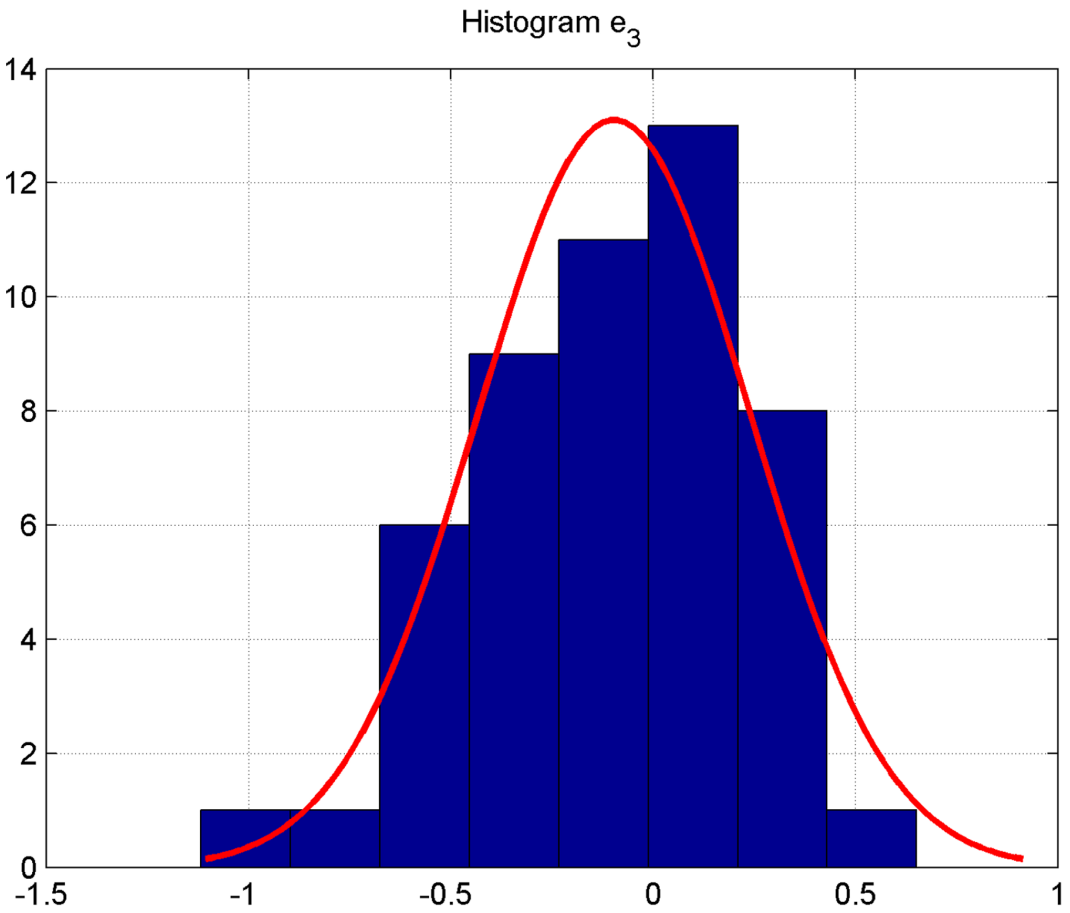


Figure 7. Histogram (distribution shape) of the simulated measurement error e_5 (left)

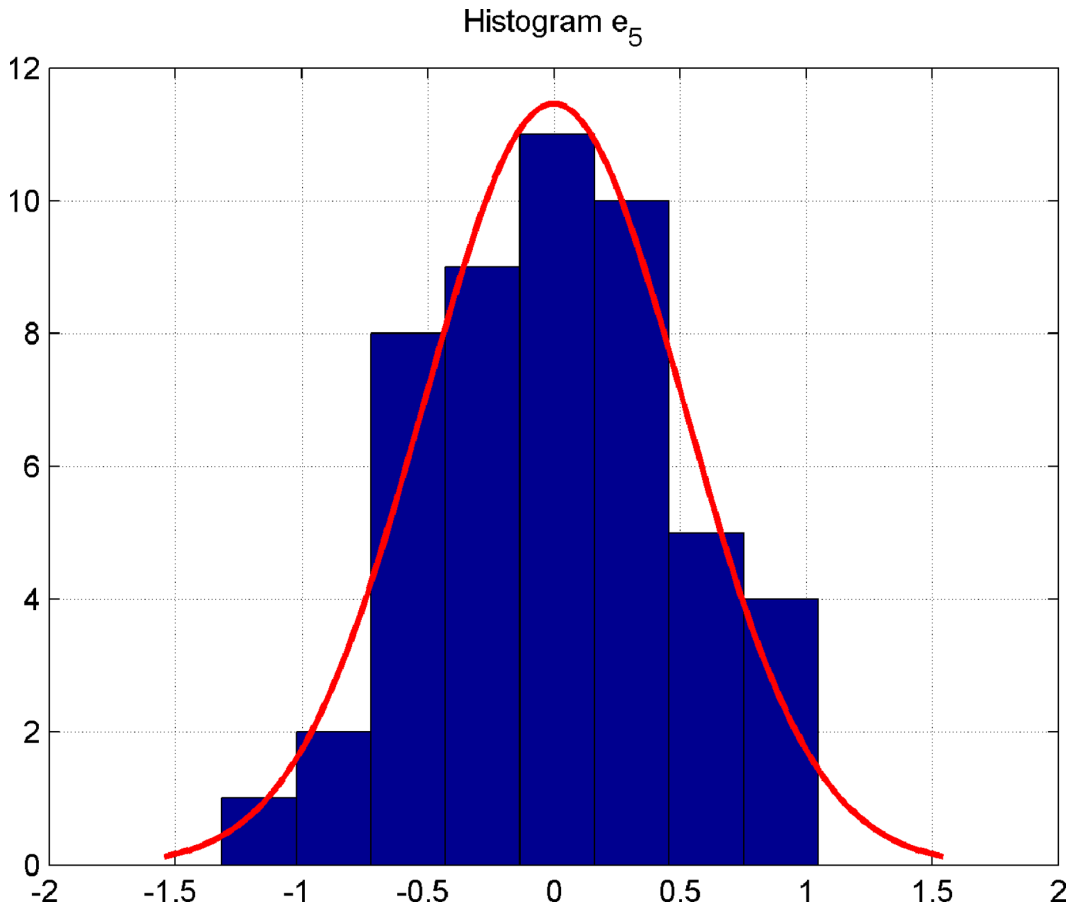


Figure 8. Histogram (distribution shape) of the simulated measurement error e_6 (right)

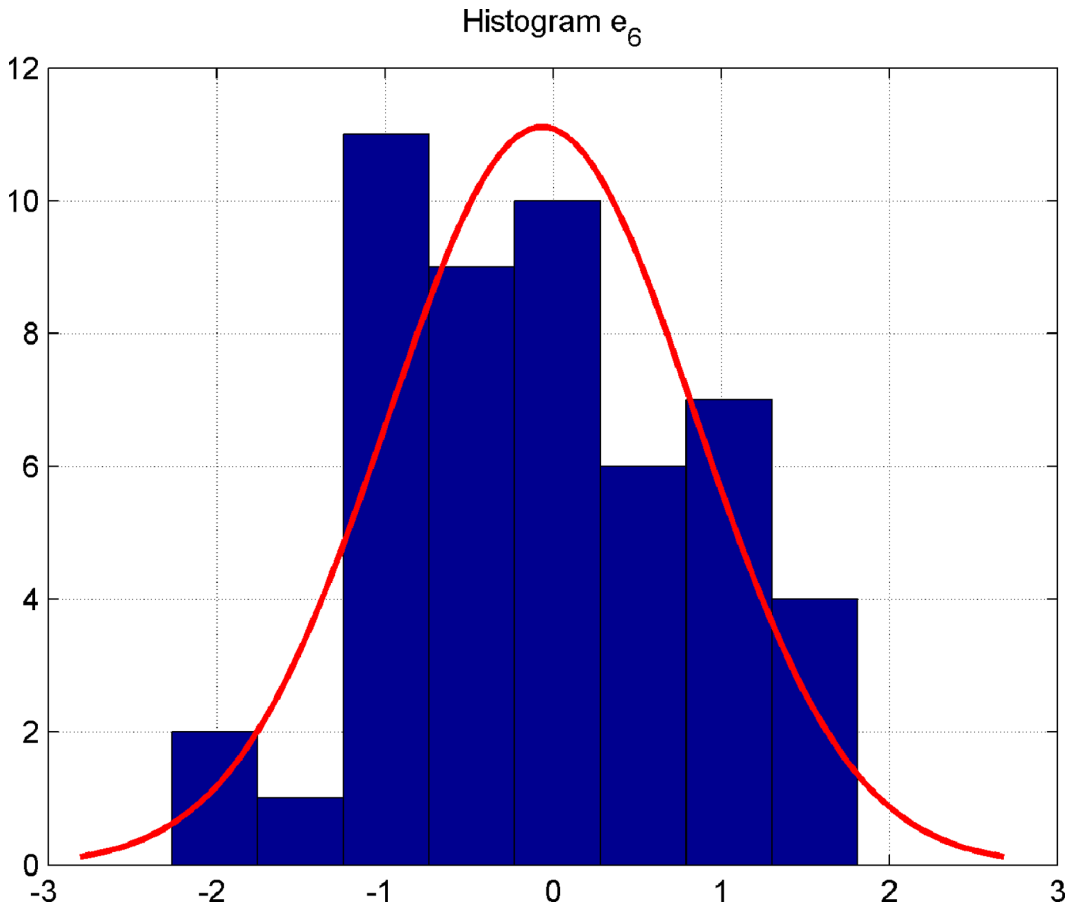


Table 1. Simulated data used to examine the operational workability and estimating accuracy of the calculation procedures and methods and noise levels of t and L

$t = t_{sim}$	$L = L_{sim}$	e_{01}	e_{05}	e_1	e_{15}^*	e_2^*	e_3^*	e_5^*	e_6^*
0.1000	0.5044	-0.0070	-0.0108	-0.1701	-0.0403	0.1269	-0.7347	0.0121	-0.6466
0.1400	2.0249	0.0039	0.1376	0.0791	0.2156	0.0639	-0.5375	1.0478	1.4042
0.1700	3.1344	-0.0083	-0.0708	0.1387	-0.0809	-0.0696	-0.2500	-0.0942	-0.7391
0.2100	4.5741	-0.0125	-0.0122	-0.0183	-0.0645	-0.0007	-0.2621	-0.3221	-0.2809
0.2600	6.3116	-0.0140	0.0020	-0.0908	0.0976	0.1236	-0.4801	-0.5039	0.0681
0.2900	7.3220	0.0028	-0.0687	0.1227	-0.1353	0.4657	-0.2035	-0.9055	-1.4299
0.3300	8.6330	0.0195	0.0540	-0.0511	0.1413	0.1773	-0.2794	-0.2447	-0.8577
0.3900	10.5244	0.0020	0.0372	-0.0818	-0.2525	-0.0488	0.3301	0.1507	-1.0996
0.4100	11.1354	0.0049	0.0771	-0.1262	-0.3631	-0.3946	-0.1830	-0.4920	-2.2647
0.4500	12.3293	0.0014	0.0344	-0.1468	-0.0238	0.0498	-0.3828	0.1267	1.8131
0.5100	14.0517	0.0023	-0.1775	0.0276	0.0493	0.0522	-0.1249	0.3940	-0.6208
0.5300	14.6081	0.0237	-0.0484	-0.0332	0.1913	-0.1583	-0.2055	-0.2966	0.7141
0.5700	15.6953	-0.0078	0.0156	0.0788	0.0180	0.2019	0.0786	0.2461	0.0146
0.6400	17.5181	-0.0016	-0.0408	-0.0057	0.1432	0.0782	-0.0694	0.2827	-0.8450
0.6900	18.7605	-0.0067	0.0142	-0.0210	0.0361	0.1174	-0.4457	-0.0214	-0.0343
0.7200	19.4830	0.0071	-0.0033	-0.1721	0.1019	-0.0972	0.1741	0.9782	0.3440
0.7800	20.8783	0.0026	-0.0274	-0.0775	-0.1453	-0.1368	-0.1092	-0.6361	-0.9714
0.8500	22.4257	-0.0033	0.0034	0.0104	-0.0115	-0.0935	-0.1800	0.1375	1.0184
0.8900	23.2728	0.0074	0.0256	-0.0428	-0.0897	-0.2221	-0.2555	-0.4634	1.0173
0.9300	24.0938	-0.0101	0.0572	0.1275	-0.0355	-0.1045	0.2382	-0.3609	-0.0828
1.1000	27.3109	0.0012	-0.0447	-0.1831	0.3445	0.1454	-0.0327	0.2205	0.5222
1.1600	28.3483	0.0020	-0.0286	-0.0181	-0.2106	-0.4511	-0.4658	0.5464	-1.0358
1.2100	29.1765	0.0157	-0.0360	0.0115	0.0051	-0.3544	0.6512	0.0851	0.4861
1.2600	29.9730	0.0054	0.0424	-0.0722	-0.0481	0.0053	-0.1025	0.6524	-0.1091
1.2900	30.4362	0.0003	0.0786	0.0112	0.0569	0.0657	0.0813	-0.4651	1.3672
1.3200	30.8886	-0.0134	0.0528	0.1661	0.2689	-0.1898	0.1064	0.1691	-0.8135
1.4000	32.0447	-0.0147	0.0671	-0.1050	0.1368	0.1073	-0.3920	-0.2654	1.7098
1.4700	32.9988	0.0059	0.0125	-0.0351	-0.0985	0.2518	0.3663	-0.3787	0.0171
1.5300	33.7761	-0.0070	-0.0401	-0.1419	-0.2151	0.1367	0.0753	-1.3158	0.3787
1.6500	35.2258	-0.0024	0.0340	0.0728	0.1272	-0.3127	0.0431	0.3801	1.2321
1.7200	36.0108	-0.0087	0.0863	-0.1206	-0.1488	-0.1461	0.4052	0.9374	0.9375
1.8500	37.3598	0.0018	0.0253	-0.0858	-0.1073	-0.1708	0.0477	0.2498	-0.7445
1.9200	38.0314	0.0023	0.0465	0.1423	0.0698	-0.0678	-0.5167	0.6882	-0.7780
2.1000	39.5992	0.0174	-0.0772	0.0823	-0.2284	-0.1343	-0.0048	0.0295	-0.7776
2.1700	40.1519	0.0182	0.0286	-0.0756	0.2334	-0.1683	0.3501	-0.6152	0.9135
2.3200	41.2392	0.0106	-0.0077	0.1017	0.0154	0.3820	0.3638	-0.4362	-0.1153
2.4500	42.0840	0.0108	0.0277	-0.0659	-0.0351	-0.0666	0.0794	-0.4013	-0.3780
2.5200	42.5046	-0.0117	0.0638	0.0106	0.1212	-0.0305	-0.5080	0.5512	0.0701
2.6400	43.1744	0.0128	-0.0320	0.1834	0.1209	0.0573	0.3289	-0.2402	0.8804
2.7800	43.8805	-0.0094	0.0753	0.2257	-0.0374	-0.3464	-0.0172	0.0457	-0.1351
2.8600	44.2507	-0.0160	-0.0472	-0.0443	0.1653	-0.2283	0.0576	-0.6666	0.2629
2.9700	44.7234	0.0317	0.0020	-0.0030	0.0551	0.0940	0.3844	-0.0915	-1.9681
3.0100	44.8855	0.0032	0.0153	0.1039	0.1650	0.0703	0.1208	-0.8129	-0.5674
3.1500	45.4146	-0.0066	0.0533	0.0608	0.0395	-0.2322	-1.1167	-0.3512	-0.9066
3.2100	45.6242	0.0001	-0.0162	-0.0233	-0.0287	0.2251	-0.4991	0.3797	-0.5433
3.4400	46.3429	0.0007	-0.0468	-0.0209	0.2623	-0.0133	-0.2484	0.8548	-0.6477
3.9300	47.5045	-0.0207	0.0708	-0.0784	-0.1175	0.0283	-0.3874	0.2427	-0.3588
4.1500	47.8980	-0.0072	0.0181	-0.2272	0.0950	0.0219	0.0460	0.0486	-0.4176
4.2000	47.9784	0.0053	-0.0957	-0.0157	0.3034	-0.1100	0.0456	0.5366	0.5591
4.3500	48.2016	0.0004	0.0160	-0.1198	-0.0620	-0.3663	-0.2079	0.3640	1.2207

*the noise levels e_{15} , e_2 , e_3 and e_5 are added to $L(t)$ measurements only, considering the fact that in relation to t this appear to be approximately 10% error in age determination – in this case it is recommended age determination by using analytical methods or repetition of the experiment.

Table 2. Simulation results L_∞ , K and t_0

Two-steps procedure for simultaneous estimation of the individual growth parameters L_∞ , K and t_0 based on the LS and IV methods (proposed in the present material)	Consecutive execution of: 1. Gulland-Holt method for estimation of K and L_∞ 2. Estimation of t_0 by the von Bertalanffy model (uses the calculated value for L_∞ delivered upon the execution of the preceding method)
Model values: $t_0 = 0.087$; $K = 0.78$; $L_\infty = 50.00$ cm	
1. Simulation data t_{sim} и L_{sim} - graphical representation is depicted in Figure 9	
1.1 After the execution of the LSM $L_\infty = 49.9925$; $K = 0.7801$; $t_0 = 0.0863$ 1.2 After the implementation of the IV $L_\infty = 49.9925$; $K = 0.7801$; $t_{01} = 0.0863$ $S_{otn_model} = \frac{S_e}{S_L} = 7.1075 * 10^{-4} \approx 0$	1.1. Gulland-Holt method for estimation of K and L_∞ $L_\infty = 49.9925$; $K = 0.7801$ 1.2. Von Bertalanffy method for estimation of t_0 $(L_\infty = 49.9925)$ $t_0 = 0.0876$
2. Simulation data: t_{sim} and $L_{01} = L_{sim} + e_{01}$	
2.1 After the execution of the LSM $L_\infty = 49.9380$; $K = 0.7823$; $t_0 = 0.0845$ 2.2 After the implementation of the IV $L_\infty = 49.9380$; $K = 0.7823$; $t_0 = 0.0845$ $S_{otn_model} = \frac{S_e}{S_L} = 0.0031 = 0.31\%$	2.1. Gulland-Holt method for estimation of K and L_∞ $L_\infty = 49.9381$; $K = 0.7823$ 2.2. Von Bertalanffy method for estimation of t_0 $(L_\infty = 49.9381)$ $t_0 = 0.0914$
3. Simulation data: t_{sim} and $L_{05} = L_{sim} + e_{05}$ graphical representation is depicted in Figure 10	
3.1 After the execution of the LSM $L_\infty = 49.8952$; $K = 0.7884$; $t_0 = 0.0907$ 3.2 After the implementation of the IV $L_\infty = 49.8952$; $K = 0.7884$; $t_0 = 0.0907$ $S_{otn_model} = \frac{S_{emodel}}{S_L} = 0.0052 = 0.52\%$	3.1. Gulland-Holt method for estimation of K and L_∞ $L_\infty = 49.8930$; $K = 0.7885$ 3.2. Von Bertalanffy method for estimation of t_0 $(L_\infty = 49.8930)$ $t_0 = 0.0944$
4. Simulation data: t_{sim} and $L_1 = L_{sim} + e_1$	

continued on following page

Table 2.Continued

<p>4.1 After the execution of the LSM $L_{\infty} = 50.1409$; $K = 0.7850$; $t_0 = 0.1158$ 4.2 After the implementation of the IV $L_{\infty} = 50.1389$; $K = 0.7850$; $t_0 = 0.1158$ $S_{otn_model} = \frac{S_{emodel}}{S_L} = 0.0269 = 2.69\%$</p>	<p>4.1. Gulland-Holt method for estimation of K and L_{∞} $L_{\infty} = 50.1409$; $K = 0.7850$ 4.2. Von Bertalanffy method for estimation of t_0 $(L_{\infty} = 50.1409)$ $t_0 = 0.0696$</p>
<p>5. Simulation data: t_{sim} and $L_{15} = L_{sim} + e_{15}$</p>	
<p>5.1 After the execution of the LSM $L_{\infty} = 50.8706$ $L_{\infty} = 50.8706$; $K = 0.7590$; $t_0 = 0.1285$ 5.2 After the implementation of the IV $t_0 = 0.0366$ $L_{\infty} = 50.8651$; $K = 0.7592$; $S_{otn_model} = \frac{S_{emodel}}{S_L} = 0.0460 = 4.60\%$ $t_0 = 0.1283$</p>	<p>5.1. Gulland-Holt method for estimation of K and L_{∞} $K = 0.7590$ 5.2. Von Bertalanffy method for estimation of t_0 $(L_{\infty} = 50.8706)$</p>
<p>6. Simulation data: t_{sim} and $L_2 = L_{sim} + e_2$</p>	
<p>6.1 After the execution of the LSM $L_{\infty} = 50.2648$; $K = 0.7521$; $t_0 = 0.0680$ 6.2 After the implementation of the IV $L_{\infty} = 50.2537$; $K = 0.7525$; $t_0 = 0.0680$ $S_{otn_model} = \frac{S_{emodel}}{S_L} = 0.0193 = 1.93\%$</p>	<p>6.1. Gulland-Holt method for estimation of K and L_{∞} $L_{\infty} = 50.2648$; $K = 0.7521$ 6.2. Von Bertalanffy method for estimation of t_0 $(L_{\infty} = 50.2648)$ $t_0 = 0.0595$</p>
<p>7. Simulation data: t_{sim} and $L_3 = L_{sim} + e_3$</p>	
<p>7.1 After the execution of the LSM $L_{\infty} = 49.6445$; $K = 0.8100$; $t_0 = 0.1165$ 7.2 After the implementation of the IV $L_{\infty} = 49.6637$; $K = 0.8092$; $t_0 = 0.1164$ $S_{otn_model} = \frac{S_{emodel}}{S_L} = 0.0262 = 2.62\%$</p>	<p>7.1. Gulland-Holt method for estimation of K and L_{∞} $L_{\infty} = 49.6445$; $K = 0.8100$ 7.2. Von Bertalanffy method for estimation of t_0 $(L_{\infty} = 49.6445)$ $t_0 = 0.1043$</p>
<p>8. Simulation data: t_{sim} and $L_5 = L_{sim} + e_5$</p>	

continued on following page

Table 2.Continued

<p>8.1 After the execution of the LSM $L_{\infty} = 51.6138$; $K = 0.6796$; $t_0 = 0.0233$ 8.2 After the implementation of the IV $L_{\infty} = 51.5260$; $K = 0.6822$; $t_0 = 0.0328$ $S_{otn_model} = \frac{S_{emodel}}{S_L} = 0.0583 = 5.83\%$</p>	<p>8.1 Gulland-Holt method for estimation of K and L_{∞} $L_{\infty} = 48.9194$; $K = 0.8718$ 8.2. Von Bertalanffy method for estimation of t_0 $(L_{\infty} = 48.9194)$ $t_0 = 0.2288$</p>
<p>9. Simulation data: t_{sim} and $L_6 = L_{sim} + e_6$ graphical representation is depicted in Figure 11</p>	
<p>9.1 After the execution of the LSM $L_{\infty} = 54.8288$; $K = 0.6729$; $t_0 = 0.2478$ 9.2 After the implementation of the IV $L_{\infty} = 54.7668$; $K = 0.6745$; $t_0 = 0.2478$ $S_{otn_model} = \frac{S_{emodel}}{S_L} = 0.2068 = 0.20.68\%$</p>	<p>9.1. Gulland-Holt method for estimation of K and L_{∞} $L_{\infty} = 54.8288$; $K = 0.6729$ 9.2. Von Bertalanffy method for estimation of t_0 $(L_{\infty} = 54.8288)$ $t_0 = -0.1327$</p>
<p>10. Simulation Data: $t_{01} = t_{sim} + e_{01}$ and L_{sim}</p>	
<p>10.1 After the execution of the LSM $L_{\infty} = 50.0168$; $K = 0.9079$; $t_0 = 0.3052$ 10.2 After the implementation of the IV $L_{\infty} = 50.1028$; $K = 0.9043$; $t_0 = 0.2995$ $S_{otn_model} = \frac{S_{emodel}}{S_L} = 0.2353 = 23.87\%$</p>	<p>10.1. Gulland-Holt method for estimation of K and L_{∞} $L_{\infty} = 51.6138$; $K = 0.6796$ 10.2. Von Bertalanffy method for estimation of t_0 $(L_{\infty} = 51.6138)$ $t_0 = 0.0118$</p>
<p>11. Simulation Data $t_{05} = t_{sim} + e_{05}$ and L_{sim}</p>	
<p>11.1 After the execution of the LSM $L_{\infty} = 11.0630$; $K = -0.4741$; $t_0 = 1.2886$ 3.2 After the implementation of the IV $L_{\infty} = 103.4981$; $K = 0.1124$; $t_0 = 1.2886$ $S_{otn_model} = \frac{S_{emodel}}{S_L} = 0.3192 = 31.92\%$</p>	<p>11.1. Gulland-Holt method for estimation of K and L_{∞} The model is proved statistically invalid 10.2. Von Bertalanffy method for estimation of t_0</p>

Figure 9. Growth rate $\frac{dL(t)}{dt}$ functional relationship of $L_{si}(t)$; the 2nd graph (at the bottom) represents L_{sim} – the simulation data model and L_{model} – the model, obtained by implementing the parameters estimated by the Two-step LS and IV method-based procedure in the absence of noise (added measurement error)

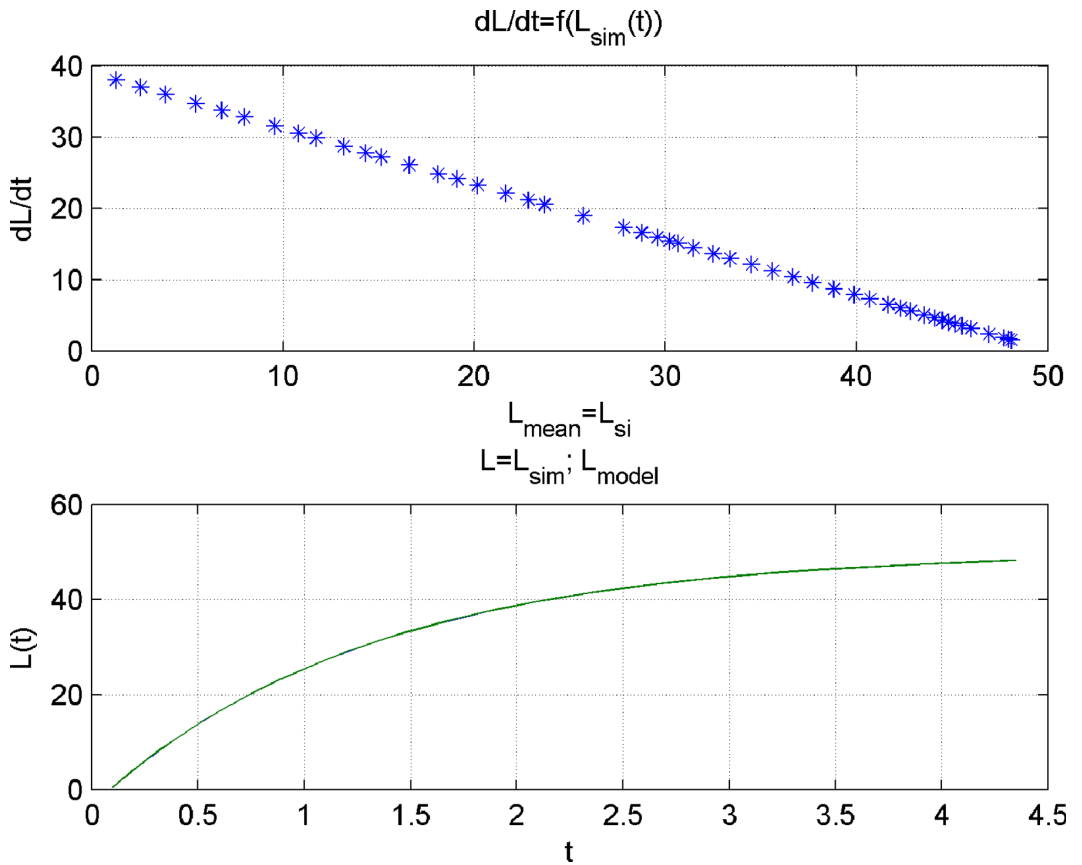


Figure 10. Growth rate $\frac{dL(t)}{dt}$ functional relationship of $L_{si}(t)$; the 2nd graph (at the bottom) represents L_{sim} – the simulation data model and L_{model} – the model, obtained by implementing the parameters estimated by the Two-step LS and IV method-based procedure in the presence of noise (added measurement error) e_2

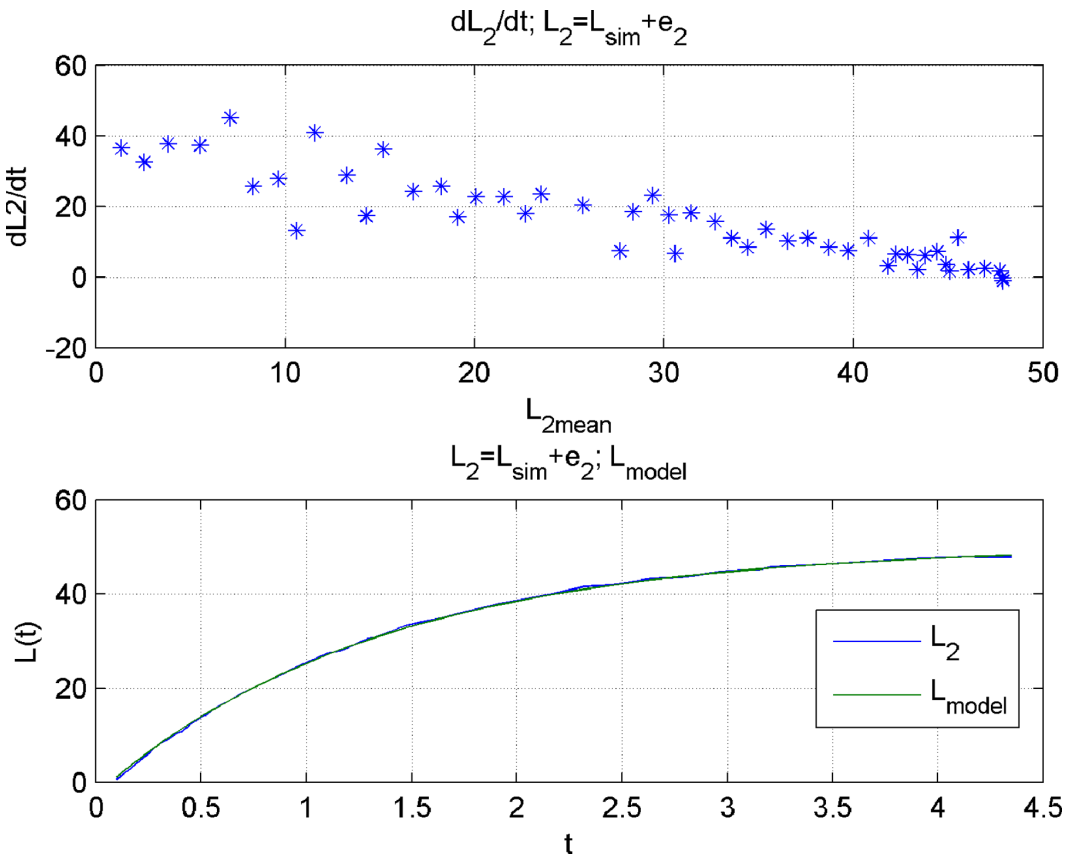
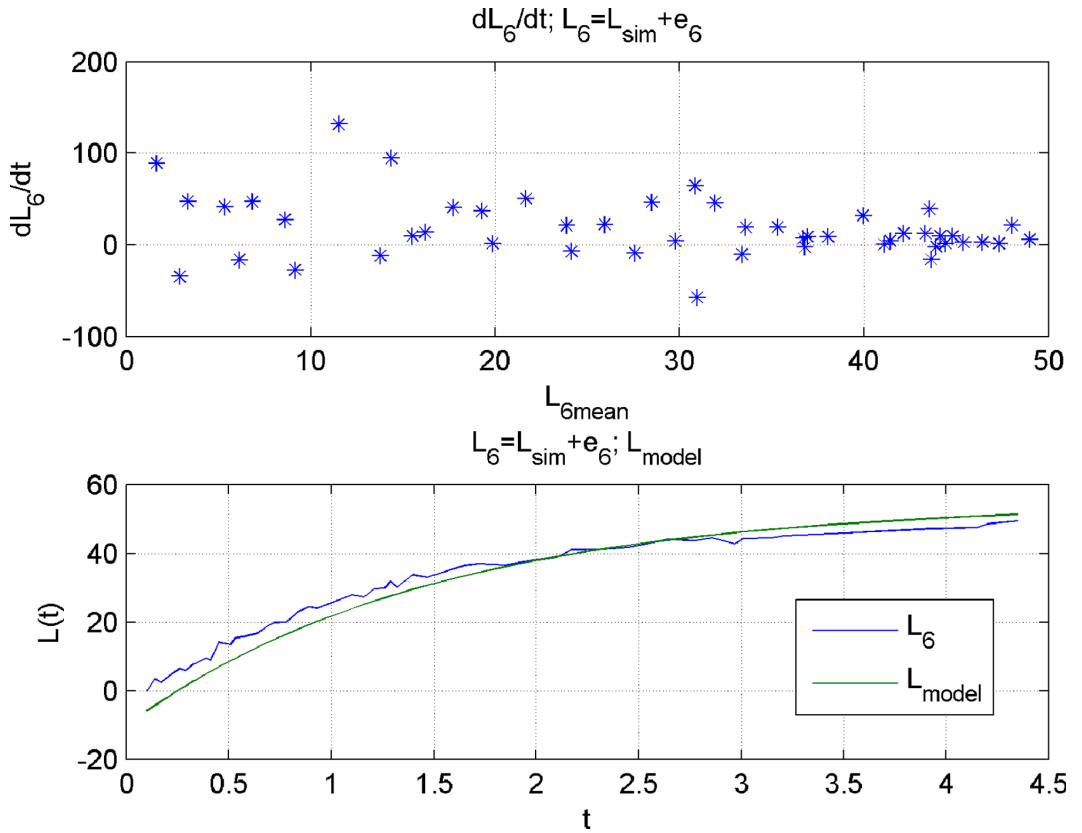


Figure 11. Growth rate $\frac{dL(t)}{dt}$ functional relationship of $L_{si}(t)$; the 2nd graph (at the bottom) represents L_{sim} – the simulation data model and L_{model} – the model, obtained by implementing the parameters estimated by the Two-step LS and IV method-based procedure in the presence of noise (added measurement error) e_6



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APPENDIX A: MATLAB-ENVIRONMENT SCRIPT

```
clc
clear
disp('Two step-procedure for simultaneous estimation of the
individual growth')
disp('parameters L_asimpt, K and t_0, based on LS and IV methods')
% Calculates the growth parameters in the von Bertalanffy model
% t - vector-row with size N - represents the age of fish in
years%
%(deltat=const and deltat=var)%
% L - vector-row with size N - represent the length of fish in cm
for ages t%
% N - number of data
N = input(' Enter the number of measurements N=');
l = importdata('C:\Users\USER\Desktop\PS\GullandH_L6.txt');%Load
.txt with length measurements%
T = importdata('C:\Users\USER\Desktop\PS\GullandH_t.txt');%load
.txt file with the corresponding ages,%
L=l'
t=T'
N=length(L)
disp('Step. 1 Claculate K and Lasimpt')
for j=1:N-1
    tsr(1,j) = (t(1,j+1) + t(1,j))/2;
end
for j=1:N-1
    Lpr(1,j) = (L(1,j+1) - L(1,j)) / (t(1,j+1) - t(1,j));
end
for j=1:N-1
    Lsr(1,j) = (L(1,j+1) + L(1,j)) / 2;
end
plot(Lsr, Lpr, 'o'), grid
for j=1:N-1
    x0(j,1) = 1;
end
X = [x0 -Lsr'];
X1 = X' * Lpr';
X2 = X' * X;
B = inv(X2) * X1
Lust = B(1) / B(2)
K = B(2)
for j=1:N
    Lmodd(1,j) = Lust * (1 - exp(-K * t(1,j)));
end
[L' Lmodd']
plot(t, [L' Lmodd']), grid
disp('Step 2. Claculate t0')
```

```

for j=1:N
    a1=abs(1-(L(1,j)/Lust)); y(1,j)=log(a1)/K;
end
Z=t+y;
t0=sum(Z)/N
for j=1:N
    Lmod(1,j)=Lust*(1-exp(-K*(t(1,j)-t0)));
end
[L' Lmod']
plot(t, [L' Lmod']), grid
disp('IV implementation: V, V1')
% the cycle klu=1:(3-10) can be run as many time as needed until
% stabilization of the estimates is reached%
for klu=1:3
for j=1:N-1
    Lsr1(1,j)=(Lmod(1,j+1)+Lmod(1,j))/2;
end
V=[x0 -Lsr1'];
X1x=V'*Lpr';
X2x=V'*X;
Bb=inv(X2x)*X1x
Lustt=Bb(1)/Bb(2)
Kk=Bb(2)
for j=1:N
    y1(1,j)=log(1-(Lmod(1,j)/Lust))/K;
end
V1=t+y1;
t01=(V1*Z')/sum(V1)
Lmod1=Lustt*(1-exp(-Kk*(t-t01)));
Lmod = Lmod1
K=Kk
Lust=Lustt
t0=t01
end
[L' Lmodd' Lmod' Lmod1']
plot(t, [L' Lmod' Lmod1']), grid
SL=std(L);
SL=std(L);
Eo=(L-Lmod); So=std(Eo); Sotn=So/SL
end
    
```

APPENDIX B: CONTENT OF THE PROGRAM SCRIPT

```

clc
clear
disp('1. Gulland and Holt Method for estimation of Lasimpt and K
')
    
```

```
r = input(' Enter the number of measurements r=');
L = importdata('C:\Users\USER\Desktop\PS\GullandH_L.txt');%Loads
.txt file with Length measurements to form the input vector for
the regression analysis%
t = importdata('C:\Users\USER\Desktop\PS\GullandH_t.txt');%Loads
.txt file with the corresponding age data%
n=r-1
for i = 1:n
    deltat(i)=t(i+1)-t(i);%calculates deltat%
    deltaL(i)=L(i+1)-L(i);%calculates the increment in length for
step deltat%
    y(i)=deltaL(i)/deltat(i);
end
Y=y'
%Calculates the mean length for interval deltat and forms the
independent variable x for the regression analysis%
for i=1:n
    Lmean(i)=((L(i)+deltaL(i))+L(i))/2;
end
x=Lmean'
disp('***a.Regression analysis***')
x0 = ones(n,1);
F = [x0,x];%Forms the regression matrix%
FF = (F'*F);
b =((FF^(-1))*F')*Y; %Calculates the regression coefficients b%
beta = b';
b0 = beta(1,1);
b1 = beta(1,2);
disp('Check the condition number of matrix FF:')
Cond_FF = cond(FF)% determines the condition factor of the
information matrix FF, which forms the covariance matrix, once
inverted - if the condition factor cond(FF) >10-10^2 it is
recommended to run the analysis with normalized/standardized
variables%
maxval = 10;
if (Cond_FF > maxval)
    disp('Cond FF>10 ==>')
    disp('Run the regression analysis with standardized/Normalized
variables')
    xmean = mean(x);%Calculates the mean value of x=Lmean%
    ymean = mean(Y);%Calculates the mean value of y=deltaL/deltat%
    sX = std(x);%Calculates the standard deviation of x%
    Xnorm =(x-xmean)/sX;%Standardizes x (the observed/measured
values)%
    f0 = ones(n,1);
    Fnorm = [f0,Xnorm];%Forms the regression matrix%
    FFnorm = Fnorm'*Fnorm;
    disp('Condition factor of FFnorm')
    Cond_norm = cond(FFnorm)%Claculates the condition factor of
the matrix FFnorm%
```

```
%formed by using the standardized x values%
B = (((FFnorm)^(-1))*Fnorm')*Y;
B1 = B';
disp('Regression coefficient b0n - standardized variables')
b0n = B1(1,1)
disp('Regression coefficient b1n - standardized variables')
b1n = B1(1,2)
disp('Regression coefficient B0 - model')
B0 = (b0n-((b1n*xmean)/sX))
disp('Regression coefficient B1 - model')
B1 = b1n/sX
disp('Calculates the estimation of the dependent variable y_
hat=GrR by using the values calculated for the regression
coefficients yhat=:')
yhat = B0+B1*x
e = Y-yhat
disp('Calculates the model error e:')
scatter(x,Y);grid
title('Regression analysis(Plot (y_m_e_a_s,x) и (y_h_a_t,x)')
hold on
y1 = yhat;
plot(x,y1)
hold off
else
disp('Condition number of matrix FF<10')
b0 = beta(1,1)
disp('Regression coefficient b0=:')
b1 = beta(1,2)
disp('Regression coefficient b1=:')
disp('Calculates the estimation of the dependent variable y_
hat=GrR by using the values calculated for the regression
coefficients yhat=:')
yhat = b0+b1*x
disp('Calculates the model error e:')
e = Y-yhat
scatter(x,Y);grid
title('Regression analysis(Plot (y_m_e_a_s,x) и (y_h_a_t,x)')
hold on
y1 = yhat;
plot(x,y1)
hold off
end
disp('***b. Model Statistical analysis***')
Qo = sum((Y-yhat).^2); %Calculates the partition of the sum of
squares%
Q = sum((Y-ymean).^2);%Calculates the total sum of squares%
Qcalc = Qo/Q;
disp('Calculate the correlation coefficient R')
R = sqrt((1-Qcalc))
disp('F-test the R value and model workability F=')
```

```

F = (R^2 * (n-2)) / ((1-R^2) * 1) % Fc is found in F-test distribution
table to be compared%
%with the calculated value F > Fcr%

Fcr = input('Enter table value for Fcr=:');
if (F > Fcr)
    disp('The model is proved functional and can be used to
predict the behavior of the BO under analysis')
    figure
    scatter(x, Y); grid
    title('Regression analysis (Plot (y_m_e_a_s, x) и
(y_h_a_t, x)') hold on
    yl = yhat;
    plot(x, yl)
    hold off
if (Cond_FF > maxval)
    disp('Calculate the growth parameters K и L_asimpt
by using the calculated regression coefficients')
    disp('K=:')
    K = -B1
    a = B0;
    disp('L_asimpt=:')
    Lasimpt = -(a/B1
else
    disp('The model is proved non-functional and respectively
cannot be used to predict the BO behavior')
end

disp('2. Von Bertalanffy method for estimation of K и t0')
disp('***Provides estimates of K and to if Lasimpt is known***')
r = input(' Enter the number of measurements (observations) r=');
Lasimpt = input(' Enter value for Lasimpt=');
x=t
y=-log(1-(L/Lasimpt))%Forms the output vector y for the regression
analysis%
disp('***a.Regression analysis***')
x0 = ones(r, 1);
F = [x0, x]; %Forms the regression matrix%
FF = (F' * F);
b = ((FF^(-1)) * F') * y; %Calculates the regression coefficients b_i%
beta = b';
b0 = beta(1, 1);
b1 = beta(1, 2);
disp('Check the condition number of the matrix FF:')
Cond_FF = cond(FF) % determines the condition factor of the
information matrix FF1, which forms the covariance matrix, once
inverted - if the condition factor cond(FF) > 10-10^2 it is
recommended to run the analysis with normalized/standardized
variables%
maxval = 10;

```

```
if (Cond_FF > maxval)
    disp('Cond FF>10 ==>')
    disp('Run the regression analysis with standardized/Normalized
variables')
    xmean = mean(x);%Calculates the mean value of x=t%
    ymean = mean(y);%Calculates the mean value of y=-log(1-(L/
Lasimpt))%
    sX = std(x);%Calculates the standard deviation of x%
    Xnorm =(x-xmean)/sX;%Standardizes x values%
    f0 = ones(r,1);
    Fnorm = [f0,Xnorm];%Forms the regression matrix by using the
standardized x values%
    FFnorm = Fnorm'*Fnorm;
    disp('Condition number of matrix FFnorm')
    Cond_norm = cond(FFnorm
    B = ((FFnorm)^(-1))*Fnorm')*y;
    B1 = B';
    disp('Regression coefficient b0n - standardized variables')
    b0n = B1(1,1)
    disp('Regression coefficient b1n - standardized variables')
    b1n = B1(1,2)
    disp('Regression coefficient B0 - model')
    B0 = (b0n-((b1n*xmean)/sX))
    disp('Regression coefficient B1 - model')
    B1 = b1n/sX
    disp('Calculates the estimation of the dependent variable y_
hat by using the values calculated for the regression coefficients
yhat=: ')
    yhat = B0+B1*x
        disp('Calculates the model error e e:')
        e = y-yhat
else
    disp('The condition number of matrix FF<10')
    disp('Regression coefficient b0=:')
    b0 = beta(1,1)
    disp('Regression coefficient b1=:')
    b1 = beta(1,2)

    disp('Calculates the estimation of the dependent
variable y_hat by using the values calculated for the regression
coefficients yhat=:yhat=:')
    yhat = b0+b1*t
    disp('Claculates the model error e:')
    e = y-yhat
end
disp('***b. Model statistical analysis***')
    Qo = sum((y-yhat).^2); % Calculates the partition of
the sum of squares %
    ymean=mean(y);
    Q = sum((y-ymean).^2);% Calculates the total sum of
```



```

squares%
    Qcalc = Qo/Q;
    disp('Correlation coefficient R')
    R = sqrt((1-Qcalc))%Calculates the correlation
coefficient%
    disp('F-test the R value and model workability F=')
    F = (R^2*(r-2))/((1-R^2)*1
    Fcr = input('Enter table value for Fcr=:');
if (F > Fcr)
    disp('The model is proved functional and can be
used to predict the behavior of the BO under analysis')
if (Cond_FF > maxval)
    disp('Estimation of the growth parameters K and t0
by using the regression coefficients estimated')
    disp('Estimation of K=:')
    K=B1
    disp('Estimation of t0=:')
    t0=- (B0/B1)
else
    disp('Estimation of the growth parameters K and
t0 by using the regression coefficients estimated')
    disp(' Estimation of K=:')
    K=b1
    disp('Estimation of t0=:')
    t0=- (B0/B1)
end
else
    disp('The model is proved non-functional and
respectively cannot be used to predict the BO behavior')
end

```

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