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Topological Properties of Multigranular Rough sets on Fuzzy Approximation Spaces

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ABSTRACT

One of the extensions of the basic rough set model introduced by Pawlak in 1982 is the notion of rough sets on fuzzy approximation spaces. It is based upon a fuzzy proximity relation defined over a Universe. As is well known, an equivalence relation provides a granularization of the universe on which it is defined. However, a single relation defines only single granularization and as such to handle multiple granularity over a universe simultaneously, two notions of multigranulations have been introduced. These are the optimistic and pessimistic multigranulation. The notion of multigranulation over fuzzy approximation spaces were introduced recently in 2018. Topological properties of rough sets are an important characteristic, which along with accuracy measure forms the two facets of rough set application as mentioned by Pawlak. In this article, the authors introduce the concept of topological property of multigranular rough sets on fuzzy approximation spaces and study its properties.

KEYWORDS

Fuzzy Approximation Spaces, Optimistic Multigranulation, Pessimistic Multigranulation, Rough Sets, Topological Properties

1. INTRODUCTION

Equivalence relations (ERs) are used to define Rough sets (RS) and are modeled to capture uncertainty in data (Pawlak, 1982). Successful application of rough sets depends upon two notions; the topological characterization and the accuracy measure as observed by Pawlak (1991). Granules are the smallest addressable units of data. The study of granules is beneficial from the point of view that the characteristics of all the elements in a granule are similar in view of the real life applications. It makes the study simpler and concise. Zadeh in 1979 introduced Granular computing (GC) in the context of fuzzy sets and again it was revived by him in 1997. Study of granules reduces the complexity of algorithms without affecting the goals of study. The first definition of rough sets uses unigranular structure from the GC point of view. However, the notion was generalised by introducing rough sets on a class of ERs. This is perhaps the first instance of mutigranularities. In many real-life applications, it is highly required to handle multiple granularity of a universe of discourse taken simultaneously. In 2006 the first type of mutigranularities, called the optimistic multigranulation based upon rough sets was introduced in and later in the second type called pessimistic multigranulation based upon rough sets was introduced and studied (Qian & Liang, 2006; Qian et al., 2007; Qian et al., 2010). Since then, multigranular rough sets for other rough set models like fuzzy rough sets, rough sets

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on incomplete approximation spaces, covering based rough sets (CBRS) and fuzzy approximation spaces (FAS) have been studied by several authors. Several types of CBRS have been introduced in the literature. Topological approach to CBRS is introduced and studied in (Tripathy & Nagaraju, 2012; Tripathy & Nagaraju, 2015; Qian et al., 2007; De, 1999; De et al., 2003; Zhu, 2002; Zhu & Wang, 2007; Zhu, 2007).

The latest among the extensions of the multigranulation concept is to rough set model is to the notion of rough sets on FAS (Tripathy & Bhambani, 2018). A formal definition is to be provided in the next section. In this case a fuzzy proximity relation is taken as the basis for defining a rough set instead of an ER. The rough set models are associated with two types of properties; the algebraic properties and the topological properties. The whole set comes under consideration with respect to topological properties, whereas the individual elements come under the algebraic properties. Two important concepts associated with the study of rough sets are; their accuracy measure and categories (termed as categories by Pawlak). Both the properties should be considered simultaneously for dealing with any application successfully. Depending upon the structure of the lower and upper approximations of a set it can be categorized under 4 categories. Here we focus to deal with MGRS on FAS. It is interesting to find the categories of set theoretic operations of union, intersection and complement of a set. It has enough applications to the classification of sets as has been shown in Tripathy (2006, 2010). So, this study will pave the way for some analysis in the direction of these two papers. Classifications are the building blocks of rough sets in its basic form and the studies in confirm the statement of Pawlak that the complement in bi-valued logic is different from the notion of complements in case of multi-valued logic cab be highlighted further (Tripathy et al., 2006; Tripathy & Mitra, 2010).

The arrangement of the different sections of this paper henceforth is that the second section presents various concepts to be used in the paper. We introduce rough sets on FAS section 3 along with their properties. Topological properties in the context of basic rough sets are discussed in section 4. Next, we deal with the topological properties of MGRS on FAS.by determining the categories of sets obtained by the three set theoretic operations of union, intersection and complement in section 5. This is followed by concluding remarks on the work done in the next section.

2. DEFINITIONS AND NOTATIONS

Below we present some of the concepts to be used in this paper along with their definitions. We shall be adopting the notations introduced here. We begin with the presentation of Fuzzy set placed by Zadeh in 1965. Unless otherwise mentioned, we take V as a universal set for all our discussions.

Definition 2.1: The notion of a fuzzy set A is equivalent to its membership function mem_A where mem_A associates every element x of V with a membership value $mem_A(x)$ lying in the interval [0, 1].

Associated with mem_A is a sister function called $non-mem_A$, which is the one's complement of mem_A, that is $non-mem_A(x)=1-mem_A(x)\ \forall$ x in V.

Pawlak introduced RS in (1982). This model is completely following the notion of uncertainty through boundary regions introduced by G. Frege. The formal definition is as follows.

R is an ER over V. As is well known, R induces a partition over V and the partition elements are called EQ classes. We denote the EQ class corresponding to an element x in V by $eq_R(x)$. As per Pawlak, for the study of RSs, we V and a set of ERs E defined over it as a knowledge base. The intersection of a set of ERs E is an ER and is denoted by IND(E). Following the standard notations, we say for any subset F of E, IND(F) as the indiscernibility relation over F.

Definition 2.2: For any set Z in V and an indiscernibility relation B, two crisp sets called the lower and upper approximations of Z with respect to B, denoted by $\underline{B}Z$ and $\overline{B}Z$ defined as $\underline{B}Z = \{z \in V \mid eq_B(z) \subseteq Z\}$ and $\overline{B}Z = \{z \in V \mid eq_B(z) \bigcap Z \neq \phi\}$

The difference between the sets $\overline{B}X$ and $\underline{B}X$ is called the b-boundary of X, $Boundary_B(X)$ represents the region of uncertainty associated with X with respect to B. In fact, $\underline{B}X$, $\overline{B}X$ and $Boundary_B(X)$ contain the certain elements, possible elements and the uncertain elements of X with respect to B.

If the $Boundary_B(X)$ is empty then there are no uncertain elements in X with respect to B and hence it reduces to a B-definable set. Equivalently, X is B-definable iff $\underline{B}X = \overline{B}X$ Otherwise X is said to be rough with respect to B.

Multigranular computing deals with more than two granulations defined over a universe simultaneously. As mentioned earlier these are the optimistic MGRS and pessimistic MGRS. The notations used for bi-multigranulation were $C+D$ for optimistic Multigranulation and $C*D$ for pessimistic multigranulation. The notations used in the original papers for the two categories of Multigranulation were different. The notations used are simpler and expressive and so we use these notations in this paper (Tripathy et al., 2010). It may be noted that for multiple ERs $R_1, R_2, \ldots R_n$, the optimistic multigranulation is denoted by $\sum_{i=1}^{n} R_i$ and the pessimistic multigranulation is denoted by $\prod_{i=1}^{n} R_i$.

Definition 2.3: Let C and D be two ERs over V. Optimistic lower, upper approximations of a subset W of V by $\underline{C+D}(W)$ and $\overline{C+D}(W)$, respectively and define them as:

(2.1) $\underline{\underline{C+D}}\, W = \{u \mid eq_C(u) \subseteq W \text{ or } eq_D(u) \subseteq W\}$

(2.2) $\overline{C+D}\, W = (\underline{C+D}(W^C))^C$

Definition 2.4: Let C, D and W be as above. Pessimistic MG lower approximation and pessimistic MG upper approximation of W by $\underline{C*D}(W)$ and $\overline{C*D}(W)$, respectively and define them as

(2.5) $\underline{\underline{C*D}}\, (W) = \{u \mid eq_C(u) \subseteq W \text{ and } eq_D(u) \subseteq W\}$

(2.6) $\overline{C*D}\, W = (\underline{C*D}(W^C))^C$

A fuzzy relation over V is any fuzzy subset of $V \times V$.

Definition 2.5: A fuzzy relation C over V is said to be a fuzzy proximity relation (FPR) over V if and only if

(2.7) C is fuzzy reflexive, that is $mem_C(u,u) = 1, \forall u \in V$.

(2.8) C is fuzzy symmetric if and only if $mem_C(u,v) = mem_C(v,u), \forall u, v \in V$.

Let us denote the unit interval [0, 1] by I.

Definition 2.6: The $\gamma - cut$ A_γ of any FPR A, where γ is in I is a subset of $V \times V$ is given by

(2.9) $A_\gamma = \{(u,v) \mid mem_A(u,v) \geq \gamma\}$.

For any FPR A on V and γ in I, we say that u and v are $\gamma - similar$ if $(u,v) \in A_\gamma$ and denote it by $uA_\gamma v$.

Definition 2.7: u and v are $\gamma - identical$ written as $uA(\gamma)v$ if $uA_\gamma v$ or for a sequence of elements $c_1, c_2, ...c_n$ in V the relation $uA_\gamma c_1 A_\gamma c_2 A_\gamma ... A_\gamma v$ holds true.

Clearly it follows that $A(\gamma)$ is an ER for any γ in I. Here, (V, A) is called a FAS. The advantage of taking the $\gamma - identical$ relation is that $(V, A(\gamma))$ is an approximation space as per the notion used in Pawlakian RSs.

Following the standard notations, $[y]_{A(\gamma)}$ denotes the similarity class of any y in V with respect to $A(\gamma)$.

Definition 2.8: Let C be a FAR on V. Then for any γ in I, the lower and upper approximations of W in V for as (Qian et al., 2007; Qian et al., 2010)

(2.10) $\underline{C(\gamma)}W = \{u \in V \mid eq_{C(\gamma)}(u) \subseteq W\}$ and

(2.11) $\overline{C(\gamma)}W = \{u \in V \mid eq_{C(\gamma)}(u) \cap W \neq \phi\}$.

If the lower and upper approximations are equal then W is said to be $C(\gamma)$ definable. Else, W is said to be $C(\gamma)$-rough.

The notion of RSs on FAS was introduced by De et al (1999). We take it further by defining the two notions of optimistic multigranular RSs (OMGRS) on FAS and pessimistic multigranular RSs (PMGRS). Also, we shall study many of their properties and provide an application in a real-life situation.

In topological properties (TPs) of PMGRS have been obtained (Tripathy, 2012). In knowledge representation using rough sets on FAS is discussed (Tripathy, 2010). The study of topological properties for RS has started (Tripathy, 2010). TPs for optimistic and pessimistic MGRS were studied respectively (Tripathy & Raghavan, 2011; Tripathy & Nagaraju, 2012). Similarly, TPs for incomplete MGRS and CB rough sets are presented in respectively (Tripathy & Nagaraju, 2015; Tripathy & Mitra, 2009). Also, some more TPs of CBRS are obtained. Algebraic properties of MGRS are discussed (Tripathy & Mitra, 2009; Tripath & Mitra, 2011). Neighbourhood based RSs (NBRSs) are extensions of basic RSs (Hu et al., 2008). TPs for NBRS are presented (Tripath & Mitra, 2014). Neighbourhood rough sets for MGRS was introduced and studied (Lina et al., 2012). An extension of MGRS to the context of fuzzy sets can be found. A beautiful application of MGRS on FA to prediction of rainfall can be found (Tripathy et al., 2018).

Let C and D be two FARs over V. Then for any subset Y of V and γ in I we define

(2.12) $\underline{C(\gamma)+D(\gamma)}Z = \{u \in V \mid eq_{C(\gamma)}(u) \subseteq Z \ or \ eq_{D(\gamma)}(u) \subseteq Z\}$

(2.13) $\overline{C(\gamma)+D(\gamma)}Z = \{u \in V : eq_{C(\gamma)}(u) \cap Z \neq \phi \ and \ eq_{D(\gamma)}(u) \cap Z \neq \phi\}$

Definition 2.9: A $(C + D)\ \gamma - multigranular$ RS Z is such that $\underline{C(\gamma)+D(\gamma)}Z \neq \overline{C(\gamma)+D(\gamma)}Z$. Otherwise, we say Z is $(C + D)\ \gamma - multigranular$ definable.

In order to maintain simplicity, the word optimistic is not used as it is evident from the notation used. We define

(2.14) $\underline{C(\gamma)*D(\gamma)}Z = \{u \in V : eq_{C(\gamma)}(u) \subseteq Z \ and \ eq_{D(\gamma)}(u) \subseteq Z\}$

(2.15) $\overline{C(\gamma)*D(\gamma)}Z = \{u \in V : eq_{C(\gamma)}(u) \cap Z \neq \phi \ or \ eq_{D(\gamma)}(u) \cap Z \neq \phi\}$

Definition 2.10: A $(C*D)\ \gamma-multigranular$ RS Z is such that $\underline{C(\gamma)*D(\gamma)}Z \neq \overline{C(\gamma)*D(\gamma)}Z$. Otherwise, we say Z is $(C*D)\ \gamma-multigranular$ definable.

For any γ in I, we define

(2.16) $\underline{C(\gamma)+D(\gamma)}Z = \{u \in V : eq_{C(\gamma)}(u) \subseteq Z\ or\ eq_{D(\gamma)}(u) \subseteq Z\ \}$

(2.17) $\overline{C(\gamma)+D(\gamma)}Z = \{u \in V : eq_{C(\gamma)}(u) \bigcap Z \neq \phi\ and\ eq_{D(\gamma)}(u) \bigcap Z \neq \phi\}$

Definition 2.11: A $(C+D)\ \gamma-multigranular$ RS Z is such that $\underline{C(\gamma)+D(\gamma)}Z \neq \overline{C(\gamma)+D(\gamma)}Z$. Otherwise, we say Z is $(C+D)\ \gamma-multigranular$ definable

Similar to above, the word optimistic is excluded from the definition. So,

(2.18) $\underline{C(\gamma)*D(\gamma)}Z = \{u \in V : eq_{C(\gamma)}(u) \subseteq Z\ and\ eq_{D(\gamma)}(u) \subseteq Z\}$

(2.19) $\overline{C(\gamma)*D(\gamma)}Z = \{u \in V : eq_{C(\gamma)}(u) \bigcap Z \neq \phi\ or\ eq_{D(\gamma)}(u) \bigcap Z \neq \phi\}$

Definition 2.12: A $(C*D)\ \gamma-multigranular$ RS Z is such that $\underline{C(\gamma)*D(\gamma)}Y \neq \overline{C(\gamma)*D(\gamma)}Y$. Otherwise, we say Z is $(C*D)\ \gamma-multigranular$ definable

Note 2.1: It is worth noting that

(2.20) $\overline{C(\gamma)+D(\gamma)}(Z) =\sim \left[\underline{C(\gamma)+D(\gamma)}(\sim Z)\right]$ and $\overline{C(\gamma)*D(\gamma)}(Z) =\sim \left[\underline{C(\gamma)*D(\gamma)}(\sim Z)\right]$.

3. PROPERTIES OF MGRS ON FAS

Several properties of MGRS on FAS were established in Tripathy et al (2012). Here, we present some of the properties which will be used in this paper.

We take C and D as two FARs over V and any γ in I in the theorems below. Let Z and W be subsets of V. Then

Theorem 3.1: We have

(3.1) $\underline{C(\gamma)+D(\gamma)}Z \subseteq Z \subseteq \overline{C(\gamma)+D(\gamma)}Z$

(3.2) $\underline{C(\gamma)+D(\gamma)}\phi = \overline{C(\gamma)+D(\gamma)}\phi = \phi$ and $\underline{C(\gamma)+D(\gamma)}V = \overline{C(\gamma)+D(\gamma)}V = V$

(3.3) $Z \subseteq W \Rightarrow \underline{C(\gamma)+D(\gamma)}Z \subseteq \underline{C(\gamma)+D(\gamma)}$W and $\overline{C(\gamma)+D(\gamma)}Z \subseteq \overline{C(\gamma)+D(\gamma)}W$

Theorem 3.2: We have

(3.4) $\underline{C(\gamma)*D(\gamma)}Z \subseteq Z \subseteq \overline{C(\gamma)*D(\gamma)}Z$

(3.5) $\underline{C(\gamma)*D(\gamma)}\phi = \overline{C(\gamma)*D(\gamma)}\phi = \phi$ and $\underline{C(\gamma)*D(\gamma)}V = \overline{C(\gamma)*D(\gamma)}V = V$

(3.6) $Z \subseteq W \Rightarrow \underline{C(\gamma)*D(\gamma)}Z \subseteq \underline{C(\gamma)*D(\gamma)}W$ and $\overline{C(\gamma)*D(\gamma)}Z \subseteq \overline{A(\gamma)*B(\gamma)}W$

Theorem 3.3:

(3.7) $\underline{C(\gamma)*D(\gamma)}(Z \bigcup W) = \underline{C(\gamma)*D(\gamma)}(Z) \bigcup \underline{C(\gamma)*D(\gamma)}(W)$

(3.8) $\underline{C(\gamma)*D(\gamma)}(Z \bigcup W) \supseteq \underline{C(\gamma)*D(\gamma)}(Z) \bigcup \underline{C(\gamma)*D(\gamma)}(W)$

(3.9) $\overline{C(\gamma)*D(\gamma)}(Z\cup W)\supseteq\overline{C(\gamma)*D(\gamma)}(Z)\cup\overline{C(\gamma)*D(\gamma)}(W)$

(3.10) $\overline{C(\gamma)*D(\gamma)}(Z\cup W)=\overline{C(\gamma)*D(\gamma)}(Z)\cup\overline{C(\gamma)*D(\gamma)}(W)$

Theorem 3.4:

(3.11) $\underline{C(\gamma)*D(\gamma)}(Z\cap W)\subseteq\underline{C(\gamma)*D(\gamma)}(Z)\cap\underline{C(\gamma)*D(\gamma)}(W)$

(3.12) $\underline{C(\gamma)*D(\gamma)}(Z\cap W)=\underline{C(\gamma)*D(\gamma)}(Z)\cap\underline{C(\gamma)*D(\gamma)}(W)$

(3.13) $\overline{C(\gamma)*D(\gamma)}(Z\cap W)=\overline{C(\gamma)*D(\gamma)}(Z)\cap\overline{C(\gamma)*D(\gamma)}(W)$

(3.14) $\overline{C(\gamma)*D(\gamma)}(Z\cap W)\subseteq\overline{C(\gamma)*D(\gamma)}(Z)\cap\overline{C(\gamma)*D(\gamma)}(W)$

Theorem 3.5: If $\gamma\geq\delta$ then, we have

(3.15) $\underline{C(\delta)+D(\delta)}Z\subseteq\underline{C(\gamma)+D(\gamma)}Z$

(3.16) $\overline{C(\delta)+D(\delta)}Z\subseteq\overline{C(\gamma)+D(\gamma)}Z$

(3.17) $\underline{C(\delta)*D(\delta)}Z\subseteq\underline{C(\gamma)*D(\gamma)}Z$

(3.18) $\overline{C(\delta)*D(\delta)}Z\subseteq\overline{C(\gamma)*D(\gamma)}Z$

4. TOPOLOGICAL PROPERTIES OF RSS

In general, the properties which deal with whole sets are called topological properties (TP). TPs are useful in any application of RSs for complete analysis. In fact, Pawlak says that it is complement to accuracy measures. Together these two concepts provide better analysis (Pawlak, 1982; Pawlak, 1991). To summarize, it was noted by him that "The accuracy coefficient expresses how large the boundary region of the set is but says nothing about the structure of the boundary whereas the topological classification of RSs gives no information about the size of the boundary region but provides us with some insight as to how the boundary region is structured" (Pawlak, 1991). Topological characterization leads to four categories of RSs. The characterization depends upon the structures of the lower and upper approximations. These are as given below. Here we take Y a subset of V and A as an ER over V. These categories are ctg-1, ctg-2, ctg-3 and ctg-4 and are defined as:

(4.1) ctg-1 (Y is roughly A-definable): If $\underline{A}(Y)\neq\phi$ and $\overline{A}(Y)\neq V$.

(4.2) ctg-2 (Y is internally A-undefinable): If $\underline{A}(Y)=\phi$ and $\overline{A}(Y)\neq V$

(4.3) ctg-3 (Y is externally A-undefinable): If $\underline{A}(Y)\neq\phi$ and $\overline{A}(Y)=V$

(4.4) ctg-4 (Y is totally A-undefinable): If $\underline{A}(Y)=\phi$ and $\overline{A}(Y)=V$

The intuitive meaning of each of these categories of elements is discussed (Pawlak, 1982).

The importance of the study of the categories of union and intersection of RSs of different categories has been discussed in several papers (Tripathy, 2012; Tripathy & Mitra, 2010; Tripathy & Raghavan, 2011; Tripathy & Nagaraju, 2012; Tripathy & Mitra, 2009). An interesting application is provided (Tripathy & Mitra, 2010). The study of distributed knowledge base systems can be handled through the categories of the union and intersection of RSs sets.

5. TOPOLOGICAL PROPERTIES OF MGRS ON FAS (MGRFAS)

Following the above definitions, we state the following topological characterisations for the MGRFAS.

Here the operation 'o' represents '$+$' or '$*$'.

Definition 5.1: Let $Y \subseteq V$ and A and B be two FPRs on V and γ be in I. Different categories for a subset Y of V with respect to multigranulation 'o' of RSs on FAS are defined below

(5.1) δ-1 (Roughly $A(\gamma)\ o\ B(\gamma)$-definable): If $\underline{A(\gamma)\ o\ B(\gamma)}(Y) \neq \phi$ & $\overline{A(\gamma)\ o\ B(\gamma)}(Y) \neq V$

(5.2) δ-2 (Internally $A(\gamma)\ o\ B(\gamma)$-undefinable): If $\underline{A(\gamma)\ o\ B(\gamma)}(Y) = \phi$ & $\overline{A(\gamma)\ o\ B(\gamma)}(Y) \neq V$

(5.3) δ-3 (Externally $A(\gamma)\ o\ B(\gamma)$-undefinable): If $\underline{A(\gamma)\ o\ B(\gamma)}(Y) \neq \phi$ & $\overline{A(\gamma)\ o\ B(\gamma)}(Y) = V$

(5.4) δ-4 (Totally $A(\gamma)\ o\ B(\gamma)$-undefinable): If $\underline{A(\gamma)\ o\ B(\gamma)}(Y) = \phi$ & $\overline{A(\gamma)\ o\ B(\gamma)}(Y) = V$

It is worth noting that substitution of 'o' by '$+$' or '$*$' make the type of multigranulation clear and also the presence of 'α' specifies the degree of the FAS.

5.1. Illustrative Example

Let us illustrate the above concepts through an example. We consider three FPRs given in tables 1-3 below.

Let $\gamma = 0.8$. Then

$C_\gamma^* = \{\{\phi_1,\phi_2\},\{\phi_3,\phi_4\},\{\phi_5,\phi_6\}\}$, $D_\gamma^* = \{\{\phi_1,\phi_2\},\{\phi_3\},\{\phi_4,\phi_5\},\{\phi_6\}\}$ and $E_\gamma^* = \{\{\phi_1,\phi_2,\phi_3,\phi_6\},\{\phi_4,\phi_5\}\}$

Optimistic Case:

1. Let us take $P = \{\phi_1,\phi_2,\phi_3,\phi_4\}$. Then $\underline{C(\gamma) + D(\gamma)}P = \{\phi_1,\phi_2,\phi_3,\phi_4\} \neq \phi$ and $\overline{C(\gamma) + D(\gamma)}P = \{\phi_1,\phi_2,\phi_3,\phi_4,\phi_5\} \neq V$. So, P is of δ-1.

Table 1. FPR A

C	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6
ϕ_1	1	.8	.6	.3	.5	.2
ϕ_2	.8	1	.7	.6	.4	.3
ϕ_3	.6	.7	1	.9	.6	.5
ϕ_4	.3	.6	.9	1	.7	.6
ϕ_5	.5	.4	.6	.7	1	.9
ϕ_6	.2	.3	.5	.6	.9	1

Table 2. FPR B

D	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6
ϕ_1	1	.8	.2	.5	.6	.7
ϕ_2	.8	1	.4	.7	.5	.2
ϕ_3	.2	.4	1	.5	.3	.6
ϕ_4	.5	.7	.5	1	.9	.7
ϕ_5	.6	.5	.3	.9	1	.6
ϕ_6	.7	.2	.6	.7	.6	1

Table 3. FPR C

E	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6
ϕ_1	1	.9	.5	.6	.7	.2
ϕ_2	.9	1	.8	.2	.2	.8
ϕ_3	.5	.8	1	.3	.4	.2
ϕ_4	.6	.2	.3	1	.8	.3
ϕ_5	.7	.2	.4	.8	1	.4
ϕ_6	.2	.8	.2	.3	.4	1

2. Let us take $Q = \{\phi_1, \phi_4\}$. Then $\underline{C(\gamma) + D(\gamma)}Q = \phi$ and $\overline{C(\gamma) + D(\gamma)}Q = \{\phi_1, \phi_2, \phi_4\} \neq V$ So, Q is of δ-2.
3. Let us take $T = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_6\}$. Then $\underline{C(\gamma) + D(\gamma)}T = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_6\} \neq \phi$ and $\overline{C(\gamma) + D(\gamma)}T = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\} = V$. So, T is of δ-3.
4. Let us take $W = \{\phi_1, \phi_3, \phi_5\}$. Then $\underline{C(\gamma) + D(\gamma)}W = \phi$ and $\overline{C(\gamma) + D(\gamma)}W = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\} = V$. So, W is of δ-4.

Pessimistic Case:

1. Let us take $P = \{\phi_1, \phi_2, \phi_3, \phi_4\}$. Then $\underline{C(\gamma) * D(\gamma)}P = \{\phi_1, \phi_2, \phi_3\} \neq \phi$ and $\overline{C(\gamma) * D(\gamma)}P = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5\} \neq V$. So, P is of δ-1.
2. Let us take $Q = \{\phi_1, \phi_4\}$. Then $\underline{C(\gamma) * D(\gamma)}Q = \phi$ and $\overline{C(\gamma) * D(\gamma)}Q = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5\} \neq V$. So, Q is of δ-2.
3. Let us take $T = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_6\}$. Then $\underline{C(\gamma) * D(\gamma)}T = \{m_1, m_2, m_3\} \neq \phi$ and $\overline{C(\gamma) + D(\gamma)}T = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\} = V$. So, T is of δ-3.
4. Let us take $W = \{\phi_1, \phi_3, \phi_5\}$. Then $\underline{C(\gamma) * D(\gamma)}W = \phi$ and $\overline{C(\gamma) + D(\gamma)}W = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\} = V$. So, W is of δ-4.

3.2. A Real life Example

We take an information system "High production" with the conditional attributes: Seed quality (S), Soil fertility (F) and Temperature (T) and decision attribute (P).

The attribute domains are as follows:

S = {Good, Average, Low}, F = {Good, Average, Low}, T = {Top, Middle, Bottom} and HP = {Y, N}

We take 6 areas $\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6$. Let the table 4 below provide the Agricultural environment of the regions as:

Let $V = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\}$. The fuzzy proximity relations Seed quality (S), Soil fertility (F) and Temperature (T) and decision attribute (P) (Tables 5-10).

Now, let us take the target set as $X = \left\{\phi_1, \phi_2, \phi_3\right\}$ corresponding to the "yes" category of "High production" and find out which areas are similar with respect to the two granularities seed quality and soil fertility.

Let us take γ as 0.8. The partitions of V induced from the above fuzzy proximity relations Seed quality and soil fertility as provided are as follows:

$S_\gamma^* = \{\{\phi_1, \phi_2, \phi_3, \phi_5\}, \{\phi_4, \phi_6\}\}$ and $F_\gamma^* = \{\{\phi_1, \phi_5\}, \{\phi_2, \phi_3, \phi_4\}, \{\phi_6\}\}$

Table 4. Agricultural information

Area	Seed quality	Soil fertility	Temperature	High Production
ϕ_1	Good	Good	Top	Y
ϕ_2	Average	Good	Top	N
ϕ_3	Average	Average	Middle	Y
ϕ_4	Low	Average	Middle	N
ϕ_5	Average	Low	Bottom	Y
ϕ_6	Low	Low	Bottom	N

Table 5. Seed quality

S	Good	Average	Low
Good	1	.8	.5
Average	.8	1	.7
Low	.5	.7	1

Table 6. Soil fertility

F	Good	Average	Low
Good	1	.7	.2
Average	.7	1	.4
Low	.2	.4	1

Table 7. Temperature

T	Top	Middle	Bottom
Top	1	.9	.5
Middle	.9	1	.8
Bottom	.5	.8	1

Table 8. Fuzzy proximity relation for Seed quality on *V*

Seed quality	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6
ϕ_1	1	.8	.8	.5	.8	.5
ϕ_2	.8	1	1	.7	1	.7
ϕ_3	.8	1	1	.7	1	.7
ϕ_4	.5	.7	.7	1	.7	1
ϕ_5	.8	1	1	.7	1	.7
ϕ_6	.5	.7	.7	1	.7	1

The OM lower and upper approximations of X for Seed quality and fertility are, $\underline{S(\gamma)+F(\gamma)}X=\{\phi_1,\phi_5\}$ and $\overline{S(\gamma)+F(\gamma)}X=\{\phi_1,\phi_2,\phi_3,\phi_5\}$. So, X is of optimistic $\delta-1$ with respect to seed quality and fertility to a degree of 0.8

Table 9. Fuzzy Proximity relation for fertility on *V*

Fertility	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6
ϕ_1	1	.7	.7	.7	1	.4
ϕ_2	.7	1	1	1	.7	.2
ϕ_3	.7	1	1	1	.7	.2
ϕ_4	.7	1	1	1	.7	.2
ϕ_5	1	.7	.7	.7	1	.4
ϕ_6	.4	.2	.2	.2	.4	1

Table 10. Fuzzy proximity relation for Temperature on *V*

Temperature	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6
ϕ_1	1	1	.9	.9	.5	.5
ϕ_2	1	1	.9	.9	.5	.5
ϕ_3	.9	.9	1	1	.8	.8
ϕ_4	.9	.9	1	1	.8	.8
ϕ_5	.5	.5	.8	.8	1	1
ϕ_6	.5	.5	.8	.8	1	1

If we change the value of γ to 0.7 then $S_\gamma^* = V$ and $F_\gamma^* = V$. So, $\underline{S(\gamma)+F(\gamma)}X = \phi$ and $\overline{S(\gamma)+F(\gamma)}X = V$. So, X is of optimistic $\delta - 2$ with respect to seed quality and fertility to a degree of 0.7.

Again, changing the value of γ to 0.9, we find $S_\gamma^* = \{\{\phi_1\},\{\phi_2,\phi_3,\phi_5\},\{\phi_4,\phi_6\}\}$ and $F_\gamma^* = \{\{\phi_1,\phi_5\},\{\phi_2,\phi_3,\phi_4,\phi_6\}\}$. So, $\underline{S(\gamma)+F(\gamma)}X = \{\phi_1,\phi_5\}$, $\overline{S(\gamma)+F(\gamma)}X = V$. So, X is of optimistic $\delta - 3$ with respect to seed quality and fertility to a degree of 0.9.

Similarly, the PM lower and upper approximations of X for seed quality and fertility are, $\underline{S(\gamma)*F(\gamma)}X=\phi$ and $\overline{S(\gamma)*F(\gamma)}X=\{\phi_1,\phi_2,\phi_3,\phi_4,\phi_5\}$. So, X is of pessimistic $\delta-2$ with respect to seed quality and fertility to a degree 0.8.

With γ as 0.7 $\underline{S(\gamma)*F(\gamma)}X=\phi$ and $\overline{S(\gamma)*F(\gamma)}X=V$. So, X is of pessimistic $\delta-4$ with respect to seed quality and fertility to a degree of 0.7.

Finally, changing γ to 0.9, $\underline{S(\gamma)*F(\gamma)}X=\{A_1\}$ and $\overline{S(\gamma)*F(\gamma)}X=V$. So, X is of pessimistic $\delta-3$ with respect to seed quality and fertility to a degree of 0.9.

4. COMPLEMENT OF A MGRFAS (OPTIMISTIC AND PESSIMISTIC)

Topological characterisations for complement of a MGRFAS X are same as that for basic RSs, which follows from their definitions and the proof of these properties in the base case (Table 11). So, we have

Proof of (2, 1): Using the properties (2.20) and (2.21) **t**he proofs of δ-2 and δ-3 follow directly. Next, we consider the case of δ-1.

Suppose an Optimistic MGRFAS Y is of δ-1. Then $\underline{C(\gamma)+D(\gamma)}(Y)\neq\phi$ and $\overline{C(\gamma)+D(\gamma)}(Y)\neq V$. From (2.20) as $\overline{C(\gamma)+D(\gamma)}(Y)=\left[\underline{C(\gamma)+D(\gamma)}(Y^C)\right]^C$, $\left[\underline{C(\gamma)+D(\gamma)}(Y^C)\right]^C\neq V$. So, $\underline{C(\gamma)+D(\gamma)}(Y^C)\neq\phi$. Also, $\overline{C(\gamma)+D(\gamma)}(Y^C)=\left[\underline{C(\gamma)+D(\gamma)}(Y)\right]^C\neq V$. Hence Y^C is of δ-1.

Similarly, the δ-4 case can be established.

Next, suppose an Optimistic MGRFAS X is of δ-2. Then $\underline{C(\gamma)+D(\gamma)}(Y)=\phi$ and $\overline{C(\gamma)+D(\gamma)}(Y)\neq V$. Now by (2.20) $\overline{C(\gamma)+D(\gamma)}(Y)=\left[\underline{C(\gamma)+D(\gamma)}(Y^C)\right]^C$. So, $\left[\underline{C(\gamma)+D(\gamma)}(Y^C)\right]^C\neq V$.

Hence $\underline{C(\gamma)+D(\gamma)}(Y^C)\neq\phi$. Also, $\overline{C(\gamma)+D(\gamma)}(Y^C)=\left[\underline{C(\gamma)+D(\gamma)}(Y)\right]^C=V$. So, Y^C is of δ-3.

Similarly, δ--3 case can be handled. Proof in the pessimistic case can be established by taking (2.21) in place of (2.20) in the above reasoning process. We shall state the other properties for the optimistic and pessimistic cases separately as the properties of lower and upper approximations for union and intersection of two MGRFASs for these two categories are different as shown in Theorem 3.3 and Theorem 3.4 above.

5. UNION OF TWO OPTIMISTIC MGRFASS Y AND Z

In Table 12 we present the types of the union of two OMGRFASs from the types of their components.

Table 11. Types of complement of a set

X	δ-1	δ-2	δ-3	δ-4
X^C	δ-1	δ-3	δ-2	δ-4

Table 12. Types of union of two OMGRFASs

Z **Y**	δ **-1**	δ **-2**	δ **-3**	δ **-4**
δ **-1**	δ -1/ δ -3	δ -1/ δ -3	δ -3	δ -3
δ **-2**	δ -1/ δ -3	δ -2/ δ -4	δ -3	δ -4
δ **-3**	δ -3	δ -3	δ -3	δ -3
δ **-4**	δ -3	δ -4	δ -3	δ -4

It may be observed that the table has a smaller number of ambiguous entries and also in the ambiguous entries the number of options is less than the unigranular RS case. In fact, there are only four ambiguous entries herein comparison to 7 such cases in the unigranular case. There is no entry with all four choices also as in the base case.

Proof:

Let us consider the entries (1, 1), which is ambiguous in nature and (3, 1), which is certain in nature.

Proof of (1, 1)

By definition of δ -1, $\underline{C(\gamma)+D(\gamma)}(Y) \neq \phi$, $\overline{C(\gamma)+D(\gamma)}(Y) \neq V$, $\underline{C(\gamma)+D(\gamma)}(Z) \neq \phi$, $\overline{C(\gamma)+D(\gamma)}(Z) \neq V$ .Nowby(3.7) $\underline{C(\gamma)+D(\gamma)}(Y \cup Z) = \underline{C(\gamma)+D(\gamma)}(Y) \cup \underline{C(\gamma)+D(\gamma)}(Z) \neq \phi$.

Also, by (3.9) $\overline{C(\gamma)+D(\gamma)}(Y \cup Z) \supseteq \overline{C(\gamma)+D(\gamma)}(Y) \cup \overline{C(\gamma)+D(\gamma)}(Z)$, may or may not be equal to V . Hence $(Y \cup Z)$ is of δ -1 or of δ -3.

Proof of (3, 1)

Let Y be of δ -3 and Z be of δ -1. Then

$$\underline{C(\gamma)+D(\gamma)}(Y) \neq \phi, \overline{C(\gamma)+D(\gamma)}(Y) = V, \underline{C(\gamma)+D(\gamma)}(Z) \neq \phi, \overline{C(\gamma)+D(\gamma)}(Z) \neq V .$$

By (3.7) $\underline{C(\gamma)+D(\gamma)}(Y \cup Z) \neq \phi$ and by (3.9) $\overline{C(\alpha)+D(\alpha)}(X \cup Y) = U$. So, $(X \cup Y)$ is of δ -1.

Example to show that (2, 2) can actually occur.

6. UNION OF TWO PESSIMISTIC MGRFASS Y AND Z

In Table 13 we present the types of the union of two PMGRFASs from the types of their components.

Proof:

We consider two entries, (2, 2) which is the most ambiguous and (3, 3) which is certain.

Proof of (2, 2)

Let Y and Z be both of δ -2. Then $\underline{C(\gamma)*D(\gamma)}(Y) = \phi = \underline{C(\gamma)*D(\gamma)}(Z)$ and $\overline{C(\gamma)*D(\gamma)}(Y) \neq V \neq \overline{C(\gamma)*D(\gamma)}(Z)$. So, by (3.8) $\underline{C(\gamma)*D(\gamma)}(Y \cup Z) \supseteq \underline{C(\gamma)*D(\gamma)}(Y) \cup \underline{C(\gamma)*D(\gamma)}(Z)$ may or may not be equal to ϕ .

By (3.10) $\overline{C(\gamma)*D(\gamma)}(Y \cup Z) = \overline{C(\gamma)*D(\gamma)}(Y) \cup \overline{C(\gamma)*D(\gamma)}(Z)$ may or may not be equal to V.

Table 13. Types of union of two PMGRFASs

Z Y	δ -1	δ -2	δ -3	δ -4
δ -1	δ -1/ δ -3	δ -1/ δ -3	δ -3	δ -3
δ -2	δ -1/ δ -3	δ -1/ δ -2/ δ -3/ δ -4	δ -3	δ -3/ δ -4
δ -3	δ -3	δ -3	δ -3	δ -3
δ -4	δ -3	δ -3/ δ -4	δ -3	δ -3/ δ -4

Hence, all the four possibilities are there. That is $Y \bigcup Z$ can be of any of the categories δ -1/ δ -2/ δ -3/ δ -4.

Proof of (3, 3)

Let Y and Z be both of δ -3. Then $\underline{C(\gamma) * D(\gamma)}(Y) \neq \phi \neq \underline{C(\gamma) * D(\gamma)}(Z)$ and $\overline{C(\gamma) * D(\gamma)}(Y) = V = \overline{C(\gamma) * D(\gamma)}(Z)$.

So, by (3.8) $\underline{C(\gamma) * D(\gamma)}(Y \bigcup Z) \neq \phi$ and by (3.9) $\overline{C(\gamma) * D(\gamma)}(Y \bigcup Z) = V$. Hence $Y \bigcup Z$ is of δ -3.

It may be observed that the entries in this table are same as it was in the union table for basic RSs. The two tables are same.

7. INTERSECTION OF TWO OPTIMISTIC MGROFASS Y AND Z

In Table 14 we present the types of the intersection of two OMGRFASs from the types of their components.

Proof: We take two cases from the table (1, 1) which is an ambiguous entry and (4, 2) which is a certain entry.

Proof of (1, 1)

Suppose both Y and Z are of δ -1. Then $\underline{C(\gamma)+D(\gamma)}(Y) \neq \phi \neq \underline{C(\gamma)+D(\gamma)}(Z)$ and $\overline{C(\gamma)+D(\lambda)}(Y) \neq V \neq \overline{C(\gamma)+D(\gamma)}(Z)$.

Then by (3.11) $\underline{C(\gamma)+D(\gamma)}(Y \bigcap Z) \subseteq \underline{C(\gamma)+D(\gamma)}(Y) \bigcap \underline{C(\gamma)+D(\gamma)}(Z)$. So, we get $\underline{C(\gamma)+D(\gamma)}(Y \bigcap Z)$ may or may not be ϕ.

Table 14. Types of intersection of two OMGRFASs

Z Y	δ -1	δ -2	δ -3	δ -4
δ -1	δ -1/ δ -2	δ -2	δ -3/ δ -4	δ -4
δ -2	δ -2	δ -2/ δ -4	δ -4	δ -4
δ -3	δ -3/ δ -4	δ -4	δ -3/ δ -4	δ -4
δ -4	δ -2	δ -2	δ -4	δ -4

Also, by (3.13) $\overline{C(\gamma)+D(\gamma)}(Y\bigcap Z)=\overline{C(\gamma)+D(\gamma)}(Y)\bigcap\overline{C(\gamma)+D(\gamma)}(Z)$. So, we get $\overline{C(\gamma)+D(\gamma)}(Y\bigcap Z)$ as not equal to V. Hence, $Y\bigcap Z$ is of δ -1 or δ -2.

Proof of (4, 2)

Suppose Y and Z are of δ -4 and δ -2 respectively. Then $\underline{C(\gamma)+D(\gamma)}(Y)=\phi=\underline{C(\gamma)+D(\gamma)}(Z)$, $\overline{C(\gamma)+D(\gamma)}(Y)=V$ and $\overline{C(\gamma)+D(\gamma)}(Z)\neq V$.

By (3.11) $\underline{C(\gamma)+D(\gamma)}(Y\cap Z)=\phi$ and by (3.13) $\overline{C(\gamma)+D(\gamma)}(Y\cap Z)\neq V$. So, $Y\cap Z$ is of δ -2.

It may be observed that the table has a smaller number of ambiguous entries and also in the ambiguous entries the number of options is less than the unigranular RS case. In fact, there are only five ambiguous entries herein comparison to 7 such cases in the unigranular case. There is no entry with all four choices also as in the base case.

8. INTERSECTION OF TWO PESSIMISTIC MGRFASS Y AND Z

In Table 15 we present the types of the union of two PMGRFASs from the types of their components.

Proof: We take two cases from the table (1, 3) which is an ambiguous entry and (2, 2) which is a certain entry.

Proof of (1, 3)

Let us take X to be of δ -1 and Y to be of δ 3. Then $\underline{C(\gamma)*D(\gamma)}(Y)\neq\phi\neq\underline{C(\gamma)*D(\gamma)}(Z)$ and $\overline{C(\gamma)*D(\gamma)}(Y)\neq V\neq\overline{C(\gamma)*D(\gamma)}(Z)$. Then by (3.12)

$\underline{C(\gamma)*D(\gamma)}(Y\cap Z)=\underline{C(\gamma)*D(\gamma)}(Y)\cap\underline{C(\gamma)*D(\gamma)}(Z)$, So, $\underline{C(\gamma)*D(\gamma)}(Y\cap Z)$ can be both ϕ or not ϕ.

Again by (3.14) $\overline{C(\gamma)*D(\gamma)}(Y\cap Z)\subseteq\overline{C(\gamma)*D(\gamma)}(Y)\cap\overline{C(\gamma)*D(\gamma)}(Z)$ is not equal to V. Hence $Y\bigcap Z$ can be of δ -1 or δ -2.

Proof of (2, 2)

Let Y and Z be both of δ -2. Then $\underline{C(\gamma)*D(\gamma)}(Y)=\phi=\underline{C(\gamma)*D(\gamma)}(Z)$ and $\overline{C(\gamma)*D(\gamma)}(Y)=V=\overline{C(\gamma)*D(\gamma)}(Z)$.

Then by (3.12) $\underline{C(\gamma)*D(\gamma)}(Y\cap Z)=\underline{C(\gamma)*D(\gamma)}(Y)\cap\underline{C(\gamma)*D(\gamma)}(Z)$. So, $\underline{C(\gamma)*D(\gamma)}(Y\cap Z)=\phi$.

Again by (3.14) $\overline{C(\gamma)*D(\gamma)}(Y\cap Z)\subseteq\overline{C(\gamma)*D(\gamma)}(Y)\cap\overline{C(\gamma)*D(\gamma)}(Z)$ is not equal to V. Hence $Y\bigcap Z$ is of δ -2.

Table 15. Types of union of two PMGRFASs

Z **Y**	δ **-1**	δ **-2**	δ **-3**	δ **-4**
δ **-1**	δ -1/ δ -2	δ -2	δ -1/ δ -2	δ -2/ δ -4
δ **-2**	δ -2	δ -2	δ -2/ δ -4	δ -2/ δ -4
δ **-3**	δ -1/ δ -2/ δ -3/ δ -4	δ -2/ δ -4	δ -3/ δ -4	δ -2/ δ -4
δ **-4**	δ -2/ δ -4	δ -2/ δ -4	δ -2/ δ -4	δ -2/ δ -4

It may be noted that in this table out of 16 entries as many as 13 are ambiguous. This is quite high in comparison to the 7 number of ambiguous entries in the intersection table for basic RSs.

9. CONCLUSION

Multigranular RSs on FAS (MGRFAS) are introduced very recently. The new additions here are the Topological characterizations of such type of RSs. Using the notion of categories of such RSs, we established the categories of complement, union and intersection of MGRFAS. It has been observed that the complement table remains same. The cardinality of entries in the different tables for union and intersection for optimistic multigranulations on FAS (OMGRFAS) got reduced in comparison to the corresponding tables for basic RSs. However, in case of pessimistic multigranular FAS (PMGRFAS) the ambiguities in the table for union remain same, whereas for intersection the cardinality of the set of ambiguous entries increases in case of intersection. So, with respect to topological characteristics OMGRFAS is better than PMGRFAS.

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