Blockchain Traceability Valuation for Perishable Agricultural Products Under Demand Uncertainty

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ABSTRACT

Various perishable agricultural products are recalled due to harmful health risks. Blockchain has been used to reduce the amount of such products wasted and disposed. Specifically, a supply chain with a wholesaler, a retailer, and customers is considered where the retailer decides when to switch from a conventional supply chain information management system (SCIMS) to a blockchain-based SCIMS. This article models the uncertain customers' demand as a geometric Brownian motion process and shows how to obtain the optimal demand threshold above which the switch occurs and the corresponding expected time. Next, the model is extended by incorporating two types of government subsidies (i.e., a fixed subsidy on the switching cost and a variable subsidy per unit demand). Through sensitivity analysis and numerical studies, the impacts of key parameters on the optimal demand threshold and expected time of switching are presented. Finally, managerial insights and policy implications are derived.

KEYWORDS

Blockchain, Demand Uncertainty, Geometric Brownian Motion (GBM), Government Subsidy, Perishable Agricultural Products, Real Options, Supply Chains, Traceability Valuation

INTRODUCTION

It has been frequently reported that various perishable agricultural products such as romaine lettuce are recalled and disposed due to harmful health risks. In such a case, in a conventional supply chain information management system (SCIMS), the traceability of the source of the harmful health risks is low and time-consuming (Blissett & Harreld, 2008). The reason is that, data is simply recorded on paper for traceability purposes by numerous stakeholders in the supply chain of perishable agricultural products, while the rest use digital methods (Yiannas, 2018). This leads to the inconsistency in the use of SCIMS, and stakeholders are not able to communicate with each other or to effectively trace the origins of products on a short notice. As a result, a large amount of perishable agricultural products that are potentially not contaminated are wasted and disposed as precaution during perishable agricultural product recalls. This situation calls for a solution for an enhanced traceability in the supply chain of perishable agricultural products.

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To address this problem, the perishable food industry has been implementing blockchain, "... a shared, immutable ledger that facilitates the process of recording transactions and tracking assets in a business network" (Gupta, 2018). In a blockchain network, timestamped transaction data is stored in blocks that are linked in a chain by hashes. This mechanism prevents the alternation or insertion of any block. In a blockchain based SCIMS, all the information throughout every step such as product identification, batch codes, purchase orders, and time codes of harvesting, processing, shipping, and receiving, is collected and shared by all stakeholders (e.g., farms, distribution centers, stores; Walmart Food Safety & Health, 2018). With Hyperledger Fabric (a blockchain framework), blockchain offers a more efficient way to precisely pinpoint where the contamination originated, and to reduce the unnecessarily broad recalls (Guo, Liu, & Zhang, 2018). For example, in a pilot study of mango products, blockchain substantially reduced the time to identify the originating farm, from nearly seven days to 2.2 seconds (Yiannas, 2018).

Also, typically, large retailers (e.g., Walmart, Sam's Club) perform like pioneers in the adoption of new technology. In 2018, Walmart and Sam's Club required all the leafy green vegetables suppliers to utilize blockchain for traceability purposes by September 2019 to reduce the loss of retailers and suppliers during recalls (Walmart, 2018).

Meanwhile, government often grants subsidies for the welfare of the public, especially when it is related to information technology. For instance, government subsidized supermarkets opened in high-need areas to improve the food environment in underserved neighborhoods (Elbel, Moran, Dixon, Kiszko, Cantor, Abrams & Mijanovich, 2015). In 2019, through the Health Resources and Services Administration (HRSA), the U.S. Department of Health and Human Services (HHS) subsidized 49 Health Center Controlled Networks (HCCNs) with almost \$42 million to expand the use of health information technology (HHS, 2019). Considering that the blockchain enhances the traceability in the supply chain and reduces the harmful health risks, it is reasonable to assume that government provides the retailers in perishable product supply chain with subsidies to facilitate their switching to a blockchain based SCIMS.

Considering the lump sum switching cost and a series of transition actions that occur at the time of switching, the retailer's decision on switching from the conventional SCIMS to the blockchain based SCIMS is large and highly irreversible. Moreover, such switch decision is often made under uncertainties such as the demand uncertainty of retail customers. That is, when the retail customers' demand is low, for the retailer, the profit saved by the blockchain based SCIMS may not offset the costs associated with the switching. Method-wise, real options approach is used in this paper as it captures the uncertainty in the decision-making process as opposed to traditional Net Present Value (NPV) approach. A real option refers to the right but not the obligation to make decisions on taking the ownership of a real asset or project at a specific time in the future (Tallon, Kauffman, Lucas, Whinston, & Zhu, 2002; Wu, Wu, & Wen, 2010). Real options originated from the finance area and has been extended to the decision making in the engineering discipline.

Under these circumstances, it is highly desirable to understand how a retailer can make economically rational decisions on switching from a conventional SCIMS to a blockchain based SCIMS, and how the government subsidies influence the retailer's decision on such a switch. Towards these goals, in this paper, under the assumption that the retail customers' demand for a single perishable agricultural product follows a Geometric Brownian Motion (GBM) process, the authors (1) valuate the traceability in the supply chain to determine the optimal time for a retailer to switch from a real options perspective in the basic model, (2) extend the basic model by incorporating two types of government subsidies, namely, a fixed subsidy on the switching cost and a variable subsidy per unit demand, and determine the new optimal time for the retailer to switch, (3) derive managerial insights and economic implications for the retailer's switch decision from analytical/numerical sensitivity analyses, and (4) provide policy implications from the government's perspective.

The critical contributions of this research include: (1) closed-form solutions for the optimal threshold of retail customers' demand above which the SCIMS switch occurs and the corresponding

expected time without/with the presentence of government subsidies, (2) an insight that, as the retail customers' demand becomes more volatile, the retailer should defer the switch of SCIMS, (3) from a government's perspective, a small amount of variable subsidy is more efficient for a rapid switch among retailers, while a fixed subsidy anticipates for a more even pace of switch. Also, the fixed subsidy is more efficient at a higher level as opposed to the variable subsidy that is more efficient at a lower level.

The remainder of this article is organized as follows. A review of literature on the blockchain and real options is presented in the next section. Then the authors present the model formulation and analysis for a basic model and an extended model with two government subsidies. After that, a numerical example of romaine lettuce is conducted to further demonstrate how the change of key parameters impacts the optimal threshold of demand and expected time of switching. Finally, conclusions, limitations and future research are presented respectively.

LITERATURE REVIEW

Blockchain

The development of blockchain has boosted a series of discussions and attempts on its application in perishable agricultural supply chains. For instance, Tian (2016) developed a conceptual framework of an agricultural product supply chain traceability system combining blockchain with RFID technology. Moreover, it is estimated that every year, around 1/3 of food is lost or wasted in the world (FAO, 2020). Among such loss and waste, 8% is caused by improper packaging and storage, especially for perishable products such as fresh produce, meat, dairy products since they require strict temperature and packaging conditions (Blockchain Guru, 2019). One promising solution is to use RFID tags and sensors to track the transportation and storage conditions along the shipping journey and to use Smart Contracts (a special feature of blockchain) to notify all stakeholders in the network whenever abnormal conditions occur. Also, according to IBM Research (2020), 45% of fruits and vegetables are spoiled and wasted because of a chaotic distribution system. This is because the imprecise nature of supply chains based on such systems forces farmers to make planting and harvesting decisions based on guesswork, and sellers to predict customer demand and behavior based on incomplete information. The solution to this problem is to implement a blockchain-enabled food supply chain that is enhanced by Internet of Things (IoT) devices and Artificial Intelligence (AI) computing. That is, IoT sensors track fruits, vegetables, or any other food items along the journey from field to grocery store, and AIenhanced, real-time data enables retailers to have better understanding on consumers eating patterns. In this way, both farmers and suppliers know the amount of perishable produce they should grow or order to meet the demand, and thus the perishable produce is fresher, and less amount is thrown away.

Regarding the reduction of the wastage and disposal in the perishable agricultural supply chain in this paper, the advantages and disadvantages of using the blockchain based SCIMS are summarized in Table 1.

Real Options

Derived from financial options, real options approach has been broadly applied in solving decision-making problems as it incorporates the flexibility the decision makers confront in operating decisions (Trigeorgis & Tsekrekos, 2017). In existing literature, there are mainly three option valuation approaches, i.e., partial differential equations (Black and Scholes, 1973), trees and lattices (Cox, Ross, & Rubinstein, 1979), and simulations (Boyle, 1977). Examples of using real options approach in investments under uncertainties are as follows. Schwartz and Zozaya-Gorostiza (2003) evaluated IT investment projects by modeling the uncertainties in project costs and cash flows simultaneously. Tauer (2006) established entry and exit decision models for dairy farmers under the milk price uncertainty. Takashima and Yagi (2009) modeled a single investment and a sequential investment using real

Table 1. Advantages and disadvantages of using the blockchain based SCIMS to reduce the wastage and disposal in the supply chain of perishable agricultural products

Advantage	- Blockchain provides end-to-end traceability which allows the stakeholders in the supply chain to access the remaining shelf life of perishable food by tracking its journey and freshness (IBM, 2018). - Blockchain invites stakeholders to trade in trusting relationship (Zhang, Lee, & van de Ligt, 2016). - Blockchain efficiently improves the traceability of food regarding its safety and transparency in agriculture and food supply chains (Kamilaris, Fonts, & Prenafeta-Boldó, 2019).
Disadvantages	- When the demand is low, the profit saved from the reduction of wastage and disposal may not offset the costs associated with the retailer's switch to the blockchain based SCIMS.

options approach and showed the influence of a catastrophic event on the flexibility of the sequential one by comparing the option values of both investments under the cash flow uncertainty. They also determined the optimal investment timing and location of the power plant given construction costs and the catastrophic event are dependent on the location. Wu and Liou (2011) evaluated enterprise resource planning (ERP) investment incorporating revenue and cost uncertainties and determined the optimal threshold of the ratio of revenue to cost.

In terms of technology transition problems, in most cases, deterministic models are used. However, they are not able to incorporate uncertainties. For instance, in 2010, Cook and Ali used the NPV approach to evaluate quality improvement projects. Woo, Kim, Sung, Lee, Shin, and Lee (2019) evaluated the biopharmaceutical technology regarding new drug development using an improved risk adjusted NPV valuation model.

To the authors' knowledge, no stochastic models can be found that have an emphasis in demand volatility where the blockchain based SCIMS reduces wastage and disposal, and the retailer in the perishable agricultural product supply chain faces the SCIMS switch decision. Although real options approach has a number of advantages such that it captures the uncertainties as opposed to deterministic models, there are circumstances where it is not worthy (see Table 2).

Table 2. Advantages and disadvantages of using real options approach to solve for the switching problem

Advantages	- Real options approach captures uncertainties and provides straightforward closed-form solutions (Miller & Park, 2002). - Real options approach is not critically dependent on an accurate prediction of the retail customers' demand. Instead, economic thresholds are provided that are typically not regrettable.
Disadvantages - When the demand has little volatility, using real options approach to solve the is not well worth.	

MODEL FORMULATION AND ANALYSIS

Basic Model

In a supply chain of a single perishable agricultural product consisting of a wholesaler, a retailer, and retail customers (see Figure 1), the authors consider a switching problem of the retailer's perspective from a conventional SCIMS to a blockchain based SCIMS. The reason for this switch is that the blockchain based SCIMS facilitates the traceability of the perishable product, which in turn will

Figure 1. Supply chain of a single perishable agricultural products



reduce the wastage and disposal because, for example, in a case of virus or bacteria outbreak, the contaminated products can be pinpointed in a rapid manner.

To facilitate the modeling and analysis, the following assumptions are proposed.

Assumption 1: The retail customers' demand for a single perishable agricultural product at time point t, D_t (lb at a day), follows a GBM process where the time granularity is a day:

$$dD_{t} = \alpha D_{t} dt + \sigma D_{t} dz_{t} \tag{1}$$

where α (% per day; > 0) and σ (% per square root of day; >0) are the instantaneous growth rate and volatility of the demand, respectively. dz_t is the increment of a Wiener process, and $dz_t = \epsilon \sqrt{dt}$, $\epsilon \sim N\left(0,1\right)$.

Proposition 1: Suppose the retail customers' demand at time point 0 is D_0 , the expected value of D_t is $E(D_t) = D_0 e^{\alpha t}$ (Dixit & Pindyck, 1994, p. 71-72). See Appendix 1 for proof.

Assumption 1 is based on the observation that the retail customers' demand for a perishable agricultural product increases on average and fluctuates over time. Empirical data support can be found in Table 3, where the authors estimate the consumption of fresh lettuce (romaine and leaf) at a day in Houston, TX from 2000 to 2017. As is shown in Figure 2, the consumption of fresh lettuce at a day has a positive growth rate with fluctuations over time.

For the ease of reference, the rest notations used in this paper are summarized in Table 4.

The unit selling price P and the unit purchase price C are assumed to remain unchanged over time. Meanwhile, the costs associated with processing activities (e.g., shipping, storage, disposal) and the corresponding labor costs are not taken into consideration.

For the conventional SCIMS, the authors make the following assumptions.

Assumption 2: At time point t, w fraction of the demand D_t is wasted and disposed as precaution during recalls. Hence, the total amount of the perishable product that the retailer purchases from the wholesaler is $(1+w)D_t$ (lb at a day).

w is a constant that can be estimated from historical data by dividing the total amount of the perishable product wasted and disposed as precaution during recalls over the retail customers' demand within last year. This assumption yields the following proposition.

Proposition 2: The total amount of the perishable product that the retailer purchases from the wholesaler at time point t before switching, $(1+w)D_t$ (lb at a day) also follows a GBM process with the same growth rate and volatility as D_t . See Appendix 2 for proof.

Table 3. Estimated consumption of fresh lettuce at a day in Houston, TX from 2000 to 2017

Year	Annual per capita (lb) (Shahbandeh, 2019)	Population (million) (U.S. Census Bureau, 2019)	Daily consumption (lb) (Estimated)
2000	8.4	1.9774	45,507
2001	8.0	1.9943	43,711
2002	9.6	2.0156	53,013
2003	10.8	2.0197	59,761
2004	12.0	2.0174	66,325
2005	9.7	2.0219	53,733
2006	12.0	2.0587	67,683
2007	11.5	2.0651	65,065
2008	10.4	2.0844	59,391
2009	10.0	2.1186	58,044
2010	12.0	2.0993	69,018
2011	11.7	2.1255	68,132
2012	11.9	2.1598	70,415
2013	11.4	2.1982	68,656
2014	10.8	2.2388	66,244
2015	11.9	2.2822	74,406
2016	14.5	2.3045	91,549
2017	15.0	2.3127	95,042

Figure 2. Estimated consumption of fresh lettuce over time (Houston, TX, from 2000 to 2017)

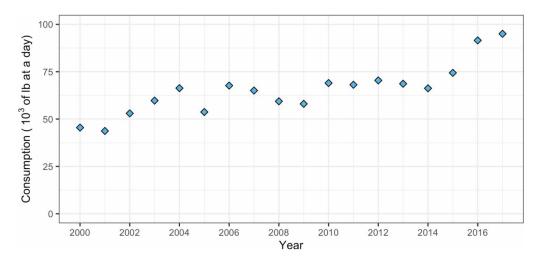


Table 4.	Notations	and	descri	ptions

Notation	Description	
P	Unit selling price that is paid by retail customers to the retailer (\$/lb)	
C	Unit purchase price that is paid by the retailer to the wholesaler (\$/lb)	
w	The ratio of the amount of the wastage and disposal as precaution during recalls over the demand at time point t	
I	Switching cost incurred to the retailer at the time of switching (\$)	
r	The ratio of the amount of wastage and disposal as precaution during recalls using the blockchain based SCIMS over that amount using the conventional SCIMS	
C_b	Payment for using the blockchain based SCIMS that is paid by the retailer to IBM (\$/day)	
ρ	Discount rate for money (% per day)	
$V_{_1}$	Project value function in phase 1 (\$)	
V_2	Project value function in phase 2 (\$)	
D^*	Optimal threshold of demand above which the SCIMS switch occurs (\$/lb)	
T^*	Expected time of switching (day)	

Assumption 3: The payment for using the conventional SCIMS (i.e., costs associated with phone calls, emails, paper copies) are ignored.

Assumption 4: At certain time point, the retailer switches from the conventional SCIMS to the blockchain based SCIMS at a switching cost of I(\$).

Referring to the definition of adoption costs of information technology upgrades in Mukherji, Rajagopalan, & Tanniru's work (2006), in this paper, the switching cost I is defined as the cost associated with purchasing or upgrading necessary equipment, as well as training and transitioning employees completely to the blockchain based SCIMS.

For the blockchain based SCIMS, the authors make the following assumptions.

Assumption 5: At time point t, the amount of wastage and disposal as precaution during recalls is reduced to r(0 < r < 1) fraction of that amount before switching. That is, the total amount of product the retailer purchases from the wholesaler is $(1 + rw)D_t$ (lb at a day).

By collecting the product information and storing it on the network, blockchain creates a more transparent supply chain where the source of contamination can be rapidly identified and thus, unnecessarily broad recalls are reduced (Guo et al., 2018). For instance, in a case of dairy products contamination, Marin, Marin and Vidu (2019) claimed that blockchain can trace the originating farm within seconds, and only a batch of dairy products needs to be removed from distribution. With the above qualitative data support, the authors assume that the blockchain based SCIMS reduces the amount of wastage and disposal of perishable agricultural products as precaution during recalls use

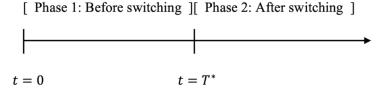
a coefficient r to denote the reduction efficiency. Notably, a smaller r indicates more amount of the perishable product is saved from being wasted and disposed. This assumption yields the following proposition.

Proposition 3: The total amount of the perishable product that the retailer purchases from the wholesaler at time point t after switching, $(1 + rw)D_t$ (lb at a day) also follows a GBM process with the same growth rate and volatility as D_t . See Appendix 3 for proof.

Assumption 6: Once the retailer switches to the blockchain based SCIMS, the retailer will use it forever.

The timeline with respect to the switching of SCIMS is divided into two phases by T^* , namely, phase 1 and phase 2 (see Figure 3).

Figure 3. The timeline with respect to the switching of SCIMS



The problem can be described as a maximization of the total expected discounted value by choosing T^* as follows:

$$maxE\bigg[\int\limits_{0}^{T^{*}}e^{-\rho t}\Big[PD_{t}-C\left(1+w\right)D_{t}\Big]dt-Ie^{-\rho T^{*}}+\int\limits_{T^{*}}^{\infty}e^{-\rho t}\Big[PD_{t}-C\left(1+rw\right)D_{t}-C_{b}\Big]dt\bigg] \tag{2}$$

where $T^* = \inf\{t \ge 0 \mid D_t \ge D^*\}$.

Phase 2: After Switching

Attimepoint t inphase 2, when operating, the retailer has a cash flow of $\max \left[PD_t - C\left(1 + rw\right)D_t - C_b, 0\right]$. This implies that when $P > \left(1 + rw\right)C$ and:

$$D_{\scriptscriptstyle t} > D_{\scriptscriptstyle min} = \frac{C_{\scriptscriptstyle b}}{P - C \left(1 + rw\right)}$$

the retailer makes profit from the selling of the perishable agricultural product. Under a technical condition of $\rho-\alpha>0$, the project value at time point t, $V_2\left(D_t\right)$, is equal to the expected value of discounted future cash flows as follows (Murto, 2007). The proof is given in Appendix 4:

$$V_{2}\left(D_{t}\right) = E\left\{\int_{t}^{\infty} e^{-\rho\left(x-t\right)} \left[PD_{x} - C\left(1+rw\right)D_{x} - C_{b}\right] dx\right\} = \frac{\left[P - C\left(1+rw\right)\right]D_{t}}{\rho - \alpha} - \frac{C_{b}}{\rho} \tag{3}$$

Phase 1: Before Switching

In phase 1, when operating, the cash flow function at time point t is given by $\max\left[PD_t-C\left(1+w\right)D_t,0\right]$. Similarly, in order for the retailer to make profit, P is supposed to be greater than $\left(1+w\right)C$, and there is no requirement for D_t . The project value at time point t, $V_1\left(D_t\right)$, must satisfy the following Bellman optimality principle equation:

$$\rho V_{1}\left(D_{t}\right)dt = \left[PD_{t} - C\left(1 + w\right)D_{t}\right]dt + E\left[dV_{1}\left(D_{t}\right)\mid D_{t}\right] \tag{4}$$

Equation (4) means that at time point t, the return for holding the switching option should be equal to the immediate profit when holding the switching option plus the expected appreciation of the project value conditioning on the demand level.

By applying Ito's Lemma on $\,dV_{\scriptscriptstyle \parallel}$, the following differential equation can be derived:

$$\frac{1}{2}\sigma^2 D_t^2 \frac{\partial^2 V_1}{\partial D_t^2} + \alpha D_t \frac{\partial V_1}{\partial D_t} - \rho V_1 + \left(P - C\right) D_t - Cw D_t = 0 \tag{5}$$

Equation (5) is subject to the following two boundary conditions (Siddiqui & Takashima, 2012):

$$V_1\left(D^*\right) = V_2\left(D^*\right) - I \tag{6}$$

$$V_1'\left(D^*\right) = V_2'\left(D^*\right) \tag{7}$$

Equation (6) and Equation (7) are the value matching condition and smooth pasting condition, respectively. The value matching condition ensures that at the time of exercising the switching option, the project value before switching is equal to the project value after switching minus the switching cost. The smooth pasting condition guarantees that the slopes of the left-hand side and the right-hand side of the value matching condition are equal at the optimal threshold of demand.

Under technical conditions of $\rho - \alpha > 0$ and $\alpha - \frac{\sigma^2}{2} > 0$ (Dixit & Pindyck, 1994), the general solution to Equation (5) is given by (see Appendix 5 for proof):

$$V_{1}\left(D_{t}\right) = A_{1}D_{t}^{\beta_{1}} + \frac{\left[P - C\left(1 + w\right)\right]D_{t}}{\rho - \alpha} \tag{8}$$

where:

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$$\beta_1 = \frac{1}{\sigma^2} \left[\frac{\sigma^2}{2} - \alpha + \sqrt{\left(\frac{\sigma^2}{2} - \alpha\right)^2 + 2\rho\sigma^2} \right], \; \beta_1 > 1$$

Using the two boundary conditions, the coefficient A_1 and the optimal threshold of demand D^* can be solved. That is:

$$A_{\!\scriptscriptstyle 1} = \frac{Cw \left(1-r\right)}{\left(\rho-\alpha\right)\beta_{\!\scriptscriptstyle 1} D^{*\beta_{\!\scriptscriptstyle 1}-1}}$$

and D^* is given by:

$$D^* = \frac{\left(\frac{C_b}{\rho} + I\right)(\rho - \alpha)\beta_1}{Cw(1 - r)(\beta_1 - 1)} \tag{9}$$

It can be verified that the expected time for the retailer to optimally switch is (Appendix 6 for proof):

$$T^* = \left(\ln \frac{\left(\frac{C_b}{\rho} + I\right) (\rho - \alpha) \beta_1}{Cw(1 - r)(\beta_1 - 1)} - \ln D_0 \right) / \left(\alpha - \frac{1}{2}\sigma^2\right)$$
(10)

Extended Model With Subsidies

Next, the basic model is extended by incorporating two types of government subsidies. That is, government provides the retailer with a one-time fixed subsidy of U(\$) on the switching cost to initiate the switch of SCIMS, and a variable subsidy S(\$/10) per unit demand for using the blockchain based SCIMS in the supply chain of the perishable agricultural product.

Phase 2: After Switching

In phase 2, when operating, the retailer's cash flow at time point t is $\max\left[PD_t-C\left(1+rw\right)D_t-C_b+SD_t,0\right]$, and it is required that $P>\left(1+rw\right)C$ and $D_t>D_{\min}=\frac{C_b}{P-C\left(1+rw\right)+S}$. Given $\rho-\alpha>0$, the project value, $V_2\left(D_t\right)$, is equal to the expected value of discounted cash flows as follows (Murto, 2007). The proof is given in Appendix 7:

$$\begin{split} &V_{2}\left(D_{t}\right)=E\left\{\int_{t}^{\infty}e^{-\rho\left(x-t\right)}\left[PD_{x}-C\left(1+rw\right)D_{x}+SD_{x}-C_{b}\right]dx\right\}\\ &=\frac{\left[P-C\left(1+rw\right)+S\right]D_{t}}{\rho-\alpha}-\frac{C_{b}}{\rho} \end{split} \tag{11}$$

Phase 1: Before Switching

When operating, the project value at time point t, $V_1(D_t)$, remains the same as Equation (8) in the basic model, i.e.:

$$V_{_{1}}\left(D_{_{t}}\right) = A_{_{1}}D_{_{t}}^{\beta_{_{1}}} + \frac{\left[P - C\left(1 + w\right)\right]D_{_{t}}}{\rho - \alpha}$$

but now $V_1\left(D_t\right)$ is subjective to the following two boundary conditions:

$$V_{1}\left(\boldsymbol{D}^{*}\right)=V_{2}\left(\boldsymbol{D}^{*}\right)-\left(\boldsymbol{I}-\boldsymbol{U}\right)\tag{12}$$

$$V_1'(D^*) = V_2'(D^*) \tag{13}$$

Equation (12) is the value matching condition, which suggests that at the time of exercising the switching option, the project value before switching should be equal to the project value after switching minus the switching cost net of the fixed subsidy. Equation (13) is the smooth pasting condition, and it ensures the slopes of both sides of Equation (12) are equal at the switching time.

Sequentially, it can be verified that $A_1 = \frac{Cw(1-r)+S}{\left(\rho-\alpha\right)\beta_1D_1^{*\beta_1-1}}$, and the optimal threshold of

demand D^* is:

$$D^* = \frac{\left(\frac{C_b}{\rho} + I - U\right)(\rho - \alpha)\beta_1}{\left[Cw(1 - r) + S\right](\beta_1 - 1)}$$
(14)

Similarly, with the two types of government subsidies, the expected time of switching becomes:

$$T^* = \left(\ln \frac{\left(\frac{C_b}{\rho} + I - U \right) (\rho - \alpha) \beta_1}{\left[Cw \left(1 - r \right) + S \right] (\beta_1 - 1)} - \ln D_0 \right) / \left(\alpha - \frac{1}{2} \sigma^2 \right)$$

$$(15)$$

Analytical Sensitivity Analysis

Among the ten parameters that determine the optimal threshold of demand D^* , the authors conduct analytical sensitivity analysis on seven of them (C, w, r, I, C_b , U, S), and numerically examine the impact of the rest three (σ , α and ρ) on D^* as the partial derivatives with respect to them cannot not explicitly obtained. Also, since in the stochastic optimal control theory, the optimal project value corresponds to timing, sensitivity analysis on T^* is included as well.

Corollary 1: Given
$$\rho > \alpha$$
 and $\alpha - \frac{\sigma^2}{2} > 0$, $\frac{\partial D^*}{\partial C} < 0$, $\frac{\partial T^*}{\partial C} < 0$, $\frac{\partial D^*}{\partial w} < 0$, and $\frac{\partial T^*}{\partial w} < 0$.

The proof is given in Appendix 8. This corollary indicates that when the unit purchase price of the perishable product increases or a larger proportion of the perishable product is wasted and disposed as precaution during recalls, the optimal threshold of demand and the expected time of switching decrease. In such cases, the retailer loses more money due to the wastage and disposal. Consequently, the retailer will switch to the blockchain based SCIMS earlier from an economic perspective.

Corollary 2: Given
$$\rho > \alpha$$
 and $\alpha - \frac{\sigma^2}{2} > 0$, $\frac{\partial D^*}{\partial r} > 0$, $\frac{\partial T^*}{\partial r} > 0$.

The proof is given in Appendix 9. The positive partial derivatives suggest that, a larger coefficient of reduction efficiency leads to a higher optimal threshold of demand and the expected time of switching. This is because a larger r implies that less amount of perishable agricultural product is saved from being wasted and disposed by the blockchain based SCIMS. As a result, it is economically rational for the retailer to defer the switching option.

Corollary 3: Given
$$\rho > \alpha$$
 and $\alpha - \frac{\sigma^2}{2} > 0$, $\frac{\partial D^*}{\partial I} > 0$, $\frac{\partial T^*}{\partial I} > 0$, $\frac{\partial D^*}{\partial C} > 0$, and $\frac{\partial T^*}{\partial C} > 0$.

The proof is given in Appendix 10. Corollary 3 suggests that as the switching cost or the payment for using the blockchain based SCIMS increases, the optimal threshold of demand and the expected time of increase. This makes economic sense because, under such circumstances, the retailer benefits less from the SCIMS switch, so there is less incentive for the retailer to switch. Therefore, the retailer will wait longer before exercising the switching option.

Corollary 4: Given
$$\rho > \alpha$$
 and $\alpha - \frac{\sigma^2}{2} > 0$, $\frac{\partial D^*}{\partial U} < 0$, $\frac{\partial T^*}{\partial U} < 0$, $\frac{\partial D^*}{\partial S} < 0$, and $\frac{\partial T^*}{\partial S} < 0$.

The proof is given in Appendix 11. The interpretation of Corollary 4 is as follows. When government provides the retailer with higher fixed subsidy on the switching cost or higher variable subsidy per unit demand, the retailer has a lower switching cost or a higher cash flow after switching. Either way, the retailer will be more eager to switch from an economic perspective, so the optimal threshold of demand and expected time of switching will decrease.

NUMERICAL STUDY

In this section, the authors conduct a numerical study on romaine lettuce to further demonstrate the findings in the previous section. The parameter values and references are summarized in Table 5, where some parameter values are hypothetical due to the lack of numerical data.

Table 5. Parameters and values

Parameter	Value	References
α	0.0505	Shahbandeh (2019); U.S. Census Bureau (2019);
σ	0.1202	Method 3 in Croghan, Jackman and Min's paper (2017)
D_{0}	78,552 (lb at day)	Population USA (2019); Shahbandeh, 2019)
P	0.94 (\$/lb)	USDA (2019)
C	0.36 (\$/lb)	USDA (2019)
w	0.137	ExerciseBike (2019)
C_b	133.33 (\$/day)	IBM Cloud (2019)
ρ	0.0543	Damodaran (2019)
r	0.4	Hypothetical
I	1,000,000 (\$)	Hypothetical
U	200,000 (\$)	Hypothetical
S	0.05 (\$/lb)	Hypothetical

The key numerical results in Table 6 show that, in the basic model where no subsidies are provided, the optimal threshold of demand is 2,100,161 (lb at a day) and the corresponding expected time of switching is 76 (day). With the presentence of two types of government subsidies, in the extended model, the optimal threshold of demand is reduced to 625,047 (lb at a day), and correspondingly, the expected time of switching is reduced to 48 (day). Also, the minimum demand level for the retailer to make profit from the selling of the perishable product is reduced from 238 (lb at a day) to 218 (lb at a day) when the two government subsidies are provided.

Table 6. Numerical results

Notation	Value (basic model - no subsidies)	Value (extended model - with subsidies)	
$oxed{eta_1}$	1.0653	1.0653	
$A_{\rm l}$	2.8255	8.2255	
D_{min}	238 (lb at a day)	218 (lb at a day)	
D^*	2,100,161 (lb at a day)	625,047 (lb at a day)	
T^*	76 (day)	48 (day)	

Next, numerical sensitivity analysis is conducted on σ , α and ρ , since their impact on D^* and T^* has not been analytically examined.

Figure 4 illustrates that as the demand becomes more volatile, the optimal threshold of demand and the expected time of switching increase, meaning that the exercise of the retailer's switching option should be deferred. This is because, with a higher demand uncertainty, the flexibility to exercise the switching option at any time point becomes more valuable. Hence, it is economically rational for the retailer to hold the switching option longer and wait for more information.

In terms of the growth rate of demand, when it increases, the optimal threshold of demand and the expected time of switching decrease (see Figure 5). This is because when the retail customers' demand for the perishable product is rapidly growing, the retailer's benefit from using the blockchain based SCIMS is amplified. Therefore, the retailer prefers to exercise the switching option earlier.

Figure 4. Variation of $\operatorname{\it D}^*$ and $\operatorname{\it T}^*$ with respect to σ

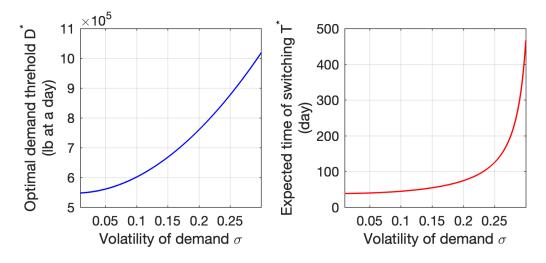
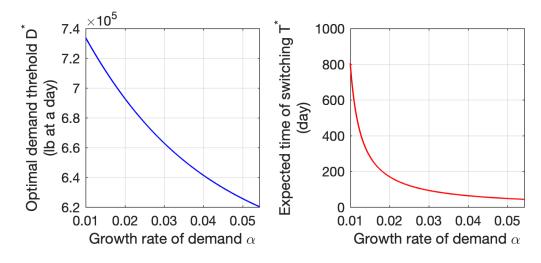


Figure 5. Variation of $\operatorname{\mathcal{D}}^*$ and $\operatorname{\mathcal{T}}^*$ with respect to α



As can be observed from Figure 6, when the discount rate for money increases, the optimal threshold of demand and the expected time of switching increase. The reason is that, when money is heavily discounted, the retailer's loss due to wastage and disposal during recalls is trivial. Consequently, there is less incentive for the retailer to switch from the conventional SCIMS to the blockchain based SCIMS.

Although the magnitude of the partial derivative of D^* and T^* with respect to U and S have been given in the sensitivity analysis section, the authors include Figure 7 and Figure 8 to discuss the convexness and concaveness of these curves. Intuitively, the optimal threshold of demand linearly decreases as the fixed subsidy on switching cost increases, and convex decreases as the variable subsidy per unit demand increases. Specifically, when the variable subsidy S increases from 0 to 0.1 (\$/lb), the optimal threshold of demand D^* substantially decreased from 1.68×10^6 (lb at a day) to 0.38×10^6 (lb at a day). However, when S increases from 0.3 (\$/lb) to 0.4 (\$/lb), D^* decreased

Figure 6. Variation of $\operatorname{\mathcal{D}}^*$ and $\operatorname{\mathcal{T}}^*$ with respect to $\operatorname{\mathcal{\rho}}$

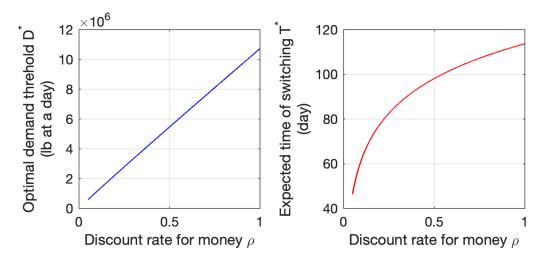


Figure 7. Variation of \boldsymbol{D}^* and \boldsymbol{T}^* with respect to \boldsymbol{U}

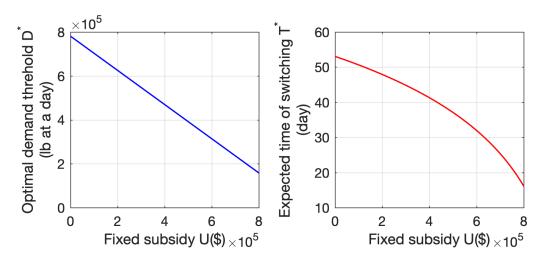
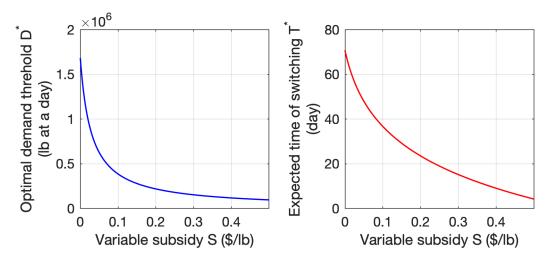


Figure 8. Variation of $\operatorname{\textbf{\it D}}^*$ and $\operatorname{\textbf{\it T}}^*$ with respect to S



from 0.15×10^6 (lb at a day) to 0.11×10^6 (lb at a day). This implies that, from a perspective of the optimal demand threshold reduction, a small amount of variable subsidy is more economically efficient than the fixed subsidy if the government expects retailers to rapidly switch to the blockchain based SCIMS. On the other hand, the fixed subsidy is more viable than the variable subsidy when government anticipates an even switch among retailers.

As for the expected time of switching, it is concave decreasing when the fixed subsidy increases, and convex decreasing when the variable subsidy increases. This means that, regarding the expected switching time reduction, fixed subsidy is more efficient at a higher level, while variable subsidy is more efficient at a lower level.

CONCLUSION

This paper considers a retailer in a supply chain of a perishable agricultural product who faces a volatile retail customers' demand and decides when to switch to a blockchain based SCIMS from a conventional SCIMS. The authors investigated how economic rational decisions can be made on such a switch from a real option perspective under the assumption that the retail customers' demand for a single perishable agricultural product follows a GBM process. Specifically, without/with the presentence of a fixed subsidy and a variable subsidy from the government, the authors constructed mathematical models and obtained the closed-form solutions of the demand thresholds for the retailer to optimally switch and the corresponding expected switching time. By analytically and numerically examining the impact of key parameters on the optimal threshold of demand and the expected time of switching, a series of managerial insights and policy implications are derived. For instance, the retailer is recommended to defer the switching option when the customers' demand is volatile. Furthermore, from the government's perspective, a small amount of variable subsidy should be promoted if the government anticipates the retailers to rapidly switch to the blockchain based SCIMS in a short time, while a fixed subsidy is recommended if an even pace of switch among retailers is expected. Also, fixed subsidy is more efficient at a higher level as opposed to variable subsidy that is more efficient at a lower level.

The novelty of this paper is to show under what conditions a retailer can switch from a conventional SCIMS to a blockchain based SCIMS as well as the expected time for the switch when the demand uncertainty is characterized by a GBM process.

LIMITATIONS AND FUTURE RESEARCH

The limitations of this paper are as follows. To start, the assumption that the blockchain based SCIMS reduces the amount of wastage and disposal to r fraction of that using the conventional SCIMS (assumption 5) is based on qualitative inference and lacks quantitative data support. Secondly, the authors assume that once the retailer switches to the blockchain based SCIMS, the retailer will use it forever (assumption 6). Nonetheless, in reality, technology innovations are commonly observed and full of uncertainties. It is likely that blockchain will require updates or be replaced by a more advanced SCIMS in the future. Thirdly, in this paper, the authors only incorporate the demand uncertainty, while the retailer's switch decision can also be impacted by other uncertainties such as the unit selling price.

In the future, the authors may incorporate the uncertainties of technology innovation and unit selling price in the models. Or, with the development of blockchain, quantitative data support can be used to justify assumption 5. Future research directions can also be focused on decision models for other stakeholders in the supply chain of perishable agricultural products such as the wholesaler or the farm cooperative. Finally, discussions can be expanded to the valuation of the blockchain based system regarding other properties such as transparency, immutability, irrefutability in various industries (e.g., financial, insurance, manufacturing industry).

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APPENDIX 1: PROOF OF PROPOSITION 1

Define $F_t = \ln(D_t)$. By Ito's Lemma, the total differential of function F_t is as follows (Dixit & Pindyck, 1994, p. 80):

$$\begin{split} dF_t &= \frac{\partial F_t}{\partial t} dt + \frac{\partial F_t}{\partial D_t} dD + \frac{1}{2} \frac{\partial^2 F_t}{\partial D_t^2} \left(dD_t \right)^2 \\ &= \frac{1}{D_t} \left(\alpha D_t dt + \sigma D_t dz_t \right) + \frac{1}{2} \left(-\frac{1}{D_t^2} \right) \left(\alpha D_t dt + \sigma D_t dz_t \right)^2 \\ &= \left(\alpha dt + \sigma dz_t \right) - \frac{1}{2} \left(\alpha^2 dt^2 + \sigma^2 dz_t^2 + 2\alpha dt \sigma dz_t \right) \end{split} \tag{1-1}$$

where $\frac{\partial F_t}{\partial t}=0$ (because the function $F_t=\ln\left(D_t\right)$ has a steady state regardless of the value of t), $\frac{\partial F_t}{\partial D_t}=\frac{1}{D_t}$, and $\frac{\partial^2 F_t}{\partial D_t^2}=-\frac{1}{D_t^2}$. Since $dz_t=\epsilon\sqrt{dt}$, $dz_t^2=\epsilon^2dt$ and $dtdz_t=\epsilon dt^{\frac{3}{2}}$. Terms in dt^2 and $dt^{\frac{3}{2}}$ go to zero faster than dt as it becomes infinitesimally small, so they can be ignored (Dixit & Pindyck, 1994, p. 80). Also:

$$dz_{_{t}}^{^{2}}=\epsilon^{^{2}}dt\cong E\left(\epsilon^{^{2}}\right)dt=\left\{ Variance\left(\epsilon\right)+\left[E\left(\epsilon\right)\right]^{^{2}}\right\} dt=\left(1+0^{^{2}}\right)dt=dt$$

Hence:

$$dF_{\scriptscriptstyle t} = \left(\alpha dt + \sigma dz_{\scriptscriptstyle t}\right) - \frac{1}{2} \left(\sigma^2 dt\right) = \left(\alpha - \frac{1}{2} \,\sigma^2\right) dt + \sigma dz_{\scriptscriptstyle t} \tag{1-2}$$

This implies that over finite time interval t, the change in F_t (the natural logarithm of D_t) is normally distributed with mean $\left(\alpha-\frac{1}{2}\sigma^2\right)t$ and variance of σ^2t . Therefore, $\ln\left(D_t\right)=\ln\left(D_0\right)+\left(\alpha-\frac{1}{2}\sigma^2\right)dt+\sigma dz_t, \text{ where } D_0 \text{ is the value of } D_t \text{ at time point 0. Stated } t$

otherwise, D_t is a lognormal process and can be written as $D_0 e^{\left[\alpha-\frac{1}{2}\sigma^2\right]dt+\sigma dz_t}$ (Luenburger, 1998, p. 308-309).

For a random variable $X \sim N(\mu, \sigma^2)$, the moment generating function (MGF) is as follows (Miller, Miller & Freund, 2014, p. 187):

$$M_X(s) = E(e^{sX}) = e^{\mu s + \frac{1}{2}\sigma^2 s^2}, -\infty < s < \infty$$
 (1-3)

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For a random variable $F_t \sim N\left[\left(\alpha - \frac{1}{2}\sigma^2\right)t, \sigma^2 t\right]$, the MGF is given by (Sigman, 2006, p. 3):

$$M_{F_t}\left(s\right) = E\left(e^{sF_t}\right) = e^{\left(\alpha - \frac{1}{2}\sigma^2\right)ts + \frac{1}{2}\sigma^2ts^2}, -\infty < s < \infty \tag{1-4}$$

Therefore, the expected value of $\,D_{\scriptscriptstyle t}\,$ can be calculated by setting $\,s=1$:

$$E\left(D_{_{t}}\right) = E\left(D_{_{0}}e^{^{F_{_{t}}}}\right) = D_{_{0}}M_{_{F_{_{t}}}}\left(1\right) = D_{_{0}}e^{\left(\alpha - \frac{1}{2}\sigma^{2}\right)t + \frac{1}{2}\sigma^{2}t} = D_{_{0}}e^{\alpha t} \tag{1-5}$$

APPENDIX 2: PROOF OF PROPOSITION 2

Define $G_t = (1+w)D_t$. By Ito's Lemma, the total differential of function G_t is given by:

$$\begin{split} dG_t &= \frac{\partial G_t}{\partial t} dt + \frac{\partial G_t}{\partial D_t} dD_t + \frac{1}{2} \frac{\partial^2 G_t}{\partial D_t^2} \Big(dD_t \Big)^2 = \Big(1 + w \Big) \Big(\alpha D_t dt + \sigma D_t dz_t \Big) \\ &= \alpha \Big[\Big(1 + w \Big) D_t \Big] dt + \sigma \Big[\Big(1 + w \Big) D_t \Big] dz_t = \alpha G_t dt + \sigma G_t dz_t \end{split} \tag{2-1}$$

where $\dfrac{\partial G_{_t}}{\partial t}=0$ (because the function $G_{_t}=\left(1+w\right)D_{_t}$ has a steady state regardless of the value of t), $\dfrac{\partial G_{_t}}{\partial D_{_t}}=1+w$, and $\dfrac{\partial^2 G_{_t}}{\partial D_{_t}^2}=0$.

Hence, G_t , i.e., $\left(1+w\right)D_t$, follows a GBM process with the same growth rate α and volatility σ as D_t .

APPENDIX 3: PROOF OF PROPOSITION 3

Similarly, define $H_t = \left(1 + rw\right)D_t$. By Ito's Lemma, the total differential of function H_t is given by:

$$\begin{split} dH_t &= \frac{\partial H_t}{\partial t} dt + \frac{\partial H_t}{\partial D_t} dD_t + \frac{1}{2} \frac{\partial^2 H_t}{\partial D_t^2} \Big(dD_t \Big)^2 = \Big(1 + rw \Big) \Big(\alpha D_t dt + \sigma D_t dz_t \Big) \\ &= \alpha \Big[\Big(1 + rw \Big) D_t \Big] dt + \sigma \Big[\Big(1 + rw \Big) D_t \Big] dz_t = \alpha H_t dt + \sigma H_t dz_t \end{split} \tag{3-1}$$

where $\dfrac{\partial H_t}{\partial t}=0$ (because the function $H_t=\left(1+rw\right)D_t$ has a steady state regardless of the value of t), $\dfrac{\partial H_t}{\partial D_t}=1+rw$, and $\dfrac{\partial^2 H_t}{\partial D_t^2}=0$.

Hence, H_t , i.e., $\left(1+rw\right)D_t$, follows a GBM process with the same growth rate $\,\alpha\,$ and volatility $\,\sigma\,$ as $\,D_t$.

APPENDIX 4: PROOF OF EQUATION (3)

$$\begin{split} &V_{2}\left(D_{t}\right)=E\left\{\int_{t}^{\infty}e^{-\rho(x-t)}\left[PD_{x}-C\left(1+rw\right)D_{x}-C_{b}\right]dx\right\}\\ &=E\left\{\int_{t}^{\infty}e^{-\rho(x-t)}\left[P-C\left(1+rw\right)\right]D_{x}dx\right\}-\int_{t}^{\infty}e^{-\rho(x-t)}C_{b}dx\\ &=\left[P-C\left(1+rw\right)\right]\int_{t}^{\infty}e^{-\rho(x-t)}E\left(D_{x}\right)dx-C_{b}\int_{t}^{\infty}e^{-\rho(x-t)}dx\\ &=\left[P-C\left(1+rw\right)\right]\int_{t}^{\infty}e^{-\rho(x-t)}D_{t}e^{\alpha(x-t)}dx-C_{b}\int_{t}^{\infty}e^{-\rho(x-t)}dx\\ &=\left[P-C\left(1+rw\right)\right]D_{t}\int_{t}^{\infty}e^{-(\rho-\alpha)(x-t)}dx-C_{b}\int_{t}^{\infty}e^{-\rho(x-t)}dx\\ &=\left[-\frac{\left[P-C\left(1+rw\right)\right]D_{t}}{\rho-\alpha}\right]e^{-(\rho-\alpha)(x-t)}\left|_{t}^{\infty}-\left(-\frac{C_{b}}{\rho}\right)e^{-\rho(x-t)}\left|_{t}^{\infty}=\frac{\left[P-C\left(1+rw\right)\right]D_{t}}{\rho-\alpha}-\frac{C_{b}}{\rho}\end{aligned} \tag{4-1}$$

APPENDIX 5: PROOF OF EQUATION (8)

A particular solution to Equation (5) can be verified to be $V_1\left(D_t\right) = \frac{\left[P-C\left(1+w\right)\right]D_t}{\rho-\alpha}$ under a technical condition of $\rho-\alpha>0$. Also, a homogeneous solution to Equation (5) can be written as $V_1\left(D_t\right) = A_1D_t^{\beta_1} + A_2D_t^{\beta_2}$ under a technical condition of $\alpha-\frac{\sigma^2}{2}>0$, where:

$$\beta_{\text{\tiny 1,2}} = \left\lceil \frac{\sigma^2}{2} - \alpha \pm \sqrt{\left\lceil \frac{\sigma^2}{2} - \alpha \right\rceil^2 + 2\rho\sigma^2} \right\rceil / \left\lceil \sigma^2 \right\rceil$$

are the two roots of the fundamental quadratic equation $\mathbb{Q}=\frac{1}{2}\sigma^2\beta\left(\beta-1\right)+\alpha\beta-\rho=0$. It can be verified that $\beta_1>1$ and $\beta_2<0$ (Dixit and Pindyck's, 1994, p. 143). So, the general solution to Equation (5) is:

$$V_{1}\left(D_{t}\right)=A_{1}D_{t}^{\beta_{1}}+A_{2}D_{t}^{\beta_{2}}+\frac{\left[P-C\left(1+w\right)\right]D_{t}}{\rho-\alpha}$$

and A_1 and A_2 are constants to be determined.

The signs of constants A_1 and A_2 can be discussed as follows. Assuming A_1 is negative, since β_1 is greater than 1, when D_t goes to positive infinity, the term $A_1D_t^{\beta_1}$ goes to negative infinity. This is against economic implications as larger demand is supposed to bring the retailer with more profit and thus, contributes to a higher project value. Therefore, A_1 cannot be negative. Similarly, if A_2 is positive, when D_t is small and approaches to zero, the term $A_2D_t^{\beta_2}$ goes to positive infinity since β_2 is negative. This also violates the economic signification because smaller demand should contribute to lower profit as well as lower project value. Hence, A_2 cannot be positive. Conversely,

if A_2 is negative, $\frac{\partial \left(A_2 D_t^{\beta_2}\right)}{\partial D_t} = A_2 \beta_2 D_t^{\beta_2 - 1} < 0$, meaning that the project value decreases as the demand increases. This does not make economic sense since the project value should increase with an increase in the demand, so A_2 cannot be negative either. Since A_2 cannot be either positive or negative, it is required to be 0. Therefore, the general solution becomes

$$V_{_{1}}\left(D_{_{t}}\right) = A_{_{\!\!1}}D_{_{t}}^{^{\beta_{_{\!\!1}}}} + \frac{\left[P - C\left(1 + w\right)\right]D_{_{t}}}{\rho - \alpha}\,. \label{eq:V1}$$

APPENDIX 6: PROOF OF THE EXPECTED TIME OF SWITCHING

In Appendix 1, the authors show that the change in F_t (the natural logarithm of D_t) is normally distributed with mean $\left(\alpha - \frac{1}{2}\sigma^2\right)t$ and variance of σ^2t . Since the natural logarithm is a monotonically increasing function, the expected time for the retailer to optimally switch can be interpreted as the expected passage time from D_0 to D^* :

$$T^* = \left(\ln D^* - \ln D_0\right) / \left(\alpha - \frac{1}{2}\sigma^2\right) = \left(\ln \frac{\left(\frac{C_b}{\rho} + I\right)(\rho - \alpha)\beta_1}{Cw(1 - r)(\beta_1 - 1)} - \ln D_0\right) / \left(\alpha - \frac{1}{2}\sigma^2\right) \tag{6-1}$$

APPENDIX 7: PROOF OF EQUATION (11)

$$\begin{split} &V_{2}\left(D_{t}\right)=E\left\{\int_{t}^{\infty}e^{-\rho(x-t)}\left[PD_{x}-C\left(1+rw\right)D_{x}+SD_{x}-C_{b}\right]dx\right\}\\ &=E\left\{\int_{t}^{\infty}e^{-\rho(x-t)}\left[P-C\left(1+rw\right)+S\right]D_{x}dx\right\}-\int_{t}^{\infty}e^{-\rho(x-t)}C_{b}dx\\ &=\left[P-C\left(1+rw\right)+S\right]\int_{t}^{\infty}e^{-\rho(x-t)}E\left(D_{x}\right)dx-C_{b}\int_{t}^{\infty}e^{-\rho(x-t)}dx\\ &=\left[P-C\left(1+rw\right)+S\right]\int_{t}^{\infty}e^{-\rho(x-t)}D_{t}e^{\alpha(x-t)}dx-C_{b}\int_{t}^{\infty}e^{-\rho(x-t)}dx\\ &=\left[P-C\left(1+rw\right)+S\right]D_{t}\int_{t}^{\infty}e^{-(\rho-\alpha)(x-t)}dx-C_{b}\int_{t}^{\infty}e^{-\rho(x-t)}dx\\ &=\left[-\frac{\left[P-C\left(1+rw\right)+S\right]D_{t}}{\rho-\alpha}\right]e^{-(\rho-\alpha)(x-t)}\left|_{t}^{\infty}-\left(-\frac{C_{b}}{\rho}\right)e^{-\rho(x-t)}\right|_{t}^{\infty}=\frac{\left[P-C\left(1+rw\right)+S\right]D_{t}}{\rho-\alpha}-\frac{C_{b}}{\rho} \end{split} \tag{7-1}$$

APPENDIX 8: PROOF OF COROLLARY 1

By Equation (14), Equation (15) and technical conditions of $\rho > \alpha$ and $\alpha - \frac{\sigma^2}{2} > 0$:

$$\frac{\partial D^*}{\partial C} = -\frac{w\left(1-r\right)\left(\frac{C_b}{\rho} + I - U\right)\left(\rho - \alpha\right)\beta_1}{\left[Cw\left(1-r\right) + S\right]^2\left(\beta_1 - 1\right)} < 0 \tag{8-1}$$

$$\begin{split} &\frac{\partial T^*}{\partial C} = \frac{\partial T^*}{\partial D^*} \frac{\partial D^*}{\partial C} = -\frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)D^*} \frac{w\left(1 - r\right)\left(\frac{C_b}{\rho} + I - U\right)\left(\rho - \alpha\right)\beta_1}{\left[Cw\left(1 - r\right) + S\right]^2\left(\beta_1 - 1\right)} \\ &= -\frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)\left[Cw\left(1 - r\right) + S\right]} < 0 \end{split} \tag{8-2}$$

$$\frac{\partial D^*}{\partial w} = -\frac{C\left(1-r\right)\left(\frac{C_b}{\rho} + I - U\right)\left(\rho - \alpha\right)\beta_1}{\left[Cw\left(1-r\right) + S\right]^2\left(\beta_1 - 1\right)} < 0 \tag{8-3}$$

$$\begin{split} \frac{\partial T^*}{\partial w} &= \frac{\partial T^*}{\partial D^*} \frac{\partial D^*}{\partial w} = -\frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)D^*} \frac{C\left(1 - r\right)\left(\frac{C_b}{\rho} + I - U\right)\left(\rho - \alpha\right)\beta_1}{\left[Cw\left(1 - r\right) + S\right]^2\left(\beta_1 - 1\right)} \\ &= -\frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)\left[Cw\left(1 - r\right) + S\right]} < 0 \end{split} \tag{8-4}$$

APPENDIX 9: PROOF OF COROLLARY 2

By Equation (14), Equation (15) and technical conditions of $\rho > \alpha$ and $\alpha - \frac{\sigma^2}{2} > 0$:

$$\frac{\partial D^*}{\partial r} = \frac{Cw\left(\frac{C_b}{\rho} + I - U\right)(\rho - \alpha)\beta_1}{\left[Cw(1 - r) + S\right]^2(\beta_1 - 1)} > 0 \tag{9-1}$$

$$\frac{\partial T^*}{\partial r} = \frac{\partial T^*}{\partial D^*} \frac{\partial D^*}{\partial r} = \frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)D^*} \frac{Cw\left(\frac{C_b}{\rho} + I - U\right)(\rho - \alpha)\beta_1}{\left[Cw(1 - r) + S\right]^2(\beta_1 - 1)}$$

$$= \frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)} \frac{Cw}{\left[Cw(1 - r) + S\right]} > 0$$
(9-2)

APPENDIX 10: PROOF OF COROLLARY 3

By Equation (14), Equation (15) and technical conditions of $\rho > \alpha$ and $\alpha - \frac{\sigma^2}{2} > 0$:

$$\frac{\partial D^*}{\partial I} = \frac{\left(\rho - \alpha\right)\beta_1}{\left[Cw(1 - r) + S\right]\left(\beta_1 - 1\right)} > 0 \tag{10-1}$$

$$\frac{\partial \boldsymbol{T}^*}{\partial \boldsymbol{I}} = \frac{\partial \boldsymbol{T}^*}{\partial \boldsymbol{D}^*} \frac{\partial \boldsymbol{D}^*}{\partial \boldsymbol{I}} = \frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)\boldsymbol{D}^*} \frac{\left(\rho - \alpha\right)\beta_1}{\left[Cw\left(1 - r\right) + S\right]\left(\beta_1 - 1\right)}$$

$$=\frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)} \frac{1}{\left(\frac{C_b}{\rho} + I - U\right)} > 0 \tag{10-2}$$

$$\frac{\partial D^*}{\partial C_b} = \frac{\left(\rho - \alpha\right)\beta_1}{\rho \left[Cw\left(1 - r\right) + S\right]\left(\beta_1 - 1\right)} > 0 \tag{10-3}$$

$$\frac{\partial \, \boldsymbol{T}^*}{\partial \, \boldsymbol{C}_{\boldsymbol{b}}} = \frac{\partial \, \boldsymbol{T}^*}{\partial \, \boldsymbol{D}^*} \frac{\partial \boldsymbol{D}^*}{\partial \, \boldsymbol{C}_{\boldsymbol{b}}} = \frac{1}{\left(\alpha - \frac{1}{2} \, \sigma^2\right) \boldsymbol{D}^*} \frac{\left(\rho - \alpha\right) \beta_{\boldsymbol{1}}}{\rho \left[Cw\left(1 - r\right) + S\right] \left(\beta_{\boldsymbol{1}} - 1\right)}$$

$$= \frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)} \frac{1}{\left(\frac{C_b}{\rho} + I - U\right)\rho} > 0 \tag{10-4}$$

APPENDIX 11: PROOF OF COROLLARY 4

By Equation (14), Equation (15) and technical conditions of $\rho > \alpha$ and $\alpha - \frac{\sigma^2}{2} > 0$:

$$\frac{\partial D^*}{\partial U} = -\frac{\left(\rho - \alpha\right)\beta_1}{\left[Cw\left(1 - r\right) + S\right]\left(\beta_1 - 1\right)} < 0 \tag{11-1}$$

$$\frac{\partial \boldsymbol{T}^*}{\partial \boldsymbol{U}} = \frac{\partial \boldsymbol{T}^*}{\partial \boldsymbol{D}^*} \frac{\partial \boldsymbol{D}^*}{\partial \boldsymbol{U}} = -\frac{1}{\left[\alpha - \frac{1}{2}\sigma^2\right]\boldsymbol{D}^*} \frac{\left(\rho - \alpha\right)\beta_1}{\left[Cw\left(1 - r\right) + S\right]\left(\beta_1 - 1\right)}$$

$$= -\frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)} \frac{1}{\left(\frac{C_b}{\rho} + I - U\right)} < 0 \tag{11-2}$$

$$\frac{\partial D^*}{\partial S} = -\frac{\left(\frac{C_b}{\rho} + I - U\right)(\rho - \alpha)\beta_1}{\left[Cw(1 - r) + S\right]^2(\beta_1 - 1)} < 0 \tag{11-3}$$

$$\begin{split} \frac{\partial T^*}{\partial S} &= \frac{\partial T^*}{\partial D^*} \frac{\partial D^*}{\partial S} = -\frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)D^*} \frac{\left(\frac{C_b}{\rho} + I - U\right)(\rho - \alpha)\beta_1}{\left[Cw(1 - r) + S\right]^2 \left(\beta_1 - 1\right)} \\ &= -\frac{1}{\left(\alpha - \frac{1}{2}\sigma^2\right)} \frac{1}{\left[Cw(1 - r) + S\right]} < 0 \end{split} \tag{11-4}$$

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