

A Fuzzy Adaptive Firefly Algorithm for Multilevel Color Image Thresholding Based on Fuzzy Entropy

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ABSTRACT

Multilevel thresholding image segmentation has been a hot issue of research in the last several years since it has a plenty of applications. The meta-heuristic search algorithm has unique advantages in solving multilevel threshold values. In this paper, a fuzzy adaptive firefly algorithm (FaFA) is proposed to solve the optimal multilevel thresholding for color images, and the fuzzy Kapur's entropy is considered as its objective function. In the FaFA, a fuzzy logical controller is designed to adjust the control parameters. A total of six satellite remote sensing color images are conducted in the experiments. The performance of the FaFA is compared with FA, BWO, SSA, NaFA, and ODFA. Some measure metrics are performed in the experiments. The experimental results show that the FaFA obviously outperforms other five algorithms.

KEYWORDS

Color Image Segmentation, Fuzzy Adaptive Firefly Algorithm, Fuzzy Kapur's Entropy, Multilevel Thresholding

1. INTRODUCTION

Image segmentation is an important method for image preprocessing, which has been widely applied to different engineering fields (Wang et al. 2018). For different image segmentation techniques, thresholding image segmentation is one of the popular techniques and has attracted a lot of attentions (Dhal et al. 2019). Generally, thresholding image segmentation is based on the gray histogram, and it is usually classified into two categories: bi-level and multilevel thresholds. If an image can be split into two regions, which is known as bi-level global thresholding. Assume that an image has multiple interesting parts, bi-level thresholding may not obtain the desirable results. However, multilevel thresholding should be taken into account to acquire the proper thresholds (Bhandari et al. 2018). Recently years, multilevel thresholding has attracted a lot of attentions, many researchers and scholars have put a plenty of effort into studying it. However, with the number of threshold value increases, the time complexity also increases exponentially since they search the optimal thresholding using exhaustive search method.

Satellite remote sensing color images have many applications including meteorological prediction, environmental protection and resource exploration. For color images, the images contain more

DOI: 10.4018/IJCNIN.20211001.0a44

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information and has three different color components, the time complexity will be exponential to the traditional exhaustive search method. Therefore, multilevel thresholding for color satellite image segmentation commonly be identified as a NP-hard optimization problem (Li and Wang. 2019). Facing multilevel thresholding problem, the computational complexity is an urgently problem to be solved. Traditional exhaustive search algorithm for multilevel thresholding are time expensive. Particularly, it will be a challenging task for multilevel thresholding image segmentation when dealing with remote sensing and satellite images (Jia et al. 2019). However, heuristic search algorithm has obvious advantages in searching the optimal value, so researchers have proposed plenty of optimization algorithms to solve multilevel thresholding problem, such as genetic algorithm (GA) (Manikandan et al. 2014), chaotic particle swarm optimization (CPSO) (Suresh et al. 2017), artificial bee colony algorithm (ABC) (Horng. 2011), modified bacterial foraging optimization (MBFO) (Tang et al. 2017) and ant colony optimization (ACO) (Khorram et al. 2019). Recently, a plenty of novel heuristic algorithms are reported, which are utilized to solve various optimization problems. Such as whale optimization algorithm (WOA) (Aziz et al. 2017), difference evolution (DE) (Li et al. 2019), improved bat algorithm (IBA) (Yun et al. 2019), Firefly algorithm (FA) (Pare et al. 2017), Salp Swarm Algorithm (SSA) (Wang et al. 2020), Electromagnetic field optimization (EFO) (Upadhyay et al. 2019), flower pollination algorithm (FPA) (Wang et al. 2015), black widow optimization (BWO) (Houssein et al. 2020), Harris hawks optimization (HHO) (Bao et al. 2019), moth-flame optimization (MFO) (Khairuzzaman et al. 2017), krill herd optimization (KHO) (Beevi et al. 2016) and so on. Among them, the FA is a popular heuristic algorithm based on the firefly flight process. Unlike the ABC, GA and the above mentioned other algorithms. The FA has few control parameters, it is very easy to implement; and has been widely used for solving different types of optimization problems, including feature selection (Zhang et al. 2017), solar radiation prediction (Ibrahim et al. 2017), multilevel thresholding (Rajinikanth et al. 2015), many objective optimization and many others (Li and Chen. 2018).

Though the standard FA has been proved to be an effective optimization algorithm, it still should be enhanced when facing complex optimization problems. At present, a plenty of improvement FA have been reported, in which most of the variant FA focus on changing the control parameters. Raja N S et al. used a Levy flight to replaced the random search strategy in the standard FA. The experimental results indicated that the LFFA provide better optimization accuracy compared with FA (Raja et al. 2013). Fister et al. proposed 12 different chaotic maps to adjust the three parameters α , β , and γ . The experimental results showed that chaotic map can effectively enhance the convergence rate and robustness of the standard FA (Fister et al. 2015). Yu et al. proposed a variable step size based firefly algorithm (VSSFA), in VSSFA, a dynamic decreasing function is utilized to model the change of parameter α , the results showed that the proposed algorithm improved the performance of the standard FA (Yu et al. 2015). Baykasoglu A et al. proposed an adaptive firefly algorithm, the results showed the performance of the AFA was apparently improved when compared with FA (Baykasoglu et al. 2015). Verma O P et al. presented a opposition and dimensional based FA (ODFA), in ODFA, a opposition based methodology was proposed to initialize the population. The results indicated that the proposed algorithm achieved high convergence speed than FA, PSO, ACO, EFO and DE (Baykasoglu et al. 2015). H.Wang et al. regarded that the firefly would attract the nearest firefly, so they proposed a neighborhood attraction (NaFA). The experimental results showed that the NaFA was better than other six FA variants in term of time complexity and accuracy (Wang et al. 2017). Aref Y et al. conceived a modified firefly algorithm (MFA), in which the tidal force formula was applied to enhance the FA, the study findings indicated the MFA outperforms other existing FA variants (Yelghi and Kose. 2018). All of the improvements were adopted to balance the exploitation and exploration capabilities of the algorithm. There is no single algorithm can solve every problem. When solving color image thresholding, The standard FA is still need to be improved.

In order to enhance the performance of the classic firefly algorithm and improve the efficiency of solving multilevel color image threshold value. In this paper, an adaptive firefly algorithm called FaFA is proposed to enhance the performance of the original FA. A fuzzy logic controller is used to

adjust the three parameters. A total of six satellite images are utilized to estimate the performance of the proposed algorithm. Moreover, FA, SSA, BWO, ODFA and NaFA are introduced to compare with the FaFA. The corresponding measure indexes including the objective function value, PSNR, FSIM, p -value test and the performance of the convergence are adopted in the experiments.

The rest of the paper is organized as follows: The theory of fuzzy Kapur's entropy is explained in Section 2. Section 3 introduces the firefly algorithm. The following, the framework of fuzzy adaptive firefly algorithm is shown in section 4. Experiments results are analyzed in section 5. Conclusions and further work are drawn in section 6.

2. THRESHOLDING IMAGE SEGMENTATION METHOD

Multilevel thresholding for color image segmentation is to search for a series of suitable thresholds based on histograms of each color channel to split three components (red, green and blue) respectively. In generally, multilevel thresholding for color image segmentation based on heuristic algorithm often adopted some criterion such as Otsu, Tsallis entropy, Renyi entropy, cross entropy and fuzzy entropy (Pare et al. 2020). To obtain a better image segmentation results, we use a fuzzy Kapur's entropy to measure the segmentation image.

2.1 Kapur's Entropy

For color image, assume that each color component with L gray levels and these gray levels are within the range of $[0, L-1]$. If the threshold vector $T = [t_1, t_2, \dots, t_{n-1}]$ is made up of $n-1$ thresholds, which can segment an image into n classes:

$$\begin{cases} I_1 = \{(x, y) \mid 0 \leq g(x, y) \leq t_1 - 1\} \\ I_2 = \{(x, y) \mid t_1 \leq g(x, y) \leq t_2 - 1\} \\ \dots \\ I_n = \{(x, y) \mid t_{n-1} \leq g(x, y) \leq L - 1\} \end{cases} \quad (1)$$

Then, the probability p_i of each gray level can be calculated by:

$$p_i = \frac{f_i}{\sum_{i=0}^{L-1} f_i}, \quad \sum_{i=0}^{L-1} p_i = 1 \quad (2)$$

where f_i is the frequency of gray-level i .

Kapur's entropy method is to calculate the maximum value of the objective entropy function based on the histogram of the image, which can be easily extended to solve multilevel thresholds image segmentation problem:

$$\begin{aligned}
 H_1 &= -\sum_{i=0}^{t_1-1} \frac{p_i}{\omega_1} \ln \frac{p_i}{\omega_1}, & \omega_1 &= \sum_{i=0}^{t_1-1} p_i \\
 H_2 &= -\sum_{i=t_1}^{t_2-1} \frac{p_i}{\omega_2} \ln \frac{p_i}{\omega_2}, & \omega_2 &= \sum_{i=t_1}^{t_2-1} p_i \\
 H_j &= -\sum_{i=t_{j-1}}^{t_j-1} \frac{p_i}{\omega_j} \ln \frac{p_i}{\omega_j}, & \omega_j &= \sum_{i=t_{j-1}}^{t_j-1} p_i \\
 H_n &= -\sum_{i=t_n}^{L-1} \frac{p_i}{\omega_n} \ln \frac{p_i}{\omega_n}, & \omega_n &= \sum_{i=t_n}^{L-1} p_i
 \end{aligned} \tag{3}$$

where H_j denotes the Kapur's entropy value of j -th part of segmented image. There are n thresholds need to be solved, so it can be deemed as a n dimensional optimization problem. The objective function is defined as:

$$T^* = \arg \max \left(\sum_{i=0}^m H_i \right) \tag{4}$$

2.2 Fuzzy Kapur's Entropy Multilevel Thresholding

In the fuzzy entropy theory, assume that the size of an image is $M \times N$, and let an image I be $D = \{(i, j) | i=0, \dots, M-1; j=0, \dots, N-1\}$. If the image is divided into three regions named as E_d , E_m , E_b by two thresholds. Usually, the histogram of the image can be computed by:

$$h_k = \frac{n_k}{M * N}, \quad \sum_{k=0}^{255} h_k = 1. \tag{5}$$

where $0 \leq k \leq 255$; n_k is the number of occurrences of the k -th pixel in D_k ; h_k is the probability of gray level k . Hence, the probability distribution of E_d , E_m , E_b can be expressed as:

$$p_d = P(E_d); \quad p_m = P(E_m); \quad p_b = P(E_b) \tag{6}$$

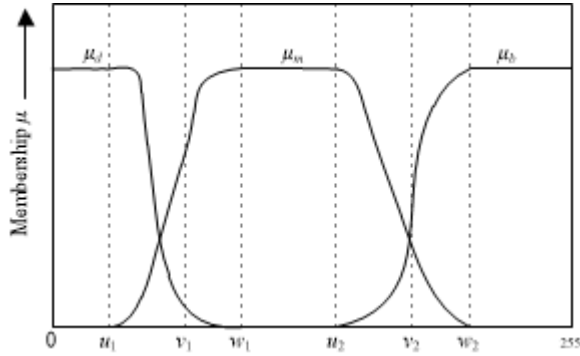
For each $(i, j) \in D$, let:

$$\begin{aligned}
 D_d &= \{(i, j) | 0 \leq g(i, j) < t_1\}, \\
 D_m &= \{(i, j) | t_1 \leq g(i, j) < t_2\}, \\
 D_b &= \{(i, j) | t_2 \leq g(i, j) \leq L-1\}
 \end{aligned} \tag{7}$$

Utilizing μ_d, μ_m, μ_b as the membership functions of E_d, E_m, E_b , which is shown in Figure 1. There are six fuzzy parameters of $u_1, v_1, w_1, u_2, v_2, w_2$ in the membership functions.

Therefore, the thresholds t_1 and t_2 depend on the six fuzzy parameters. The probability distribution of the three regions can be calculated by the following formula (Naidu et al. 2017):

Figure 1. Membership function graph



$$\begin{aligned}
 p_d &= \sum_{k=0}^{255} p_k \cdot p_{d|k} = \sum_{k=0}^{255} p_k \cdot \mu_d(k) \\
 p_m &= \sum_{k=0}^{255} p_k \cdot p_{m|k} = \sum_{k=0}^{255} p_k \cdot \mu_m(k) \\
 p_b &= \sum_{k=0}^{255} p_k \cdot p_{b|k} = \sum_{k=0}^{255} p_k \cdot \mu_b(k)
 \end{aligned} \tag{8}$$

where p_{dk} , p_{mk} , p_{bk} represent the conditional probability and satisfy the constraint of $p_{dk} + p_{mk} + p_{bk} = 1$. The corresponding mathematical formulas of the three membership functions are expressed as:

$$\mu_k(k) = \begin{cases} 1 & k \leq u_1 \\ 1 - \frac{(k - u_1)^2}{(\omega_1 - u_1) \cdot (v_1 - u_1)} & u_1 \leq k \leq v_1 \\ \frac{(k - \omega_1)^2}{(\omega_1 - u_1) \cdot (\omega_1 - v_1)} & v_1 \leq k \leq \omega_1 \\ 0 & k \geq \omega_1 \end{cases} \tag{9}$$

$$\mu_b(k) = \begin{cases} 0 & k \leq u_2 \\ \frac{(k - u_2)^2}{(\omega_2 - u_2) \cdot (v_2 - u_2)} & u_2 \leq k \leq v_2 \\ 1 - \frac{(k - \omega_2)^2}{(\omega_2 - u_2) \cdot (\omega_2 - v_2)} & v_2 \leq k \leq \omega_2 \\ 1 & k \geq \omega_2 \end{cases} \tag{10}$$

$$\mu_m(k) = \begin{cases} 0 & k \leq u_1 \\ \frac{(k - u_1)^2}{(\omega_1 - u_1) \cdot (v_1 - u_1)} & u_1 \leq k \leq v_1 \\ 1 - \frac{(k - \omega_1)^2}{(\omega_1 - u_1) \cdot (\omega_1 - v_1)} & v_1 \leq k \leq \omega_1 \\ 1 & \omega_1 \leq k \leq u_2 \end{cases} \quad (11)$$

where $u_1, v_1, w_1, u_2, v_2, w_2$ should satisfy the condition of $0 \leq u_1 < v_1 < w_1 < u_2 < v_2 < w_2 \leq 255$. Then, the fuzzy entropy of each part can be calculated by the following formulas:

$$H_d = - \sum_{k=0}^{255} \frac{p_k \cdot \mu_d(k)}{p_d} \ln \left(\frac{p_k \cdot \mu_d(k)}{p_d} \right) \quad (12)$$

$$H_m = - \sum_{k=0}^{255} \frac{p_k \cdot \mu_m(k)}{p_m} \ln \left(\frac{p_k \cdot \mu_m(k)}{p_m} \right) \quad (13)$$

$$H_b = - \sum_{k=0}^{255} \frac{p_k \cdot \mu_b(k)}{p_b} \ln \left(\frac{p_k \cdot \mu_b(k)}{p_b} \right) \quad (14)$$

The conventional fuzzy entropy function is to calculate the sum entropy of each part, and the optimal multilevel thresholding is to maximize the total fuzzy entropy (Song et al. 2019):

$$H(u_1, v_1, w_1, u_2, v_2, w_2) = \arg \max (H_d + H_m + H_b) \quad (15)$$

Hence, fuzzy entropy for bi-level thresholds color image segmentation need to solve six optimal parameters, the time consuming is expensive. However, the maximize fuzzy entropy does not take into account the detail problem of an image.

As shown in Figure 1, the optimal membership value is the crossover points of membership function which can be expressed as:

$$\begin{aligned} \mu_d(t_1) &= \mu_m(t_1) = 0.5 \\ \mu_m(t_2) &= \mu_b(t_2) = 0.5 \end{aligned} \quad (16)$$

According to the above equations, t_1 and t_2 can be calculated by the following formulas:

$$t_1 = \begin{cases} u_1 + \sqrt{(w_1 - u_1) \cdot (v_1 - u_1) / 2} & (u_1 + w_1) / 2 < v_1 < w_1 \\ w_1 - \sqrt{(w_1 - u_1) \cdot (v_1 - u_1) / 2} & u_1 < v_1 < (u_1 + w_1) / 2 \end{cases} \quad (17)$$

$$t_2 = \begin{cases} u_2 + \sqrt{(w_2 - u_2) \cdot (v_2 - u_2) / 2} & (u_2 + w_2) / 2 < v_2 < w_2 \\ w_2 - \sqrt{(w_2 - u_2) \cdot (v_2 - u_2) / 2} & u_2 < v_2 < (u_2 + w_2) / 2 \end{cases} \quad (18)$$

Fuzzy entropy thresholding also can extend to multilevel thresholds image segmentation. Due to one threshold is determined by three fuzzy parameters, so obtained n thresholds, $3n$ fuzzy parameters need to be solved. Hence, fuzzy entropy multilevel thresholding is time consuming.

3. FIREFLY ALGORITHM

The firefly algorithm simulates the behavior of the fireflies in nature searching for companions through their own light. Fireflies rely on the intensity of their own light to attract other companions. The stronger the light, the stronger the attraction. The light intensity $I(r)$ can be defined as (Yang, 2009):

$$I(r) = I_0 e^{-\gamma r^2} \quad (19)$$

where I_0 represents the initial light intensity, r is the distance between two fireflies, and γ is a fixed light absorption coefficient. The attractiveness $\beta(r)$ based on the light intensity, that can be expressed as:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \quad (20)$$

where β_0 is the attractiveness at $r = 0$.

The attractiveness β depends on the distance r , the distance between any two fireflies i and j at X_i and X_j can be computed according to Euclidian distance:

$$r_{i,j} = \|X_i - X_j\| = \sqrt{\sum_{k=1}^d (x_{i,k} - x_{j,k})^2} \quad (21)$$

where $x_{i,k}$ is the k -th element of the i -th firefly and d is the dimension of the problem. Each firefly i moves toward to firefly j , as follow:

$$X_i = X_i + \beta_0 e^{-\gamma r_{i,j}^2} (X_j - X_i) + \alpha \left(rand - \frac{1}{2} \right) \quad (22)$$

where, α is called step factor, $rand$ represents uniformly distributed random number within $[0, 1]$. The process of FA is summarized as the pseudo code shown in Table 1.

Table 1. The pseudo code of firefly algorithm

The Firefly Algorithm
1. Initialize N fireflies $\{X_1(t), X_2(t), \dots, X_N(t)\}$; 2. Evaluate the fitness values; 3. while $t \leq G_{\max}$ 4. For $i=1: N$ 5. For $j=1: N$ 6. if $f(X_j(t)) < f(X_i(t))$ 7. Move firefly i towards j in all d dimensions; 8. Calculate the fitness value of the new X_i; 9. end 10. end 11. end 12. $t++$; 13. end 14. Out the optimal solution.

4. THE FUZZY ADAPTIVE FIREFLY ALGORITHM

4.1 Analysis the Parameters

The performance of FA depends on its control parameters. Different parameters setting can lead completely different results. To tackle this problem, we will analyze the parameters of the FA, then some strategies are proposed to improve the FA. Now, let us assume that the FA convergences. Ideally, all the fireflies will cluster in the same position. Then, we can obtain the following equations:

$$\lim_{t \rightarrow \infty} X_i(t) = \lim_{t \rightarrow \infty} X_j(t) = \lim_{t \rightarrow \infty} X_i(t+1), \quad \forall i \neq j \quad (23)$$

Therefore, the following conclusions can be drawn:

$$\lim_{t \rightarrow \infty} (X_i(t+1) - X_i(t)) = 0 \quad (24)$$

$$\lim_{t \rightarrow \infty} (X_i(t) - X_j(t)) = 0 \quad (25)$$

According to Equation (22):

$$X_i(t+1) - X_i(t) = \beta_0 e^{-\gamma r_{i,j}^2} (X_j(t) - X_i(t)) + \alpha \left(rand - \frac{1}{2} \right) \quad (26)$$

Then

$$\lim_{t \rightarrow \infty} (X_i(t+1) - X_i(t)) = \lim_{t \rightarrow \infty} \left(\beta_0 e^{-\gamma r_{i,j}^2} (X_j(t) - X_i(t)) + \alpha \left(rand - \frac{1}{2} \right) \right) \quad (27)$$

So, it is easy to get the following inferences:

$$\begin{aligned}
 & \lim_{t \rightarrow \infty} \left(\beta_0 e^{-\gamma r_{i,j}^2} (X_j(t) - X_i(t)) + \alpha \left(rand - \frac{1}{2} \right) \right) = 0 \\
 & \Rightarrow \lim_{t \rightarrow \infty} \left(\beta_0 e^{-\gamma r_{i,j}^2} \right) \cdot \lim_{t \rightarrow \infty} (X_j(t) - X_i(t)) + \lim_{t \rightarrow \infty} \alpha \left(rand - \frac{1}{2} \right) = 0 \\
 & \Rightarrow \left(rand - \frac{1}{2} \right) \lim_{t \rightarrow \infty} \alpha = 0 \\
 & \Rightarrow \lim_{t \rightarrow \infty} \alpha = 0
 \end{aligned} \tag{28}$$

Hence, we can summarize that the parameter α should approach to zero at the end of generations. Generally, $\alpha \in [0, 1]$.

According to the formulas (20) and (21), the attractiveness $\beta(r)$ is determined by the distance among two fireflies. However, the distance will gradually decrease during the evolutionary process, then if the algorithm convergences, the distance will turn into zero, so we can gain:

$$\lim_{t \rightarrow \infty} r_{i,j} = 0 \tag{29}$$

then

$$\lim_{t \rightarrow \infty} \beta(r) = \lim_{t \rightarrow \infty} \beta_0 e^{-\gamma r^2} = \beta_0 \lim_{t \rightarrow \infty} e^0 = \beta_0 \tag{30}$$

From the above analysis, the attractiveness $\beta(r)$ will gradually close to β_0 during the later phase. Generally, $\beta_0 = 1$. In the following, we will discuss the parameter γ . If $\gamma \rightarrow \infty$, then, the following formula can be obtained:

$$\lim_{\gamma \rightarrow \infty} \beta_0 e^{-\gamma r_{i,j}^2} = \lim_{\gamma \rightarrow \infty} \beta_0 e^{-\infty} = 0 \tag{31}$$

As a results, the Equation (22) can turn into the following formula:

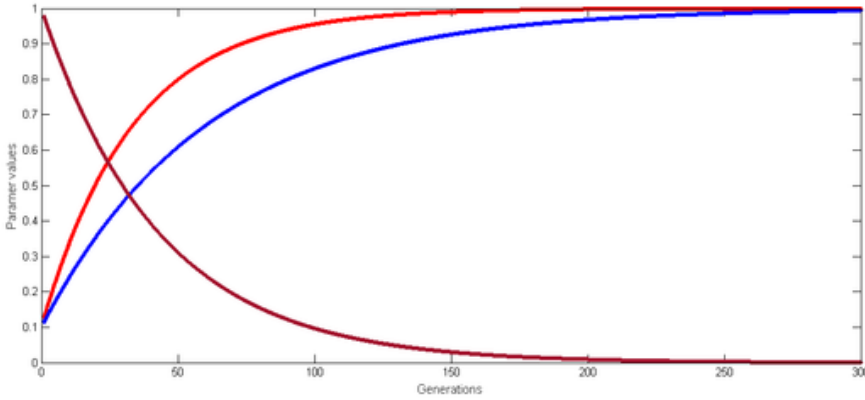
$$X_i = X_i + \alpha \left(rand - \frac{1}{2} \right) \tag{32}$$

This means the FA just can use random steps for search the space problem, it can be considered as a local search. And also, if $\gamma \rightarrow 0$.

$$\lim_{\gamma \rightarrow 0} \beta_0 e^{-\gamma r_{i,j}^2} = \lim_{\gamma \rightarrow 0} \beta_0 e^0 = \beta_0 \tag{33}$$

Then, the Equation (22) can turn into the following expression:

Figure 2. The curve of three parameters with growth of generations



$$X_i = X_i + \beta_0 (X_j - X_i) + \alpha \left(rand - \frac{1}{2} \right) \quad (34)$$

This Means that all the fireflies in the population are basically together. In this situation, it can be regarded as a global search. Hence, they are two movements in the FA. These two movements indicate local and global search in firefly algorithm. So, keeping the balance between local and global search during the evolution process, which is determined by the parameter γ . At the beginning of iterations, the setting of γ should be small, it is helpful to find the globally optimal solution. On the contrary, the setting of γ should be large, it is helpful to find the locally optimal solution.

In theory, $\gamma \in [0, \infty)$. In fact, its setting depends on the optimization problem, it typically varies from 0.1 to 10. After the above analysis, when $\beta(t) \in [0, 1]$, $\gamma(t) \in [0, 1]$, $\alpha(t) \in [0, 1]$, the curve of the the three parameters are given in Figure 2.

4.2 The Design of Fuzzy Logic Controller

After the above analysis, and from the Figure 2, we can draw the following conclusions:

- (1) when the maximum objective function value is found at the end of the generation, low the step factor α and high light absorption coefficient γ and attractiveness β are often preferred.
- (2) when the local optimal value kept at one value for a long time, and it's close to satisfying the termination condition. The light absorption coefficient γ and attractiveness β should decrease, the step factor α should increase.

Therefore, a fuzzy system is designed to tune the three control parameters. The best fitness value (*BFV*) and the number of generations (*G*) as the input variables, and the step factor α and light absorption coefficient γ and attractiveness β as the output variables. Different optimization problems have different ranges of *BFV* and *G*. So, we normalize the parameters. The *BFV* can be normalized using the following formula:

Figure 3. Membership functions of inputs and outputs (a) NBF or NU, (b) α , (c) β , (d) γ

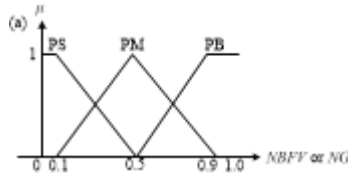


Table 2. Fuzzy rules for step factor α

α		NG		
		PS	PM	PB
NBF	PS PM PB	PS PM PM	PM PM PS	PS PS PS

$$NBFV = \frac{BFV - BFV_{\min}}{BFV_{\max} - BFV_{\min}} \quad (35)$$

where, $BFV_{\max}$ and $BFV_{\min}$ is the maximum and minimum values of BFV , respectively. The G can be normalized as:

$$NG = \frac{G - G_{\max}}{G_{\max} - G_{\min}} \quad (36)$$

where, $G_{\max}$ and $G_{\min}$ is the maximum and minimum values of G , respectively. The bound values for the parameters $\alpha(t) \in [0, 1]$, $\beta(t) \in [0, 1]$, $\gamma(t) \in [0.1, 10]$.

The membership functions of every input and output are shown in Figure 3. The fuzzy rules of the step factor (α), the attractiveness (β), the light absorption coefficient (γ) are shown in Tables 2-4. According to Tables 2-4, PS (positive small), PM (positive medium) and PB (positive big) are the linguistic values for all of the input and output variables.

According to the above description, the framework of our proposed FaFA is presented in Table 5.

Table 3. Fuzzy rules for the attractiveness β

β		NG		
		PS	PM	PB
NBF	PS PM PB	PS PM PM	PM PM PM	PM PM PB

Table 4. Fuzzy rules for the light absorption coefficient γ

γ		NG		
		PS	PM	PB
NBF	PS PM PB	PS PM PM	PM PM PB	PB PB PB

5. EXPERIMENTS AND ANALYSIS

In this section, the test color images and the corresponding histograms are presented firstly, then the parameters setting of five heuristic algorithms and the experimental conditions are listed. Finally, a set of evaluation indicators including objective function value, PSNR, FSIM, convergence performance and Wilcoxon rank sum test are utilized to evaluate the performance of the proposed algorithm.

5.1 Experimental Setting

To verify the superiority of the proposed algorithm, other five evolutionary algorithms, namely, NaFA, BWO, ODFA, SSA, and FA are selected to compare with the FaFA, these algorithms were utilized in previous works and proved perfect performance. Particularly, NaFA, ODFA are the FA variants, BWO and SSA are recently proposed. The parameters setting of these algorithms are derived from the corresponding references. The maximum number of iterations of all the algorithms used in the experiment is set to 300 with a total of 30 times independently runs per algorithm for each test image, and the population size also sets to 30. Each test image carries out in the experiments with the following number of thresholds: 5, 8, 10 and 12.

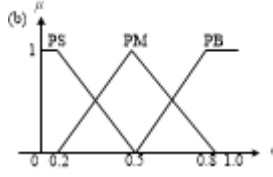
5.2 Test Images

The test satellite color images used in the experiment are labeled ‘Image 1’, ‘Image 2’, ‘Image 3’, ‘Image 4’, ‘Image 5’ and ‘Image 6’. The sizes of all images are 512×512. These original test images and their histograms are shown in Figure 4.

Table 5. Framework of the proposed FaFA

The Fuzzy Adaptive Firefly Algorithm
1. Initialize N fireflies $\{X_1, X_2, \dots, X_N\}$; 2. Evaluate the fitness values; 3. while $G \nless G_{\max}$ 4. Update the parameters according the fuzzy controller; 5. For $i=1: N$ 6. For $j=1: N$ 7. if $f(X_i) < f(X_j)$ 8. Move firefly i towards j in all d dimensions; 9. Calculate the fitness value of the new X_i; 10. end 11. end 12. end 13. $G++$; 14. end 15. Out the optimal solution.

Figure 4. Test images and the corresponding histograms for each of color channel



5.3 Results on the Objective Function Value

The proposed algorithm is utilized to solve the optimal multilevel thresholding for color images which regards the fuzzy Kapur's entropy as its objective function. It means the larger the objective function value, the better. Other five algorithms are introduced to compared with the FaFA. Then, comparing the performance of the algorithms. In detail, the mean objective function values and the corresponding standard deviation values are shown in Table 6, where 'T' is the number of the thresholds. The largest objective function value in each case are shown in bold.

In Table 6 the FaFA obtains the best results in 27 cases (30 in total) which shows the best performance in algorithm accuracy, nevertheless other five algorithms do not show superiority in any case when compared with the FaFA, the NaFA get the second rank followed by FaFA, and the BWO and ODFA perform nearly equally, and the performance of FA is the worst. The standard deviation of objective function values, also support the conclusion is that the proposed algorithm is much better than the other five algorithms.

5.4 Segmentation Image Quality Metrics

To access the performance of the proposed algorithm for color image segmentation, and avoiding the randomness in the results, Two image quality measurement indicators including Peak Signal-to-Noise Ration (PSNR) and Feature Similarity Index (FSIM) are utilized to estimate the segmented image quality. The PSNR metric is defined as (Khairuzzaman et al. 2017):

$$PSNR = 10 \log_{10} \left(\frac{255^2}{MSE} \right), \quad (dB) \quad (37)$$

where, MSE is the mean square error.

$$MSE = \frac{1}{M \cdot N} \sum_{i=1}^M \sum_{j=1}^N \left(I(i, j) - C(i, j) \right)^2 \quad (38)$$

where, C represents the original image and I stands for the segmented image, $M \cdot N$ is the size of the image. The larger the PSNR value, the better the image segmentation effect.

The FSIM is adopted to evaluate the feature similarity between two images (Elaziz et al. 2019). It is calculated by:

$$FSIM = \frac{\sum_{x \in \Omega} S_L(x) PC_m(x)}{\sum_{x \in \Omega} PC_m(x)} \quad (39)$$

Table 6. Comparison of the average objective function values and standard deviation

Images	T	Average Objective Function Values and Standard Deviation					
		NaFA	SSA	ODFA	FA	BWO	FaFA
Image 1	5	21.5762(5.69E-06)	21.4328(6.89E-06)	21.5016(9.86E-07)	21.3254(9.46E-06)	21.4896(5.97E-06)	21.6134(1.26E-07)
	8	29.2130(7.86E-05)	28.6348(8.12E-04)	29.0899(6.48E-05)	28.4698(5.27E-04)	29.1538(3.54E-05)	29.2864(3.14E-05)
	10	33.5256(8.56E-04)	33.4542(1.65E-03)	3.2263(9.82E-04)	33.1256(6.38E-02)	33.4527(7.98E-04)	33.6548(6.25E-04)
	12	37.7184(5.26E-03)	37.1981(2.98E-02)	37.3238(2.68E-02)	37.1253(9.22E-02)	37.5891(8.62E-03)	37.7221(2.18E-03)
Image 2	5	22.4570(1.14E-08)	22.3423(6.85E-07)	22.4046(7.36E-09)	22.3122(8.97E-07)	22.3864(8.93E-09)	22.4353(5.23E-09)
	8	29.8583(8.29E-06)	29.7016(4.56E-05)	29.8014(4.63E-07)	29.5893(7.98E-05)	29.7865(6.28E-06)	30.0236(4.18E-07)
	10	34.3364(5.96E-05)	34.2238(7.39E-04)	34.1912(7.21E-05)	34.1365(4.33E-03)	34.3012(7.37E-05)	34.5292(3.86E-06)
	12	38.4682(5.57E-03)	38.2287(2.01E-02)	38.2880(3.22E-03)	38.2056(6.67E-02)	38.3451(8.86E-03)	38.6469(1.94E-05)
Image 3	5	20.5713(6.67E-07)	20.2212(3.99E-06)	20.5124(8.96E-07)	20.1354(5.91E-06)	20.4326(5.77E-07)	20.6187(9.87E-08)
	8	27.6569(5.41E-06)	27.2401(4.48E-04)	27.4059(4.67E-05)	27.1365(3.35E-04)	27.7634(6.89E-06)	27.7380(6.59E-07)
	10	32.3372(4.33E-05)	31.6709(8.29E-04)	32.1320(3.37E-04)	31.5569(9.01E-02)	32.2869(7.61E-05)	32.3376(3.68E-05)
	12	36.5716(8.62E-04)	36.4026(7.02E-03)	36.4086(1.64E-04)	36.0347(4.57E-01)	36.4102(8.13E-04)	36.7373(9.54E-05)
Image 4	5	21.4959(9.29E-07)	21.3901(3.96E-06)	21.6143(4.31E-07)	21.2356(8.69E-07)	21.3693(6.99E-07)	21.5841(5.72E-07)
	8	29.2448(7.61E-05)	28.6814(5.92E-04)	29.1488(1.57E-04)	28.5642(7.20E-05)	29.1245(6.78E-06)	29.3541(2.39E-06)
	10	33.7535(9.25E-05)	33.0666(8.63E-03)	33.5273(6.33E-04)	33.0354(5.63E-04)	33.6524(8.96E-05)	34.1201(6.59E-05)
	12	37.7927(4.62E-03)	36.8502(6.31E-02)	37.1648(8.97E-03)	37.2659(2.28E-02)	36.7952(1.24E-05)	37.9542(5.91E-04)
Image 5	5	21.3851(8.86E-08)	21.1634(8.65E-07)	21.1943(6.37E-09)	21.1024(8.10E-07)	21.2302(3.26E-09)	21.4365(5.36E-10)
	8	28.5824(4.59E-07)	28.5578(9.86E-06)	28.4251(8.95E-07)	28.3695(6.07E-05)	28.4867(5.43E-07)	28.6953(7.78E-08)
	10	32.8165(7.11E-07)	32.5090(4.67E-04)	32.6598(6.12E-06)	32.4967(4.99E-03)	32.7243(9.77E-06)	32.9846(2.33E-07)
	12	36.7466(3.67E-04)	36.3301(3.94E-03)	36.5673(7.73E-04)	36.2861(2.27E-02)	36.6645(1.67E-05)	36.8697(8.54E-05)
Image 6	5	0.9151(2.28E-07)	20.6474(1.29E-07)	20.7599(8.66E-07)	20.5634(5.38E-07)	20.8651(3.95E-07)	21.2138(6.29E-08)
	8	28.0419(8.63E-06)	27.3719(6.48E-05)	27.8685(5.97E-06)	27.2563(4.91E-05)	27.8967(5.46E-06)	28.2657(9.11E-07)
	10	32.5295(3.23E-05)	31.1992(1.12E-04)	32.2689(8.02E-05)	31.1161(3.90E-04)	32.4681(6.85E-05)	32.6753(5.57E-06)
	12	36.4846(6.11E-04)	35.8871(8.37E-03)	36.1254(9.20E-04)	35.4652(2.04E-02)	36.3956(9.69E-04)	36.5124(4.39E-04)

where

$$\begin{aligned}
 S_L(x) &= S_{PC}(x) S_G(x); \\
 S_{PC}(x) &= \frac{2PC_1(x)PC_2(x) + T_1}{PC_1^2(x) + PC_2^2(x) + T_1}; \\
 S_G(x) &= \frac{2G_1(x)G_2(x) + T_2}{G_1^2(x) + G_2^2(x) + T_2}; \\
 PC_m(x) &= \max\{PC_1(x), PC_2(x)\}.
 \end{aligned} \tag{40}$$

where T_1 and T_2 are constants. $G(x)$ is the gradient magnitude of an image, and $PC(x)$ represents the phase congruence of an image. Here, we choose $T_1=0.85$ and $T_2=160$ in the experiments. A higher value of FSIM implies better performance.

The PSNR and FSIM results of the six test images are calculated by the involved algorithms, which are listed in Table 7 and Table 8. It can be easily to find that the FaFA wins 27 times in total 30 comparisons on PSNR, and Table 8 shows that the FaFA wins 28 times in total 30 comparisons on FSIM. All the experimental results show that the proposed FaFA is the best algorithm in comparison with the quality of the segmented images.

5.5 Convergence Performance

To further access the FaFA, the convergence performance on 12 levels of thresholding is also studied. For the sake of fairness, All the parameters and settings keep it the same as the previous setting. In the same way, the six test images are tested to exemplify the performance. The results are shown in Figure 5. it can be found in Figure 5 that most of the algorithms like SSA, ODFA, BWO, NaFA and

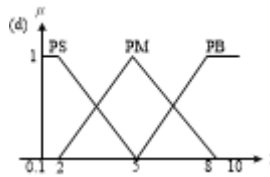
Table 7. Comparison of the average PSNR

Test Images	T	The Average PSNR					
		NaFA	SSA	ODFA	FA	BWO	FaFA
Image 1	5	20.6676	20.4376	20.4567	20.1123	20.5568	21.2641
	8	24.3047	23.4907	23.7378	23.0659	24.1236	24.3827
	10	25.1320	24.1958	25.0864	23.7536	25.0124	25.2841
	12	26.5284	25.9120	26.4550	25.1568	26.4851	26.6124
Image 2	5	20.2866	18.9355	20.2763	19.2954	20.1986	20.3012
	8	23.5917	23.3285	23.0385	22.0245	23.4893	23.5890
	10	25.2743	24.8816	24.7235	24.1369	25.1698	25.8642
	12	26.8006	24.1615	26.5675	25.8032	26.7851	26.9142
Image 3	5	20.5375	19.4044	20.4606	19.1564	20.4865	20.6012
	8	22.3450	20.9912	22.3012	21.3134	22.2639	22.4325
	10	24.6083	24.1121	23.8006	22.5124	24.5123	23.7273
	12	26.2609	26.1128	26.1728	25.9213	26.1352	26.3124
Image 4	5	19.5021	18.7714	19.1242	18.3879	19.4568	19.6897
	8	24.1292	21.7885	23.0387	22.6854	24.0986	24.3589
	10	25.5346	23.0229	24.1923	22.7576	25.3652	25.6354
	12	26.9231	26.2084	26.6812	25.8365	26.7956	27.1243
Image 5	5	19.9503	19.8559	19.9124	19.0865	19.8567	20.1234
	8	23.1050	22.4145	22.8451	21.3142	22.8659	23.4265
	10	25.5621	23.8770	24.6983	23.1983	25.3659	25.7698
	12	27.0504	25.5482	26.8457	26.0546	26.8956	27.2568
Image 6	5	19.9213	18.8301	19.4521	18.7356	19.8653	19.9854
	8	22.6605	21.2094	21.8647	21.1568	22.8693	22.8564
	10	25.0969	23.1463	24.6983	23.0012	24.8698	25.2689
	12	26.2698	25.3228	25.9752	25.2598	26.0243	26.3546

Table 8. Comparison of the average FSIM

Test Images	T	The Average FSIM					
		NaFA	SSA	ODFA	FA	BWO	FaA
Image 1	5	0.8813	0.8782	0.8795	0.8698	0.8769	0.8898
	8	0.9343	0.9075	0.9275	0.9021	0.9256	0.9366
	10	0.9507	0.9378	0.9486	0.9265	0.9457	0.9532
	12	0.9669	0.9453	0.9658	0.9356	0.9586	0.9670
Image 2	5	0.9413	0.9314	0.9403	0.9286	0.9365	0.9522
	8	0.9713	0.9694	0.9705	0.9536	0.9772	0.9764
	10	0.9792	0.9761	0.9768	0.9689	0.9752	0.9861
	12	0.9853	0.9821	0.9831	0.9810	0.9810	0.9892
Image 3	5	0.8668	0.8404	0.8654	0.8354	0.8569	0.8673
	8	0.9027	0.9072	0.9012	0.8968	0.9012	0.9095
	10	0.9315	0.9257	0.9260	0.9186	0.9286	0.9412
	12	0.9620	0.9509	0.9502	0.9486	0.9586	0.9645
Image 4	5	0.9155	0.9021	0.9286	0.9015	0.9102	0.9315
	8	0.9792	0.9664	0.9681	0.9568	0.9685	0.9786
	10	0.9810	0.9546	0.9692	0.9489	0.9789	0.9854
	12	0.9869	0.9732	0.9797	0.9625	0.9812	0.9880
Image 5	5	0.9394	0.9324	0.9345	0.9256	0.9345	0.9401
	8	0.9667	0.9612	0.9642	0.9586	0.9652	0.9710
	10	0.9862	0.9792	0.9812	0.9702	0.9801	0.9875
	12	0.9907	0.9855	0.9874	0.9842	0.9892	0.9924
Image 6	5	0.8194	0.8176	0.8188	0.8102	0.8186	0.8213
	8	0.9079	0.8844	0.8957	0.8712	0.8989	0.9086
	10	0.9418	0.9158	0.9356	0.9086	0.9345	0.9486
	12	0.9572	0.9449	0.9489	0.9386	0.9496	0.9601

Figure 5. The performance of convergence on 12-level thresholding



FA encounter the premature convergence whereas only FaFA overcomes the problem, and obtains the best objective function values. It also demonstrates that the success of the fuzzy adaptive strategy is effective. In conclusion, the proposed algorithm achieves significant improvement based on the standard FA, and the other five algorithms seem a little difference in convergence performance.

5.6 Statistical Results Analysis

A Wilcoxon rank sum test is performed to verify the experimental results, which has been conducted with 5% significance level (Tan et al. 2021). The null hypothesis is expressed as: there is no significant difference between the FaFA algorithm and other five heuristic algorithms. Nevertheless, the alternative hypothesis deems a significant difference among them. Based on the Wilcoxon rank sum test, the p value is used to judge the null hypothesis whether to accept the null hypothesis or not. If the p value is greater than 0.05, the null hypothesis will be rejected, it implies there is no significant difference between all algorithms. On the contrary, if the p value is less than 0.05, the alternative hypothesis will be accepted. Table 9 shows the p value results, which is executed on the objective function value, PSNR and FSIM. It can be summarized from Table 9 that the FaFA is significantly better than FA in terms of PSNR and FSIM. However, there is no significant difference between FaFA and SSA, ODFA, BWO, NaFA for all metrics. Analyzing the whole results, the FaFA has a remarkable improvement, it is feasible and effective for multilevel thresholding color image segmentation.

6. CONCLUSIONS AND FUTURE WORK

The traditional exhaustive search method for solving the optimal multilevel thresholding of satellite remote sensing images is time consuming. To address this problem, a fuzzy adaptive firefly algorithm is proposed to search the multilevel thresholding for color satellite image. In order to enhance the performance of the original FA, a fuzzy logic controller is designed. First, the ranges of the parameters are deduced. Then analyzing the change processes of the parameters. At last, a fuzzy rule tables is established and fuzzy controller is realized. The experiments results indicate the improvement of FaFA is successful. In the experiments, the proposed algorithm shows obvious improvement when compared with the other five well-known optimization algorithms for multilevel thresholding in terms of the objective function value, standard deviation, PSNR, FSIM and convergence performance. Besides,

Table 9. p -value test results

Metrics	Versus FaFA p Value				
	NaFA	SSA	ODFA	FA	BWO
OF	0.7807	0.5847	0.6575	0.5706	0.7028
PSNR	0.7182	0.0889	0.4393	0.0402	0.6575
FSIM	0.5990	0.1147	0.2882	0.0328	0.3223

the Wilcoxon's rank sum test further proves the validity of the proposed algorithm. However, the advantages of the proposed algorithm is not very obvious. Future works of this study will focus on studying the performance of the proposed algorithm for higher numbers of thresholds, and adopting different entropy functions.

ACKNOWLEDGMENT

This work is supported by the Natural Science Foundation of Guangdong Province of China with the Grant No.2020A1515010784.

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