

Specifications of a Queuing Model-Driven Decision Support System for Predicting the Healthcare Performance Indicators Pertaining to the Patient Flow

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ABSTRACT

This article has developed specifications for a new model-driven decision-support system (DSS) that aids the key stakeholders of public hospitals in estimating and tracking a set of crucial performance indicators pertaining to the patients flow. The developed specifications have considered several requirements for ensuring an effective system, including tracking the performance indicator on the level of the entire patients flow system, paying attention to the dynamic change of the values of the indicator's parameters, and considering the heterogeneity of the patients. According to these requirements, the major components of the proposed system, which include a comprehensive object-based queuing model and an object-oriented database, have been specified. In addition to these components, the system comprises the equations that produce the required predictions. From the system output perspective, these predictions act as a foundation for evaluating the performance indicators as well as developing policies for managing the patient flow in the public hospitals.

KEYWORDS

Decision Support Systems, Model-Driven DSSs, Object-Oriented Database, Patient Flow, Performance Indicator, Queuing Model

1. INTRODUCTION

The recent years have witnessed a great amount of research efforts to augment and enhance the management practices in the healthcare industry. A considerable part of these efforts has been devoted to the patients flow among the provided healthcare processes and services in the health units (hospitals, health centers, etc.) (e.g., Armony et al., 2015; Bean, Taylor, & Dobson, 2017; Dong & Perry, 2018; Fitzgerald, Pelletier, & Reznick, 2017; Vass & Szabo, 2015). These processes and services represent facilities that should be managed carefully in order to improve the patients flow. Improving such flow can result in providing a sufficient care to the admitted patients as well as achieving their satisfaction (Armony et al., 2015). An important aspect of the careful management of these facilities is tracking their performance with respect to many dimensions relating to the patients flow. These dimensions comprise the patient waiting times to benefit from these facilities, response times of these facilities, number of patients waiting in the queues of these facilities, number of patients served, utilization of these facilities, and patient satisfaction (Aziati & Hamdan, 2018; Hall, Belson, Murali, & Dessouky, 2013; Hu, Barnes, & Golden, 2018). Most of these dimensions are linked to the healthcare delivery

quality. For instance, Cerda', Pablos, & Rodriguez (2013) included the usage of the length of the waiting lists as a measure of the quality of the health system.

Accordingly, effective tools that aid the healthcare decision makers in tracking and improving the aspects of these dimensions are highly needed. Fulfilling this need has not received a wide attention on the level of the patients flow in the public hospitals. This is because the majority of the previous studies have focused on proposing tools on the level of the clinical decision making, indicating that these tools are directed to aid in the diagnosis and treatment of diseases (e.g., Cánovas-Segura, Campos, Morales, Juarez, & Palacios, 2016; Gudmundsson, Hansen, Halldorsson, Ludviksson, & Gudbjornsson, 2019; Sim, Ban, Tan, Sethi, & Loh, 2017; Yılmaz & Ozdemir, 2017).

Consequently, the present study responds to this need by developing specifications for a new decision support system (DSS) that provides the key stakeholders of the public hospitals with the required estimates for a set of crucial performance indicators pertaining to the patients flow. Producing these estimates is specifically vital for predicting the performance dimensions, supporting the decision making process, and improving the administrative processes in these hospitals. Moreover, these estimates are highly required due to the difficulty in knowing and tracking the actual values of those performance indicators. This difficulty stems from an observed lack in many public hospitals, which is that the events pertaining to the patients' movements among the provided facilities as well as those related to their treatments and tests are either not well registered and time stamped or not given attention at all. In this regard, Hall et al. (2013) pointed out that many hospitals have encountered difficulties in making these estimates due to using inadequate information systems or not having the required resources to develop and implement the needed information systems.

To ensure the effectiveness of the proposed DSS, the present study considers a set of requirements as follows. The first requirement is that the DSS should be based on an accurate comprehensive model for representing the patients flow through the hospital's facilities. Comprehensive means that it should include all facilities that the patients encounter during their treatment path. The second requirement is that the system should consider the diversity of the patient categories. In the healthcare environment, the patients are generally classified into several categories, including emergency patients and non-emergency patients (Ferrand, Magazine, Rao, & Glass, 2018; Lin, Patrick, & Labeau, 2014; Siddharthan, Jones, & Johnson, 1996; Tan, Wang, & Lau, 2012). Consequently, the system should track the performance dimensions for these categories. The third requirement is considering the variability of the patients flow and treating among the provided healthcare facilities. This variability implies deriving generic estimations for the performance indicators that can be applied for each individual facility, and then aggregating these estimations to produce their mean values on the collective level of these facilities. The last requirement is that providing accurate estimates requires considering the dynamic change of the arrival and service patterns of the patients, including the change of their arrival and service rates over time.

These requirements originate from several gaps in the literature of the patients flow performance. Among these gaps is that most of the previous studies focused on assessing the performance indicators in specific healthcare facilities (e.g., emergency department (Vass & Szabo, 2015), primary healthcare clinic (Azraii, Kamaruddin, & Ariffin, 2017), and cancer treatment center (Suss, Bhuiyan, Demirli, & Batist, 2017)). Hence, providing a generic solution for tracking these indicators that can be applied to a wide variety of facilities was not given the required attention. In this context, Armony et al. (2015) and Creemers, Lambrecht, and Vandaele (2007) pointed out the wide concentration of the patients flow literature on only some parts of the hospital rather than its entire system, thereby indicating the ignorance of the influences of interactions among these parts on the hospital's performance indicators. A second lack is that the possibility that the patterns of patients' arrival and service rate may change over the time was not considered in the majority of the prior studies (Fomundam & Herrmann, 2007; Singh, 2006). Lastly, many previous studies have not considered any classification of patients, which goes against the common practice in the healthcare area that the patients are assigned to different

priority classes on the basis of the urgency of their care needs (Hagen et al., 2013; Jiang, Abouee-Mehrizi, & Diao, 2018).

To comply with the four requirements, the components of the proposed DSS are specified as follows. The first component is a comprehensive object-based queuing model to represent the flow of different categories of patients among the queuing systems of various processes and services (i.e., the model's objects) in the hospital. The model is accompanied by derivation of the equations that facilitate the estimation of the key performance indicators pertaining to these objects. The estimation process is conducted by considering the working shift (e.g., morning or afternoon shifts) as a time unit. In spite of this determination, the estimation can also be applied for any time unit (e.g., hour). The second component is an object-oriented database to store the details of the objects included in the queuing model and their associated operations. The last component is a user interface for manipulating the object's data and displaying the outputs (e.g., predictions of the performance indicators for each patient's category).

The detailed specifications of these components, which are presented in sections 3 to 5, provide a response to the main question of this study: What are the specifications of an effective DSS that satisfies the afore-identified requirements?

2. LITERATURE REVIEW

This section presents a background to the DSSs with a focus on their components and usages in the healthcare area. Also, it provides an overview for the queuing models and their implementation for performance evaluations in such area. Additionally, the related studies are presented through this section.

2.1 DSSs Overview

Like many terms in the field of information technology, the term DSS has no universally accepted definition (Sharda, Delen, & Turban, 2015). Accordingly, many views and descriptions for this term are found in the literature. Among the common definitions of this term is that it represents "an interactive computer-based system that helps decision makers utilize data and models to solve unstructured problems" (Gorry & Scott-Morton, 1971). The unstructured problems are those that do not have specific algorithms for obtaining optimal solutions (Oz, 2008). In addition to the specific definitions, the term DSS can be used as a broad term to denote any computerized system that supports the decision making process in an organization (Sharda et al., 2015).

Most of the DSSs contain three major components: a data management sub-system, a model management sub-system, and a user interface (or dialog) sub-system. The data management sub-system includes a database that stores the relevant data for the situation. The model management sub-system consists of a single or collection of models that are utilized by the system to process data into useful information. These models are the primary sources for the analytical capabilities of the system. The user interacts with the DSS through the user interface sub-system (Haag & Cummings, 2013; Oz, 2008; Sharda et al., 2015). The three components can be identified simply as a database, model(s), and user interface (Turban & Volonino, 2011). Among these components, the models represent a crucial resource for the DSS and a distinguished feature that differentiates these systems from many other types of information systems (e.g., management information systems) (Marakas & O'Brien, 2013).

DSSs are classified into several broad categories, including data-driven, knowledge-driven, communications-driven, and model-driven DSSs (Power, 2008; Sharda et al., 2015). The context of this study imposes concentration on providing an overview for the model-driven DSSs. Model-driven DSSs mainly carry out quantitative analysis (Stair & Reynolds, 2012). Their distinguished purpose from the other categories is providing the decision makers with the required models for analyzing and handling complex problems (Liang, Lee, & Turban, 2008). Towards achieving this purpose, they emphasize access to and manipulation of a diverse kinds of models. The model can be a graphical

representation or a set of mathematical formulas for representing the possible associations between the identified variables of a specific problem. Among the different forms of models, the mathematical models have been mostly used in these systems. Another characteristic of these systems is that they generally rely on limited data and parameters to support the decision making process. Hence, they do not require large data sets (Power, 2008; Power & Sharda, 2007).

2.2 DSSs in the Healthcare Area

DSSs represent one of the eminent instances of information technology implementations in the healthcare area (Nykanen, 2000). They are characterized as one of the information technology systems that are designed to aid the healthcare professionals in the aspects of the problem solving process (Rajalakshmi, Mohan, & Babu, 2011). They have been believed as having a growing contribution to improve making of both clinical and administrative decisions (Kohli & Piontek, 2008). In this context, Landrum, Huscroft, Peachey, and Hall (2008) included the confirmation of the earlier researches that the usage of DSSs in the healthcare area has a great promise. Such promise can be stemmed from the capabilities of these systems, which include the ability to provide advices to medical professionals about possible patient complications, interpret investigations, assist in the administration of medical facilities, and measuring patient satisfaction.

A vast amount of the previous researches in the area of the healthcare DSSs were devoted to developing systems that aid the physicians and medical professionals in the diagnosis and treatment of diseases. These systems are broadly known as Clinical Decision Support Systems (CDSSs) and identified generally as those developed to aid healthcare professionals in making clinical decisions (Schuh, De Bruin, & Seeling, 2018). Examples of these systems include the Osteoporosis Adviser (OPAD) (Gudmundsson, et al., 2019; Halldorsson et al., 2015), TSM-CDS for predicting the cutaneous melanoma metastases (Schuh et al., 2018), Appy CDS for supporting decisions pertaining to specific pediatric patients (Ekstrom et al., 2019), and Diabetes-specific CDSS for supporting the management of diabetes (Sim et al., 2017).

Accordingly, developing decision support systems for predicting the performance indicators pertaining to managing patients flow and other healthcare management aspects (e.g., ensuring efficient resources management) has not received a considerable attention in the previous researches. Among the few studies that focused on these aspects, Carmen, Defraeye, and Nieuwenhuyse (2015) proposed a DSS for enhancing the patients flow in emergency departments. Their system enables analyzing the effects of different capacity changes on the patients flow (e.g., changing the number of beds). As outputs, this system produces values for several indicators related to the patients flow, such as the average length of stay and waiting time for beds. In the same direction, Saoud, Boubetra, and Attia (2018) developed a DSS for aiding healthcare managers in enhancing the quality of care provided in the emergency departments. They focused on several key performance indicators for measuring the emergency department's performance, which include the medical resources utilization and the average length of stay of admitted patients. By focusing on a different healthcare department, Lin (2013) developed a decision support tool for enhancing patients flow and service capacity at an eye outpatient department. This system serves several specific purposes, including identifying bottlenecks and predicting time-based and congestion performance measures (e.g., average waiting time).

By concentrating on the model component of the DSSs developed in the healthcare area (i.e., either clinical DSSs or others), there are several models have been proposed for these systems. Sartipi, Archer, and Yarmand (2011) categorized these models into four main categories: deterministic models (e.g., linear programming), stochastic models (e.g., queuing models), artificial intelligence (e.g., artificial neural networks), and practical (e.g., simulation).

Among these models, the queuing models represent the core of the proposed DSS in this study. The role of the queuing theory in supporting the decision making process has been emphasized in a considerable number of studies. For instance, Fitzgerald et al. (2017) included the queuing theory in their analysis for supporting decision making for implementing a separate unit for low-acuity patients.

Cochran and Roche (2008) developed a decision support methodology on the basis of the queuing theory for predicting the demand for hospital inpatient beds.

2.3 Using the Queuing Models for Estimating the Performance Indicators

The queuing models represent a major component of our proposed DSS. Therefore, the following review is devoted to their definition, benefits, and implementation in the healthcare area.

A queuing model is a mathematical description of a queuing system on the basis of certain assumptions about the system processes and elements (e.g., assumptions pertaining to the probabilistic nature of the arrival and service processes, the number servers, and the queuing discipline) (Green, 2013). The origin of the queuing models is in a study conducted in the context of telephone networks by Agner Krarup Erlang, a Danish engineer and mathematician, in the early years of the 19th century. Afterward, they have been extensively implemented in a wide variety of service industries to analyze how resource-constrained systems respond to a variety of demand levels (Hu et al., 2018; Peter & Sivasamy, 2019). Such implementation has motivated the healthcare industry to adopt these models.

In the healthcare area, the queuing models are essentially adopted for modeling patients flow through the health system (Hu et al., 2018; Singh, 2006). In this regard, several usages of these models in the healthcare area were outlined in the prior studies, including analysis of the patients arrival patterns over time, plan the healthcare resources for disaster management, quantify the proper service capacity to satisfy the patient need, and identifying opportunities for service improvement (Palvannan & Teow, 2012; Peter & Sivasamy, 2019; Singh, 2006). In these usages and others, the commonly evaluated healthcare performance indicators by using the queuing theory encompass the expected waiting time, length of stay, utilization of a resource, mean number of patients waiting in a queue, and number of patients who leave before receiving treatment (Aziati & Hamdan, 2018; Hu et al., 2018; Vass & Szabo, 2015; Yousefi, Yousefi, Fogliatto, Ferreira, & Kim, 2018).

From the perspective of the outcomes of the different usages of these models in the healthcare, the evaluations produced by them can lead to enhancements in the waiting time reduction, medical performance, patient satisfaction, healthcare cost reduction, and resources allocation (Afrane & Appah, 2014; Creemers et al., 2007; Green, 2013). As compared to the other performance evaluation methods (e.g. simulation techniques), queuing models are easier and cheaper to implement for predicting the system performance. This is because they rely on very little data and produce relatively simple formulas for estimating a variety of performance indicators (Green, 2013).

2.3.1 Related Work

A part of the related studies that used the queuing theory for performance evaluation in the healthcare sector are included in this section. The focus in this review is firstly given to the studies that considered patients prioritization or classification for assessing the performance indicators, then it is directed to those studies that did not consider any classification for the patients. Also, the review concentrates on identifying the performance indicators that have been assessed in these studies.

The related studies considered a variety of healthcare facilities. Among these facilities, the emergency department has received a wide focus in the prior studies. Lin et al. (2014) developed a queuing model to predict the average waiting time of multi-priority patients to access the emergency department as well as the necessary resources to realize the wait time targets for the different priority classes. By focusing on assessing the increased waiting time costs caused by non-emergency patients who inappropriately use the emergency department of the public hospitals, Siddharthan et al. (1996) proposed a priority queuing model to reduce the average waiting times.

With respect to the other healthcare facilities (i.e., other than the emergency department), Hagen et al. (2013) investigated several different queuing models for intensive care units (ICU) and the impacts on multiple performance measures, including waiting times, average utilization rate, and number of patients served. They considered several categories of patients on their investigation. Griffiths,

Williams, and Wood (2013) constructed a queuing model for a specialist neurological rehabilitation unit. They focused on estimating the mean length of stay for different groups of patients.

In many studies, assigning priorities or classes to patients was not considered. Some of these studies are included as follows. Kembe, Onah, and Iorkegh (2012) used a multi-server queuing model to analyze the queuing characteristics at a specialist clinic of a medical center. The main considerations of this model include that the patients are served on the basis of First-Come-First-Served discipline. They focused on determining the expected waiting and service costs. Bahadori, Mohammadnejhad, Ravangard, & Teymourzadeh (2014) focused on examining a First-Come-First-Served queuing network in an outpatient pharmacy of a hospital. The exact performance indicators that were calculated in their study include the average number of patients referred to the pharmacy and the system utilization rate. Bahadori, Teymourzadeh, Hosseini, & Ravangard (2017) targeted the performance optimization of a magnetic resonance imaging (MRI) department in a hospital using queuing theory and simulation. For assessing the performance measures pertaining to the queuing model (including the queue length and average waiting time), they considered the different workstations that the patients face in this department.

2.3.2 Literature Gap

According to the conducted review, a large part of the previous studies have focused on developing queuing models for individual departments in the hospitals. This is without looking to the whole hospital system and its effects on the performance indicators pertaining to the patients flow among its entities (i.e., departments, staff, or equipment). Moreover, these studies focused on assessing a limited number of performance indicators for their selected departments, indicating the need to extend the scope to cover a wide variety of measures that reflect the healthcare performance from many perspectives, including the perspective of the patients flow among the various facilitates in the hospital.

Also, it has been noticed that the majority of the previous studies considered only the preemptive discipline in their calculation of the performance measures. This indicates the ignorance of the situations that require the adoption of the non-preemptive discipline.

Another gap is related to calculating the value of an important performance indicator, which is the average waiting time. It has been realized that the reviewed studies did not include the essential components for calculating this indicator for the various patient classes. These components have been mentioned in section 4, which include the mean remaining service time of the currently serviced patient and the mean number of patients of a specific priority class already in the queue ahead of the new arriving patient.

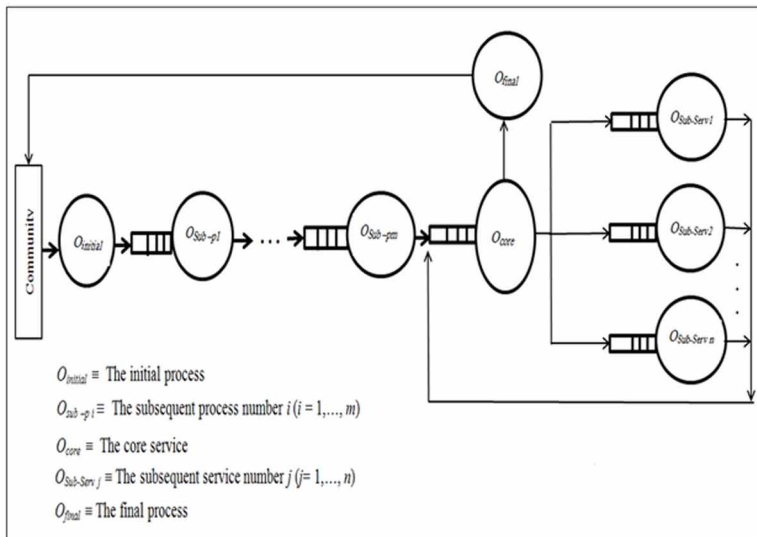
3. THE SPECIFICATIONS OF THE PROPOSED DSS

A detailed description of the components of the proposed DSS is presented in this section. These components are specified according to four steps: Developing a comprehensive object-based queuing model to represent the flow of the patients among the entire processes and services (i.e., the model's objects) provided by the hospital, deriving the formulas that facilitate the prediction of the key performance indicators pertaining to the model's objects, designing an object-oriented database to store the details of the objects included in the queuing model and their parameter's values, and specifying the user interface that will be used for performing the input and output activities of the system.

3.1 The Queuing Model Component

A multi-object queuing model (see Figure 1) has been developed to produce estimations for a set of key healthcare performance indicators. These indicators are pertaining to the patients flow among the extant processes and medical services in the public hospitals. The model consists of an initial process, m subsequent processes, a core medical service, n subsequent medical services, and a final process.

Figure 1. The multi-object queuing model



The initial process, which is also called as the sorting out process, is conducted upon an arrival of a new patient. It classifies the patient as either a high priority patient (i.e., should receive an urgent care) or a low priority patient (i.e., can follow the ordinary care procedures) on the basis of the severity of the patient's condition. The subsequent process is any process that follows the initial process or any other subsequent process, and it is defined precisely here as a set of routine tasks for carrying out specific procedures for a patient. An example of a subsequent process is the patient registration.

The medical service is defined here as performing a set of actions that contribute to the diagnosis of the patient status or his treatment. The model identifies one of the medical services as a core service and the rest as subsequent services. The core medical service is the diagnosis service, which comprises the well-known diagnosis actions, such as assessing the patient medical history, evaluating the patient's complaints and symptoms, requesting diagnostic tests and interpreting of their results, and issuing the discharge order ("The Sullivan Group", n.d.). It is identified as a core because it acts as a source for requesting subsequent medical services for the patient. Also, it acts as a destination for evaluating the results of these services. This indicates that the outputs resulted from the provision of the subsequent medical services are submitted to the core medical service for finalizing the diagnosis process.

The subsequent medical service is any service requested during providing the core service (i.e., the diagnosis service). Examples of the subsequent medical services include lab investigations, radiology tests, and medical operations. Each subsequent service represents an object that can have a finite set of categories of instances. For example, the categories of instances of the lab investigations subsequent service include blood investigations, serology investigations, chemical investigations, etc. Accordingly, the subsequent service can be viewed as a set of instances that belong to different categories.

The final process is the discharge process and it is conducted when a patient receives a leave order at the end of the diagnosis process. This order is issued after evaluating the results of the requested subsequent services for the patient and finalizing his/her diagnosis and treatment.

Consequently, the proposed model can be represented in a mathematical notation as a collection of objects as follows:

$$MOQM = \{O_{initial-process}, O_{subsequent-process\ 1}, \dots, O_{subsequent-process\ m}, O_{core-service}, O_{subsequent-service\ 1}, \dots, O_{subsequent-service\ n}, O_{final-process}\}$$

Where:

MOQM $\equiv$ Multi-Object Queuing Model

$O_{initial-process}$ $\equiv$ The sorting out process

$O_{subsequent-process\ i}$ $\equiv$ The subsequent process number i ($i=1, \dots, m$)

$O_{core-service}$ $\equiv$ The diagnosis service

$O_{subsequent-service\ j}$ $\equiv$ The subsequent service number j ($j=1, \dots, n$)

$O_{final-process}$ $\equiv$ The discharge process

In this model, each subsequent process or medical service has a queue. Joining a queue of a subsequent process can be mandatory for some processes, such as the registration process. This is in the sense that the details of each patient should be recorded. With respect to the medical services, joining the queue of the core medical service (i.e., the diagnosis) is mandatory, since the health status of each patient should be diagnosed. On the other hand, joining a queue of a subsequent medical service is not necessary for all patients. This is because the physician may not request any subsequent medical service for the patient. Consequently, the patient may not join any queue of the provided subsequent services and leaves directly at the end of the diagnosis process (i.e., moves to the discharge process).

Each queue in the model may consist of the two identified classes of the patients, which are the high and low priority patients. This combination emphasizes that the patients are served on a priority basis (i.e., a priority-based discipline), indicating that the higher priority patients are always selected for service before the low priority patients. On the level of each class, the patients are served according to a First-Come-First-Served discipline. Each patients' class has a different arrival and service rates at each queue. This stems from several factors, including the type of the object (i.e., process or service) that the class joins its queue, the obligatory of requesting the object (i.e., requested for all patients or some of them), and the number of the requested service instances for each class.

Accordingly, the queuing model consists of a set of servers, which represent the processes and services that involve the patients' queuing. These servers provide their services to a varied numbers of patients who proceed through these servers (i.e., moving from a server's queue to another) during their treatment journey, which finishes by undergoing the discharge process. Thus, the type of this queuing network is an open network in which the patients come from the community (i.e., outside the queuing system) are served according to the queuing system and then leave the system to the community (i.e., discharge).

3.2 Estimating the Mean Values of the Performance Indicators

To derive the equations of the performance indicators under focus (i.e., listed below) for the objects presented in the developed queuing model, the analysis of the priority queues conducted in Bolch, Greiner, De Meer, and Trivedi (2006), Bose (2002), and Gross, Shortle, Thompson, and Harris (2008) was adopted. In the present study, this analysis starts by deriving the equations of the mean values of the waiting times of the considered patient's priority classes. Then, it advances to calculating the mean values of the other performance measures on the basis of the obtained mean values of the waiting times. The analysis was conducted by considering the afore-identified two priority classes, which are the high priority patients (i.e., class 1) and low priority patients (i.e., class 2). In addition to this consideration, the analysis adopts the following widely used assumptions for priority queues:

- (i) The arrivals of both patient classes at an object's queue are according to independent Poisson processes with rates λ_1 for class 1 and λ_2 for class 2.

- (ii) The service distribution for each priority class is exponential with mean $1/\mu_i$ ($i=1,2$), where μ is the service rate.
- (iii) The patients are selected for service on the basis of the First-Come, First-Served discipline within their respective priority classes.
- (iv) Among the disciplines of the priority queues, the Non-preemptive Priority and Preemptive Resume Priority disciplines were assumed in the analysis due to their commonality and suitability for the considered case (i.e., queues consisting of two classes of patients). The difference between the two disciplines revolves around whether the high priority patient interrupts the service of the low priority patient (i.e., preemptive resume priority) or not (i.e., non-preemptive priority) (Bhat, 2008).

3.2.1 Description of the Performance Indicators

The exact performance indicators under focus in this analysis are as follows. The description of these indicators and their equations were adapted from Bolch et al. (2006) and Chee-Hock & Boon-Hee (2008).

- (i) Utilization of the object by class i patients [ρ_i]: This indicator is necessary to calculate the other considered indicators. Each priority class occupies an object (i.e., process or service) with an amount of time given by:

$$\rho_i = \frac{\text{arrival rate of class } i}{\text{service rate of class } i} = \frac{\lambda_i}{\mu_i}$$

This amount of time represents the utilization of the object by class i (i.e., the object is busy with class i). The overall object utilization is: $\rho = \rho_1 + \rho_2$.

- (ii) Mean Waiting Time of class i patients [$E(W_{qi})$]: It is the average time that a patient of class i waits in an object's queue in order to be served.
- (iii) Overall Mean Waiting Time in an object's queue ($E(W_q)$): It is calculated on the basis of $E(W_{qi})$ using the relation:

$$E(W_q) = \sum_{i=1}^2 \frac{\lambda_i}{\lambda} E(W_{qi})$$

- (iv) Mean Sojourn Time of class i patients [$E(T_i)$]: It is also known as the response time. It is the average total time that a patient of class i spends in the queuing system of a specific object (i.e., queue plus service). Accordingly it is calculated as follows.

$$\begin{aligned} E(T_i) &= \text{Mean Waiting Time of class } i + \text{Mean service time of class } i \\ &= E(W_{qi}) + \frac{1}{\mu_i} \end{aligned}$$

- (v) Mean Number of class i patients in the queue [$E(L_{qi})$]: It is given using a crucial formula in the queuing theory, which is called Little's formula (Little, 1961):

$$E(L_{qi}) = \lambda_i E(W_{qi})$$

(vi) Total Expected Queue Size $E(L_q)$: It is calculated using the summation formula:

$$E(L_q) = E(L_{q1}) + E(L_{q2}) = \lambda_1 E(W_{q1}) + \lambda_2 E(W_{q2})$$

(vii) Mean Number of class i patients in the queuing system of a specific object ($E(E_i)$): It is given using Little's formula:

$$E(E_i) = \lambda_i E(T_i)$$

To calculate the mean values of these performance measures, the following notation has been used:

$E(N_i) \equiv$ The mean number of patients of class i ($i=1,2$) already in the queue ahead of the new arriving patient.

$E(N_i) \equiv$ The mean number of high priority patients (i.e., class 1) who arrive later at the queue while the arriving low priority patient (i.e., class 2) in the queue and go ahead of him (i.e., served before him).

$E(S_0) \equiv$ The mean remaining service time of the currently served patient.

$E(S_i) \equiv$ The mean service time of the N_i high priority patients in the queue that are served before the arriving patient (i.e., either a low priority or high priority patient).

$E(S_2) \equiv$ The mean service time of the N_2 low priority patients in the queue that are served before the arriving low priority patient.

$E(S_i) \equiv$ The mean service time of the N_i high priority patients who arrive later at the queue and go ahead of the arriving low priority patient.

3.2.2 Non-Preemptive Priority Discipline

In this discipline, once a patient of one of the two priority classes begins the service, his/her service will not be interrupted by the arrival of a patient of the other class. This indicates that the arrival of a high priority patient does not halt the started service of a low priority patient.

The estimation of the mean waiting time in the queue $E(W_{qi})$ for the two classes of patients under this discipline is as follows. For the arriving high priority patient, the mean waiting time $E(W_{q1})$ is calculated using the following equation (Gross et al., 2008):

$$E(W_{q1}) = E(S_1) + E(S_0) \quad (1)$$

This is because such patient should wait during the time required to serve the patients of the same class already in the queue and the remaining service time of the patient in service.

The first component is given by: $E(S_1) = \frac{E(N_1)}{\mu_1}$. Using Little's formula:

$$E(N_1) = \lambda_1 E(W_{q1}). \text{ Therefore, } E(S_1) = \frac{\lambda_1 E(W_{q1})}{\mu_1} = \rho_1 E(W_{q1}).$$

Accordingly, equation (1) becomes:

$$E(W_{q1}) = \rho_1 E(W_{q1}) + E(S_0) = \frac{E(S_0)}{1 - \rho_1} \quad (2)$$

The second component in equation (1) is given by:

$$E(S_0) = p \times E[S_0 \mid \text{object (system) is busy}] \quad (3)$$

Where p is the probability that the object is busy at the time of arrival and it is found as:

$$p = \lambda \times \text{expected service time} = \lambda \times \sum_{i=1}^2 \frac{\lambda_i}{\lambda} \frac{1}{\mu_i} = \rho, \text{ and the second part is calculated as follows.}$$

$$E[S_0 \mid \text{object (system) is busy}] =$$

$$\sum_{i=1}^2 E[S_0 \mid \text{object is busy with priority class } i \text{ patient}] \times \text{probability} [\text{object is busy with priority class } i \text{ patient} \mid \text{object is busy}] = \sum_{i=1}^2 \frac{1}{\mu_i} \frac{\rho_i}{\rho}$$

Accordingly, equation (3) becomes:

$$E(S_0) = \rho \times \sum_{i=1}^2 \frac{1}{\mu_i} \frac{\rho_i}{\rho} = \sum_{i=1}^2 \frac{\rho_i}{\mu_i} \quad (4)$$

Inserting this into equation (2) gives:

$$E(W_{q1}) = \frac{\sum_{i=1}^2 \frac{\rho_i}{\mu_i}}{1 - \rho_1} \quad (5)$$

For the arriving low priority patient, the mean waiting time $E(W_{q2})$ consists of four components:

$$E(W_{q2}) = E(S_1) + E(S_2) + E(S_1) + E(S_0) \quad (6)$$

This is because the arriving patient with low priority should wait during the service time of both low and high priority patients already in the queue ($E(S_1)$ and $E(S_2)$) as well as high priority patients who arrive later at the queue and go ahead of this patient ($E(S_1)$). This is in addition to waiting during the remaining service time of the currently served patient ($E(S_0)$).

$$\text{The first and second components are given by: } E(S_i) = \frac{E(N_i)}{\mu_i} \quad (i=1,2)$$

$$\text{Using Little's formula, } E(N_i) = \lambda_i E(W_{qi}), \text{ implies that: } E(S_i) = \frac{\lambda_i E(W_{qi})}{\mu_i} = \rho_i E(W_{qi})$$

Similarly, the third component is calculated as follows.

$$E(S_1) = \frac{E(N_1)}{\mu_1} = \frac{\lambda_1 E(W_{q2})}{\mu_1} = \rho_1 E(W_{q2})$$

Inserting these equations of the first three components in equation (6) yields:

$$E(W_{q2}) = \frac{E(S_0)}{(1-\rho_1)(1-\rho_1-\rho_2)} = \frac{\sum_{i=1}^2 \frac{\rho_i}{\mu_i}}{(1-\rho_1)(1-\rho_1-\rho_2)} \quad (7)$$

3.2.3 Preemptive Resume Priority Discipline

In this discipline, the service of the low priority patient is interrupted upon an arrival of a high priority patient. The interrupted service is resumed upon the completion of the service of all of the high priority patients in the queue (Bose, 2002).

A notable difference between this discipline and the prior one is that the extant of low priority patients in the queuing system does not affect the waiting time of the high priority patient (Bolch et al., 2006; Chee-Hock & Boon-Hee, 2008). This is because the arrival of this patient interrupts the service of those low priority patients by directly entering the service. Accordingly, to calculate the mean waiting time of the high priority patients $E(W_{q1})$, we need to consider only the existence of the patients of this class in the queuing system and ignore the existence of the low priority patients. Thus,

for class 1, $E(S_0)$ in equation (5) will be reduced to $\frac{P_1}{\mu_1}$ and consequently the mean waiting time for this class under this preemptive discipline will be:

$$E(W_{q1}) = \frac{\frac{P_1}{\mu_1}}{1 - P_1} \quad (8)$$

For class 2, the mean waiting time $E(W_{q2})$ is calculated using the same equation produced for the non-preemptive discipline (i.e., equation 7). This is because an arriving patient of this class will get the service only after the patients of the same class (i.e., class 2) and the other class (i.e., class 1) ahead in the queuing system.

However, the mean sojourn time of class 2 patients (i.e., $E(T_2)$) cannot be computed by adding the mean waiting time of this class (i.e., $E(W_{q2})$) to its mean service time ($\frac{1}{\mu_2}$). This is because the service of a patient of this class may be interrupted several times by the arrivals of class 1 patients. Consequently, the time period from which the patient of class 2 starts the service until completion should be considered (Chee-Hock & Boon-Hee, 2008). This time is given by:

$$\mathbb{T}_2 = \frac{1}{\mu_2} + \frac{1}{\mu_1} \lambda_1 \mathbb{T}_2 = \frac{1}{\mu_2(1 - P_1)}$$

Where $\lambda_1 \mathbb{T}_2$ is the average arrival of patients of class 1 during the time $\mathbb{T}_2$. Adding this time to the mean waiting time of class 2 gives the mean sojourn time of this class as follows.

$$E(T_2) = E(W_{q_2}) + T_2 = E(W_{q_2}) + \frac{1}{\mu_2(1 - P_1)}$$

3.2.4 Estimating the Rest of the Performance Indicators

On the basis of the obtained values of $E(W_{qi})$ under the two disciplines, the other performance indicators of interest can be computed for both high and low priority patients as shown in Table 1.

Table 1. Formulas of the Rest of the Performance Indicators

Performance Measure	Equation
Overall Mean Waiting Time [$E(W_q)$]	$E(W_q) = \sum_{i=1}^2 \frac{\lambda_i}{\lambda} E(W_{qi})$ (9)
Mean Sojourn Time of class i patients [$E(T_i)$]	For the non-preemptive discipline: $E(T_i) = E(W_{qi}) + \frac{1}{\mu_i}$ (10) For the preemptive discipline: $E(T_i) = E(W_{qi}) + \frac{1}{\mu_1}$ (11) $E(T_2) = E(W_{q_2}) + \frac{1}{\mu_2(1 - P_1)}$ (12)
Mean Number of class i patients in the queue [$E(L_{qi})$]	$E(L_{qi}) = \lambda_i E(W_{qi})$ (13)
Total expected queue size [$E(L_q)$]	$E(L_q) = E(L_{q1}) + E(L_{q2})$ $= \lambda_1 E(W_{q1}) + \lambda_2 E(W_{q2})$ (14)
Mean Number of class i patients in the queuing system of a specific object [$E(L_i)$]	$E(L_i) = \lambda_i E(T_i)$ (15)

3.2.5 Generalization of Equations to All Objects and Working Shifts

As aforementioned, the purpose of this analysis is to extract the equations of a set of crucial performance measures for all objects available in a public hospital ($o = 1, \dots, n+m$), where o is the object's number, n is number of subsequent processes, and m is number of services. To achieve this purpose, the obtained equations for the mean waiting time at a specific object's queue under the two priority disciplines (i.e., equations 5, 7, and 8) can be generalized to all objects as follows.

For the non-preemptive discipline:

$$E(W_{qi}^o) = \frac{\sum_{i=1}^2 \frac{P_i^o}{\mu_i^o}}{1 - P_1^o} \quad (16)$$

$$E(W_{q2}^o) = \frac{\sum_{i=1}^2 \frac{P_i^o}{\mu_i^o}}{(1 - P_1^o)(1 - P_1^o - P_2^o)} \quad (17)$$

For the preemptive discipline:

$$E(W_{q1}^o) = \frac{\frac{P_1^o}{\mu_1^o}}{1 - P_1^o} \quad (18)$$

The equation of $E(W_{q2}^o)$ under this discipline is the same as equation (17) for the non-preemptive discipline.

On the basis of these equations of the mean waiting time, the equations of the other performance measures (i.e., those listed in Table 1) can be generalized to all objects in the same manner.

By considering all visited objects by a patient of class i , the total mean waiting time $E(W_{qi}^o)$ and the total mean sojourn time $E(F_i)$ of this patient are obtained using the following equations:

$$E(W_{qi}^o) = \sum_{o=1}^R E(W_{qi}^o) \quad (19)$$

Respecting $E(F_i)$, for the non-preemptive discipline:

$$E(F_i) = \sum_{o=1}^R E(W_{qi}^o) + \frac{1}{\mu_i^o} \quad (20)$$

Where R is the number of the objects that the patient has joined their queuing systems.

For the preemptive discipline:

$$E(F_i) = \sum_{o=1}^R E(W_{q1}^o) + \frac{1}{\mu_1^o} \quad (21)$$

$$E(F_2) = \sum_{o=1}^R E(W_{q2}^o) + \frac{1}{\mu_2^o(1 - P_1^o)} \quad (22)$$

For generalizing the obtained equations to the working shifts system that is currently implemented in so many health units, the present study identifies a working shift as a period of time that is characterized by having its own values for the patients arrival rate and service rate. This characterization is represented by two symbols: $[\lambda_i]_{shift j}$ and $[\mu_i]_{shift j}$, where $i = 1, 2$ (i.e., the class number) and $j = 1, 2, \dots, S$ (i.e., the shift number). Thus, considering this characterization in the equations 16 to 22 as well as those included in Table 1 yields the mean values of the performance indicators for each shift, which will have symbols such as $[E(W_{qi}^o)]_{shift j}$, $[E(W_{qi}^o)]_{shift j}$, and $[E(F_i)]_{shift j}$.

3.3 The Database and User Interface Components

The proposed queuing model can be developed as an object-type model. Objects make it easier to model complex business objects and rules as well as enable fast and efficient development of database applications (Arora, 2005; Kannan, Preethy, & Das, 2019). Accordingly, the database component of the proposed DSS contains the processes and medical services of the developed queuing system as object types. Each of these object types has finite sets of attributes and methods. The attributes contain the data, while the methods specify the operations that can be performed on the data. The application part of the DSS (i.e., containing the code and user interface) interacts with these object types and their methods in order to produce the estimations of the considered performance indicators, and eventually displays these estimations to the end users.

The details of the object types' representation in the database component as well as their methods are as follows. The subsequent processes are represented as a single object called Subsequent-Process-Object-Type. The attributes and methods of this type are shown in Table 2.

Table 2. Specification of the subsequent-process-object-type

Subsequent-Process-Object-Type	
Attributes	Methods
Process-ID	Calculate the values of the performance indicators for each class
Process-Name	
Process-Purpose	
Class1-Average-Arrival-Rate	
Class2-Average-Arrival-Rate	
Class1-Average-Service-Rate	
Class2-Average-Service-Rate	
* The average arrival and service rates are derived attributes from the following object type (i.e., Subsequent-Process-Working-Shift-Rates-Object-Type)	

At the queue of each subsequent process, the arrival and service rates in the individual working shifts are included in an object called Subsequent-Process-Working-Shift-Rates-Object-Type. The parts of this type are shown in Table 3.

Table 3. Specifications of the subsequent-process-working-shift-rates-object-type

Subsequent-Process-Working-Shift-Rates-Object-Type	
Attributes	Methods
Process-ID	Calculate the values of the performance indicators of each working shift for each class
Working-Shift-ID	
Class1-Arrival-Rate	
Class2-Arrival-Rate	
Class1-Service-Rate	
Class2-Service-Rate	

The core service is modeled as a single object called Core-Service-Object-Type. The attributes and methods of this type are shown in Table 4.

Table 4. Specifications of the core-service-object-type

Core-Service-Object-Type	
Attributes	Methods
Working-Shift-ID	_ Calculate the values of the performance indicators for each class (#) _ Add the mean waiting time of class 1 patients to the mean waiting time calculated in (#) above for each patient that returns to the core service after finishing from the subsequent services (*)
Class1-Arrival-Rate	
Class2-Arrival-Rate	
Class1-Service-Rate	
Class2-Service-Rate	
* In practice, it has been noticed that the patients that have completed their requested services are given priority to see the physician for finalizing their diagnoses	

The subsequent medical services are modeled as a single object called Subsequent-Services-Object-Type. The components of this type are detailed in Table 5.

Table 5. Specifications of the subsequent-services-object-type

Subsequent-Services-Object-Type	
Attributes	Methods
Service-ID	_ Find the average arrival rate of each service type for each class (*) _ Find the average service rate of each service type for each class (**) _ Calculate the values of the performance indicators of each service for each class
Service-Name	
Service-Purpose	
Number-of-categories-of-instances	
* It is found for each service by dividing the summation of the average arrival rates of all underlying categories of instances over the number of categories of instances. These average arrival rates are obtained from the categories object type (shown next).	
** The above note (*) is applicable for finding the average service rate	

As aforementioned, the subsequent service can have several categories of instances (such as serology, blood, and chemical categories of investigations for the lab investigations service). The categories of instances for each subsequent service are represented as a single object called Subsequent-Service-Categories-Object-Type. The parts of this type are depicted in Table 6.

The instances of each category of instances (such as HP, TWPC, and malaria test instances of the blood investigations category) are modeled as a single object called Subsequent-Service-Instances-Object-Type. The specifications of this type are shown in Table 7.

The arrival and service rates in the individual working shifts for each instance are represented in an object called Instance-Working-Shift-Rates-Object-Type. The components of this type are shown in the Table 8.

Table 6. Specifications of the subsequent-services-categories-object-type

Subsequent-Service-Categories-Object-Type	
Attributes	Methods
Service-ID	_ Find the average arrival rate of each category of instances for each class (*) _ Find the average service rate of each category of instances for each class (**) _ Calculate the values of the performance indicators of each category of instances for each class
Category-ID	
Category -Name	
Number-of-instances	
* It is found by dividing the summation of the arrival rates of all underlying instances over the number of instances. These arrival rates are obtained from the instances object type (shown next).	
** The above note (*) is applicable for finding the average service rate	

Table 7. Specifications of the subsequent-service-instances-object-type

Subsequent-Service-Instances-Object-Type	
Attributes	Methods
Category-ID	_ Calculate the values of the performance indicators of each instance for each class
Instance-ID	
Instance-Name	
Class1-Average-Arrival-Rate	
Class2-Average-Arrival-Rate	
Class1-Average-Service-Rate	
Class2-Average-Service-Rate	
* The average arrival and service rates are derived attributes from the following object (i.e., Instance-Working-Shift-Rates-Object-Type)	

Table 8. Specifications of the instance-working-shift-rates-object-type

Instance-Working-Shift-Rates-Object-Type	
Attributes	Methods
Instance-ID	Calculate the values of the performance indicators of each working shift for each class
Working-Shift-ID	
Class1-Arrival-Rate	
Class2-Arrival-Rate	
Class1-Service-Rate	
Class2-Service-Rate	

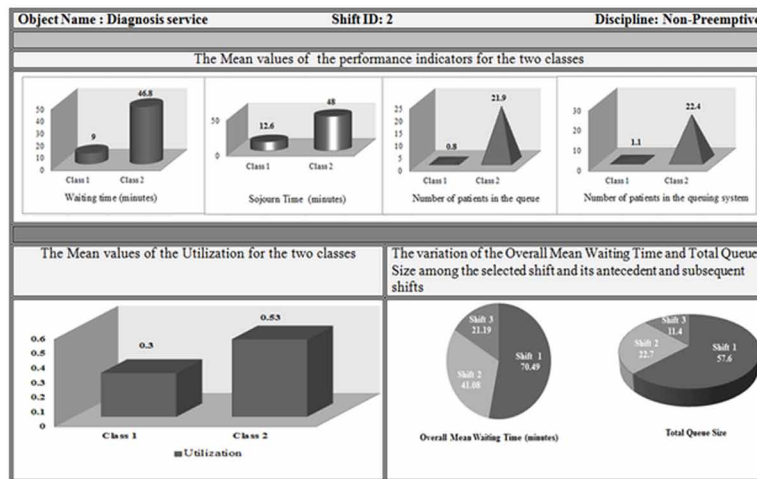
The aforesaid description reveals that for each subsequent service with instances, the performance indicators can be calculated on the service level as well as its underlying levels (i.e., the category and instance levels). Moreover, they can be calculated on the level of the individual working shifts.

Each of the object types that contain the arrival and service rates (i.e., the model parameters) is associated with an analogue object type called archive-object-type. The purpose of this object type is to track the updates performed on the values of these parameters through storing the temporal details of each update. It contains timestamp attributes to store the date and time when the update occurred as well as the same attributes of its associated original object type. Thus, this object type enables tracing the variations in the mean values of the performance indicators on the basis of the updates conducted on the values of their associated parameters. For the objects that do not include these parameters (e.g., Subsequent-Services-Object-Type), the archive-object-type contains only the attributes of the original object type.

Both the initial and final processes were not included in the definitions of the objects. This is because these processes do not have queues (i.e., they are not a part of the queuing system).

The above-mentioned database objects and their associated attributes and operations (i.e., methods), as well as the derived equations (in section 4) for performing the object operations (i.e., estimations of the performance indicators) represent the main elements that should be considered by the DSS application for calculating the mean values of the performance indicators. Displaying these values and manipulating their parameters (e.g., arrival and service rates) are performed through the user interface component, which is specified as a graphical interface containing two elements: forms for entering the values of the object attributes and dashboards consisting of graphical reports for showing the predictions of the performance indicators and their variations. A sample layout of one of these dashboards is depicted in Figure 2.

Figure 2. A sample dashboard layout for displaying the values of indicators for specific object and working shift



Since the proposed DSS is not a data-intensive system, the user interface component will be used mainly for displaying the system outputs (i.e., the predictions of the indicators) and representing them diagrammatically. The major inputs of the system, which are the arrival and service rates per a time unit (e.g., the working hours in the morning shift) at each queue, can be derived directly (i.e., by the system) from the ordinary patients' registration systems. This is instead of entering their values

manually (i.e., handwritten input) through the user interface. But, this implies that these ordinary systems should capture the details of the patients flow through the extant processes and medical services in the health unit (such as the registration time at the desk of each service and the service time).

4. CONCLUSION

This article has developed specifications for a model-driven DSS that provides estimates for a set of crucial healthcare performance indicators in the public hospitals. These estimates were firstly derived on the level of the individual processes and services that the patients encounter during their flow in such hospitals. Then, they were accumulated to produce a collective prediction on the thorough level of all processes and services that form the entire patients flow system. The developed specifications have considered several requirements for ensuring an effective DSS, including tracking the performance indicator's values on the level of the entire hospital system (i.e., covering all healthcare facilities associated with the flow of the admitted patients) as well as the level of the individual facilities, paying attention to the dynamic change of the values of the parameters used to predict these indicators, and considering the heterogeneity of the patients (i.e., different classes) on assessing these indicators.

According to these requirements, the components of the proposed system were specified. The model component represents the entire patients flow system as a set of objects that act as service centers for handling the requests of two common patient classes. The database component contains the attributes of these objects and the details of their associated operations. Some of these attributes are included to track the variations of the values of the object's parameters (e.g., arrival rates) over time. The user interface component is directed to perform the input and output activities of the system.

The present study differs from the previous studies in many aspects. With respect to the model component, the present study has focused on developing an extensive model for representing the different processes and services carried out in the hospital. This is unlike the models of the related studies, which represent mainly individual healthcare departments. Also, the present study has considered deriving equations for a considerable number of the performance indicators relating to the patients flow. This is in contrast to a large number of the previous studies that considered only a few of these indicators. Many of the considered indicators in this study are among those identified in the prior studies (e.g., Vass & Szabo, 2015) as should be considered by the healthcare managers when assessing the service systems. Regarding the database component, to our best knowledge, an object oriented database was not specified in the prior studies. Moreover, this component was not given a considerable attention in the literature of the healthcare DSSs. Another aspect differentiating this study is that it considers both the preemptive and non-preemptive disciplines in predicting the performance indicators, while most of the prior studies considered only one discipline.

In the light of these distinctions and the provided specifications of the proposed DSS, the major contributions of this study can be more elucidated as follows. It is one of the fewer studies that focus on proposing a DSS for supporting the decision making process pertaining to the patients flow in the public hospitals. The proposed DSS supports this process by tracking and estimating the key healthcare performance indicators relating to such flow in a manner that improves the prediction accuracy. This helps the decision makers to identify best alternatives and develop effective plans for managing the flow and service needs of the various patient classes. The study specifies a highly representative queuing model for the patients flow among the entire processes and services encountered by the patients, thereby indicating that it provides a more realistic representation for the patients flow than the representations provided in the prior studies. In addition to the public hospitals, the generic nature of modeling the patient flow system as a set of processes and services increases the applicability of the model for describing the patients flow in the other types of the health units (e.g., health centers). To the best of our knowledge, there is no a comprehensive queuing model for modeling the various processes and services available in the health units and assessing their performance collectively. Another uniqueness of this study is the implementation of the object-oriented paradigm for developing the specifications

of the database component of the DSS. From the system output perspective, the estimates provided by the system act as a foundation for both evaluating the key performance dimensions and developing policies for managing the patients flow in these health units.

The limitations of this study include the following. The specifications of the actual implementation of the proposed DSS have not been included. This is because this prolonged phase is not finalized yet and it has been forwarded to the future work. The study has focused on only a commonly applied classification of patients in the public hospitals, which consists of two generic categories of patients (i.e., high and low priority patients), but in some health units, there is a possibility for classifying the patients into more categories. Also, the possibility that some health units may have specific performance indicators has not been considered in this study, since it has focused on estimating the commonly identified indicators for such units. Accordingly, these possibilities should be considered in the future studies.

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