International Journal of Applied Metaheuristic Computing
Volume 13 • Issue 1

An Efficient Constructive Heuristic for the Cutting Stock Problem Applied in a Foam Mattress Industry

Mariem Baazaoui, Imam Abdulrahman Bin Faisal University, Saudi Arabia*
Souhir Elleuch, Qassim University, Saudi Arabia
Hichem Kamoun, FSEG Sfax, Tunisia

ABSTRACT

The cutting and packing problem belongs to the combinatorial optimization problems; it covers a wide range of practical cases in industries. The present paper investigates a new real-world problem that needs to be solved through daily operations of cutting foam blocks in an industrial company. The problem is considered as one of the non-classical problems in the cutting and packing area. It represents a variant of the three-dimensional cutting stock problem. The originality of the studied problem is indicated by a specific set of constraints related to the production process and the cutting ways. A constructive heuristic was developed to provide cutting patterns in advance. All possible combinations established from the ways of cutting right rectangular prisms from foam blocks define the cutting patterns. This heuristic performs well and shows promising results in reasonable computational times to provide efficient cutting plans in order to reduce the total material loss.

KEYWORDS

Constructive Heuristic, Cutting Patterns, Guillotine Cut, Optimization, Three-Dimensional Cutting Stock Problem

1. INTRODUCTION

In general, companies have a strong desire to be more successful than others. They usually try to maximize their productivity while minimize their costs. The world is characterized by a rapid technological advancement; so that, companies invest in sophisticated machines and obtain new technologies. To maintain his competitivity, the company can focus and rely on some optimization techniques as tools of decision-making.

A regional industrial company in Tunisia fabricates foam mattresses and always complains about the high level of leftover resulting from cutting patterns choosing. These patterns are not predetermined through scientific approach but are developed manually based on the judgment and the experience of the employee. The principle goal of this study paper is to find a solution of the presented foam company and provide the final ordered items (mattress in foam) to customers.

DOI: 10.4018/IJAMC.298317
*Corresponding Author

This article published as an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/) which permits unrestricted use, distribution, and production in any medium, provided the author of the original work and original publication source are properly credited.

We propose an optimization plan leads to generate efficient cutting patterns that will be able to minimize the current level of waste and therefore increase the satisfaction level of the owners. The final items (mattress in foam) made by guillotine cuts from bins (foam blocks), these bins in turn are made by tunnel. The bins of the same tunnel have the same weight and the same height but different lengths, so the objective is to minimize the total number of used bins (foam blocks) and therefore minimize the total length of tunnel (see Fig.1).

The treated problem represents one of the non-classical problems of the three dimensional (C&P) with additional constraints. Some constraints are solely determined by the industrial applications in (C&P) problem. This type of problem is important since it can help solve real industrial problems and minimize their costs. However, solving such practical problems is rather difficult because of the additional constraints encountered in the industry, such as the guillotine constraint presented in this paper. The considered problem presents specifications in the cutting process. Actually, the extraction of the final items (mattresses in foam) is made only by cuts from edge to edge. Indeed, literature defined this way of cut as guillotine cuts. These cuts are made by levels. The level is the number of phases necessary for the extraction of all the items. Fig.2. (a), (b), and (c).

Figure 1. Getting blocks from a tunnel

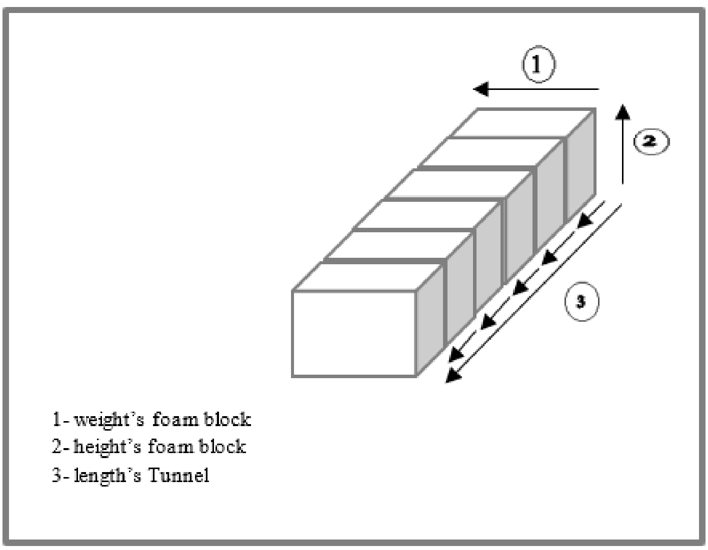
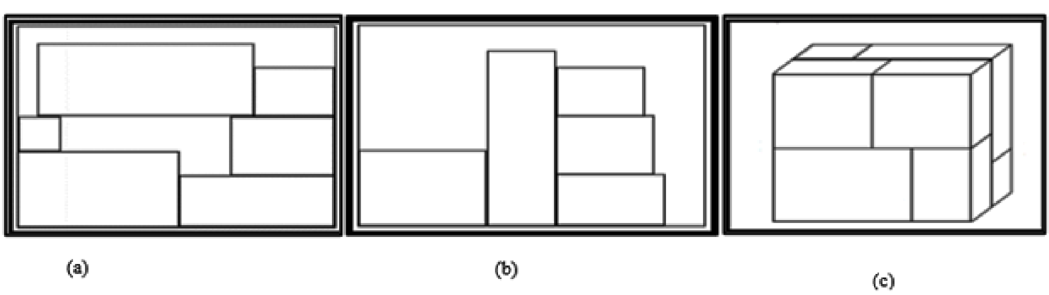


Figure 2. Example of cutting patterns. (a) Non-guillotine cuts. (b) 3-levels 2D guillotines cut. (c) 3-levels 3D guillotines cut.



Based on the last typology offered by Wäscher et al., (2007), the treated problem can be conceived as a variant of the three dimensional cutting stock problems when dimensions are not predetermined in advance (variable sizes). It is a strongly NP-hard problem, which cannot be easily solved in real cases. It has been less studied until very recently, even though it has a wide range of real applications.

Real problems can arise as a combination of several problems, including a range of specific technical constraints. This accentuated the difficulty of solving these problems. As such, the studied problem in this paper is a novel practical case belonging to the non-classical problems. It is an industrial foam-cutting problem that presents particular constraints linked, on the one hand, to the blocks production process using big Tunnels. On the other hand linked to the cutting ways, which follow the guillotine constraint. Solving such a type of problem in practical cases was and is still a big challenge.

Some algorithms are proposed in the literature for resolving these problems. In Bang-Jensen and Larsen (2012), the variable size bin packing problem was treated and the authors reveal the efficiency of their proposed algorithms through real-life instances. Zhu *et al.* (2012) applied a prototype column generation strategy for the multiple container-loading problem. Crainic et al. (2011) presented GASP greedy adaptive search approach: metaheuristics able to efficiently address two and three-dimensional multi-container packing problems. Paquay et al. (2018) described a tailored constructive heuristic for real-life instances of three-dimensional (MBSBPP) in Aircraft Company. Sumetthapiwat et al. (2020) deal with multi stock sizes and fixed-size usable leftovers for the two-stage two-dimensional cutting stock problem.

The aim of the present paper is to find a good possible solution in acceptable computation time to a real world problem, which considers additional cutting constraint. This cut must satisfy certain geometric constraints and aims to reduce losses and costs.

The rest of the paper is organized as follows. In section 2, a literature review on connected problems is presented. In section 3, a description of the studied industrial problem of the local foam company is provided and reviews of some results of previous study in (Baazaoui et al. (2017)) are presented. In section 4, a novel constructive heuristic named levels algorithm (LA) is proposed. It is composed of two complementary sub-algorithms, which is appropriated to the specificities of the real problem. In section 5, outlines of the computational results and some comparisons are made.

2. LITERATURE REVIEW

The cutting and packing (C&P) problem is one of the well studied combinatorial optimization problems and it was proved as a NP-hard problem in the strong sense. The literature is rich with theoretical and practical studies for the classical problems of (C&P) where the bins have fixed dimensions. Kantorovich (1960) was the first who initiated the (C&P) problem. One year after, the famous method "column generation" for the (C&P) problem for the one dimentional problem was introduced in Gilmore and Gomory (1961). In 1965, Gilmore and Gomory extended their proposed method for the two dimensional problem. Subsequently, several articles have been published about various versions and variants of the (C&P) classical problem; Beasley (1985), Coffman et al. (1980). The notion of arc flux for modelling the (C&P) problem was initially presented by Carvalho (1999). Fekete and Schepers (1997) were the first ones who used the graph theory technique to introduce a new modelling approach for the orthogonal bin packing problems. They continued their studies in 2001, 2004, and 2006 and they integrated other notions such as scheduling. Lodi et al. (2002), presented a brief state of the art concerning the modelization and the resolution of two dimensional problems.

Studies about the classical problem have been conducted up to the present time, namely (Martello et al., (2000), Fekete et al., (2007), Crainic et al., (2008), Parreño et al., (2008), Crainic et al., (2009), Bortfeldt and Wascher (2012), A.deQueiroz et al., (2012) and Hifi et al., (2014)), to mention a few.

Bortfeldt and Wascher (2012) made a State of the art for different studies in the classical case, whereas few studies are investigated for the opposite case.

In fact, the literature presents some current papers regarding the non-classical problem of (C&P), where the dimensions are not predefined; the multiple-bin-size bin packing problem (MBSBPP), the multiple container-loading problem (MCLP) and the multiple stock sizes cutting stock problem (MSSCSP) are but a few examples. For a description of these problems, the reader can refer to the last typology of Wäscher et al., (2007). This typology represents a new coherent system of proposed names for different categories of (C&P) problems.

Treating and solving these practical problems are far easier in the classical problems of (C&P) than in the non-classical ones. In this paper Chen et al., (1995), the authors stated an analytical model; they proposed a MILP model with consideration of multiple object sizes, multiple containers, object orientations, and the non-overlapping of items in the container.

In the literature, there are a few studies for the (MBSBPP), as well as for one and two-dimensional cases. In Alvarez-Valdes et al., (2015), lower bounds were presented in the case of the three-dimensional (MBSBPP). The optimization problem defined in Saraiva et al., (2015) can be considered as a lexicographically optimization including more than one objective: minimization of the total volume of used containers, minimization of the stacks volume inside each container, minimization of the lengths of the container patterns and minimization of the container compactness. The authors proposed an efficient constructive algorithm named layer-building (CLB) algorithm to solve the problem submitted as a challenge of Renault/ESICUP. In Paquay et al., (2018), MIP-based constructive heuristics was proposed for solving a real world problem of three dimensional (MBSBPP) proposed in air transportation.

Real problems can arise as a combination of several problems, including a range of specific technical constraints. This accentuated the difficulty of solving these problems. In Christofoletti et al., (2020), a real world problem in a small Brazilian mattress industry has been studied and solved (via optimization package) to reduce charges and leftover of the company. In their paper, Christofoletti et al.,(2020), the authors represented and integrated problem of cutting stock problem and lot-sizing problem.

Overall, the practical problems have specifications related to the production process, packaging, and sales. The studied paper presents a specification in the cutting process; more specifically, the extraction of the final items (mattresses in foam) are made only by cuts from edge to edge. All cuts must be of guillotine-type. During the last decade, some practical problems were devoted to industrial case studies, namely for the three-dimensional case, Silveira et al., (2013) who proposed a new Ant Colony Optimization (ACO) algorithm. This algorithm was proposed for a real world problem in a steel industry where the guillotine constraint is imposed by the cutting machine. Hongteng et al., (2016) proposed a three-stage heuristic algorithm for the three dimensional irregular packing problem and they tested it for real-life cases. Ha et al., (2017) developed a novel online packing algorithm, which is applicable in several real world context as the example of automated cargo loading in warehouses.

In the case of two-dimensional problem with multiple stock sizes, Park et al., (2013) illustrated an effective heuristic for the two-stage guillotine cut while permitting the rotation of items. In Martin et al., (2020) and Martin et al., (2021), models and constraint programming based algorithm are proposed in the case of two-dimensional guillotine cutting problems, which can be presented in a Real world application of glass industry.

3. PROBLEM DESCRIPTION AND PREVIOUS RESULTS

3.1 Problem Description

A Tunisian local foam company offers different sizes of mattresses in foam. Based on the information collected after an on-site visit, the production of foam mattresses passes through two phases. These two phases have been described in Baazaoui et al. (2017). The authors defined: the production of the bins (foam blocks) in the sizing phase and the final extraction of the right rectangular prism (mattress in foam) in the cutting phase.

The production phase: foam blocks are produced by tunnel. Each block has three parameters, namely: the first parameter is the width of the block which is exactely same as of the tunnel; since all the blocks of the same tunnel have the same width. The production must be a continuous process, so to change its width, we have to stop the process and re-enter a new value of width. This value varies between 1.6 meters and 2.10 meters.

The second parameter is the height of the blocks. The tunnel has no height but it acts on the height of the block through its speed. This height also depends on the chemical formula used during the process. The blocks composed the same tunnel should have the same height. Overall, the height varies from 1.4m to 1.9m.

The third parameter is the length of the block, this length is not predetermined in advance before starting the process as the height and the width, but it is to decide within the process. We choose the appropriate lengths; that will be cut into sub-blocks then into matresses in the cutting phase; then, we cut these blocks according to these lengths. The sum of the block's lengths is the tunnel length. The length of the tunnel is never a constraint of production: in fact, the production process is done continuously to produce a set of blocks according to the available storage space. And for economic reasons, this production can start only for a minimum number of blocks. Fig.3.

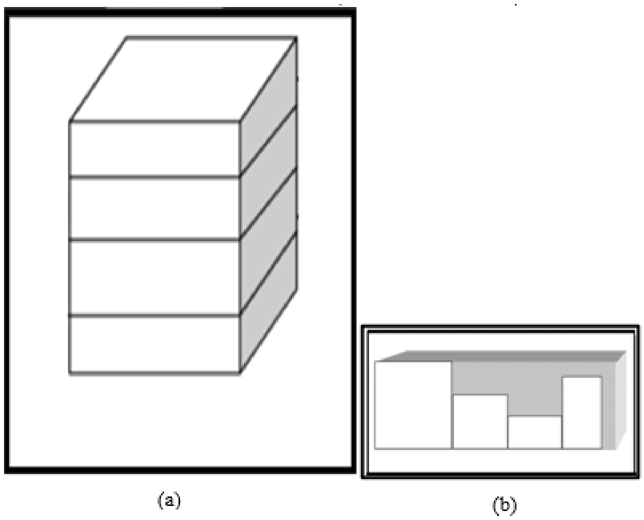
The cutting phase: Each produced block has to be cut into several sub-blocks. Fig.4. (a). Additionally, these sub-blocks must be cut into a right rectangular prism to get the final mattresses in foam Fig.4. (b). So, the aim of all cuts is to minimize the trim loss in each sub-block that composes the blocks that, in turn, compose the tunnels.

During all cuts, the following requirements must be satisfied:

- Only guillotine cuts are permitted; a succession of cuts go from edge to edge constitute the cutting configuration.
- There are no limits on the number of stages of the guillotine cuts.
- No overlapping between items (foam mattresses) which must fulfill the dimension of the foam blocks.

Figure 3. The production of blocks by tunnel

Figure 4. Cutting a block and sub-block. (a) Cut a foam block in sub-blocks. (b) An example of 4 mattresses obtained from cutting one sub-block.



The studied problem can be considered as an integrated problem of two optimization problems. The 3D strip packing problem (SPP) and the 3D Cutting Stock Problem with Multiple Stock Sizes. The 3D-SPP can be defined from the first phase; in fact, the width and the height are entered in advance before starting the production process while the length is to decide within the process. The aim is to minimize the used material so that to minimize the total length used. Wascher et al., (2007) defined the Strip Packing Problem similar to the packing problem. However, instead of packing the items in a set of bins of finite dimensions, the authors defined a single object having only one infinite dimension. For example, in the 3D case, this is a unique strip with determined height and width but a very high length assumed infinite. Therefore, it is called strip-packing problem. Therefore, the goal is to find feasible patterns consisting of all items that minimize the occupied area in the strip.

The 3D-CSPMSS can be defined from the second phase, so the problem is to know how to cut a set of small objects (foam mattresses), according to a backlog to meet, from large objects (foam blocks) available in the stock (produced from a tunnel). Therefore, cutting problems present two sub-problems: first, determine the objects of the stock to use to satisfy the order, secondly, find the patterns that minimize the waste.

Based on the last typology stated by Wascher et al., (2007), the CSPMSS is determined as items are grouped in classes then, each class with a specified order has to be extracted from different types of stock sheets. So that the objective consists in the minimization of the total number of used sheets.

In this problem, the optimization is stated in two steps. A first decision must be taken to choose the length of blocks in the production phase. A second decision must be taken to choose the cutting pattern corresponding to the minimum waste on the cutting phase in order to cut the foam mattresses from the foam blocks. It should be noted that these decisions in the two phases are interdependent. In fact, the choice of the length of the blocks is a consequence of the cutting patterns in the second phase (mattresses) thus, the optimal patterns depend on the dimensions of the foam blocks.

3.2 Previous Work

This sub-section represents and defines an outline of the different variables and the mathematical formulation elaborated by Baazaoui et al.,(2017). In fact, the constructive heuristic proposed in this paper is based on the extension of the method used for the formulation of the upper bound proposed by Baazaoui et al. (2017). The feasible solution of the MILP able to provide an upper bound for the problem, but the attained results are only presented for properly small size instances compared with the company demand. Baazaoui et al. (2014) also proposed a lower bound. They used these two bounds to restrict the optimum value between two values and get it for some small instances.

The mathematical formulation gives a feasible solution, which take in to consideration all constraints, including those used for positioning and non-overlapping items. The feasible solution provided by this formulation constitutes a special case of placements.

Using these notations: N is a set of n pieces (items) ∀ iÎN, M is a set of m tunnels ∀ kÎM, N_k is a set of maximal number of n_k blocks in a tunnel k, ∀ jÎN_k. The length of the shorter piece i defined as L_{min} while the length of the longest piece i defined as l_{max}. B_{max} and B_{min} correspond to the highest and the lowest number of sub-block j for each tunnel k. Each piece has three dimension l_i is the length of piece i, w_i is the width of piece i and h_i is the height of piece i. Each sub-block j of tunnel k has W_{jk} as width and H_{jk} as height.

Decision Variables:

$$S_k = \begin{cases} 1 & \text{if the tunnel } k \text{ is produced} \\ 0 & Otherwise \end{cases}$$

$$Y_{jk} = \begin{cases} 1 & \text{if the sub} - \text{block } j \text{ is assigned to tunnel } k\left(is\,used\right) \\ 0 & Otherwise \end{cases}$$

$$X_{ijk} = \begin{cases} 1 & \text{if the piece } i \text{ is assigned to sub} - \text{block } j \text{ of tunnel } k \\ 0 & Otherwise \end{cases}$$

- L_{jk} ϵN, is the length of sub-block j of tunnel k.
- $Z_{jk} = L_{jk}\, Y_{jk}$, is a variable used for the linearization of the objective function.

As established in Baazaoui et al.,(2017), the following formulation is donated to find an upper bound of the predefined industrial problem:

$$\text{Minimize Z} = \sum_{k=1}^{m}\sum_{j=1}^{n_k} Z_{jk} \tag{1}$$

s.t.

$$\sum_{j=1}^{n_k}\sum_{k=1}^{m} X_{ijk} = 1 \quad \forall i \in N \tag{2}$$

$$\sum_{i=1}^{n} X_{ijk} \le nY_{jk} \quad \forall k \in M, j \in N_k \tag{3}$$

$$X_{ijk} \leq S_k \quad \forall i \in N, k \in M, j \in N_k \tag{4}$$

$$S_{k+1} \leq S_k \quad \forall k \in M \tag{5}$$

$$Y_{j+1,k} \leq Y_{jk} \quad \forall k \in M,\ j \in N_k \tag{6}$$

$$B_{\min} * S_k \leq \sum_{j=1}^{n_k} Y_{jk} \leq B_{\max} * S_k \quad \forall k \in M \tag{7}$$

$$l_{min} \leq L_{j\ k} \leq l_{max} \quad \forall k \in M, j \in N_k \tag{8}$$

$$\sum_{i=1}^{n} l_i w_i h_i X_{ijk} \leq H_{j\ k} W_{j\ k} L_{j\ k} \quad \forall k \in M, j \in N_k \tag{9}$$

$$l_i X_{ijk} \leq L_{jk} \quad \forall i \in N, k \in M, j \in N_k \tag{10}$$

$$\sum_{i=1}^{n} h_i X_{ijk} \leq H_k \quad \forall k \in M, j \in N_k \tag{11}$$

or:

$$\sum_{i=1}^{n} w_i X_{ijk} \leq W_k \quad \forall k \in M, j \in N_k \tag{12}$$

$$Z_{jk} \geq L_{jk} - B\left(1 - Y_{jk}\right) \quad \forall k \in M, j \in N_k \tag{13}$$

$$Z_{jk} \leq B Y_{jk} \quad \forall k \in M, j \in N_k \tag{14}$$

$$Z_{jk} \leq L_{jk} \quad \forall k \in M, j \in N_k \tag{15}$$

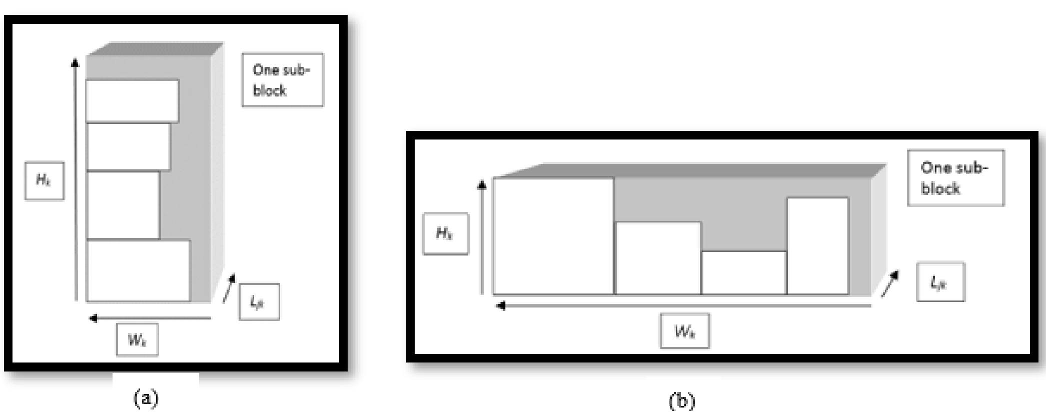
$$L_{jk} \quad \forall k \in M, j \in N_k \tag{16}$$

$$S_k, Y_{jk}, X_{ijk} \in \left\{0,1\right\} \quad \forall i \in N, k \in M, j \in N_k \tag{17}$$

(1) Represents to the objective function, which aims to minimize the total volume of produced tunnels. (2) Are the constraints ensuring that each item (piece) must be cut. (3) Are the constraints ensuring that no item is assigned to an unused sub-block. (4) Are the constraints ensuring that an item can be cut only if the tunnel is produced. (5) Are the constraints ensuring that the tunnel $k+1$ is produced only if the tunnel k is produced. (6) Are the constraints ensure that sub-block $j+1$ is created only if the sub-block j is created. (7) Are the constraints ensuring that the number of sub-block in each tunnel must be between two bound values B_{min} and B_{max}. (8) Are the constraints ensuring that no sub-block has a length less than l_{min} and no sub-block length is bigger than $l_{max;}$ (l_{min} is the length of the shorter piece; l_{max} is the length of the longest pieces). (9) Are the constraints stating that the sum of the volume of all pieces assigned to a sub-block must be lower than the volume of this sub-block. (10) Are the constraints ensuring that the length of the piece should not exceed that of the sub-block. (11) Are the constraint ensuring that if we have a maximum of one piece along the width and all the pieces in this sub-block are superimposed (the one upon the other), the summation of heights of the pieces should not exceed that of the sub-block. As shown in Fig.5. (a). (12) Are the constraint ensuring that if we have a maximum of one piece along the height and all the pieces in this sub-block are contiguous (one side by side). As shown in Fig.5. (b), the sum of widths of the pieces should not exceed that of the sub-block. (13) to (15) Are constraints used for the linearization of the quadratic term: $Z_{j\,k} = L_{jk}\, Y_{jk}$. (8) are the constraints used in order to respect the cut by sub-block and each sub-block corresponds to a unique layer of superimposed items.

This linear program with an objective function aims to the minimization of the total material loss and constraints meet all specifications of the practical problem. The presented modelization

Figure 5. Possible extreme placement cases; Baazaoui et al. (2017) (a) The pieces are one upon the other. (b) The pieces are one beside the other.



examines the two possible extreme cases stated by constraints (11) and (12) as shown in (Fig.5). The upper bound also respects the cutting made by sub-blocks (as a specification of the practical problem) where each sub-block coincides with a unique layer of the superimposed items.

This formulation provides results for fifty pieces with a run time fewer than two minutes. The described bound provides outputs in a reasonable runtime in a small instances case.

In Baazaoui et al. (2017), the comparison between results from the lower and the upper bound for the same instances shows equal values of length but different run times. This experiment has shown that the optimal value is obtained for instances composed of up to 10 pieces regardless of their types.

The effectiveness of this bound still limited in the case of larger instances, which exceeding fifty items. The purpose of the next section is to propose a constructive heuristic inspired from the upper bound in order to extend the resolution for larger instances and meet the demands of the company for different periods.

4. CONSTRUCTIVE LEVELS ALGORITHM

The representation of the industrial problem with its cutting specificity have inspired us to propose the method of levels placement. As described before, to meet the nature of cutting process, the cuts follow the guillotine constraint. In addition, the cut is performed using some parallel horizontal blades to obtain the sub-blocks and additional blades to obtain the final ordered items. In fact, all cuts should be guillotine.

The principle contribution in this paper manifested in the presentation of a new efficient and appropriate constructive heuristic based on two principal algorithms to find a good pattern that takes into account all the specifications of the real problem. The idea is to fragment the initial three-dimensional guillotine-cutting problem into one-dimensional guillotine problem and two-dimensional guillotine problem.

The literature presents some heuristics based on levels. These heuristics, like wall building and layer building, are known as a construction heuristic. They involve arranging the same type of boxes in rows or columns and placing them in the empty spaces in the bins. For all possible patterns (placement positions), a list of empty spaces is established; after each placement, new spaces are generated. In fact, the packing of boxes is accompanied by a sorting criterion in order to define an order for different types. In the literature, in 1980, George and Robinson were the first to present the example of wall building placement; this heuristic is based on a predetermined procedure to select the boxes. Zhao et al. (2014) presented a general review for the heuristics that derives from that presented by George and Robinson (1980).

In Correcher et al., (2017), GRASP algorithm was proposed, including constructive approaches to obtain solutions meet all the constraints for a real world problem in the car manufacturing industry. In this approach, firstly, pieces are loaded into layers, then these layers are loaded into stacks and finally stacks are put inside containers.

The proposed constructive heuristic of placement starts with a clustering phase, where all similar pieces are grouped into the same cluster. Pieces in the same cluster have similar lengths, which is useful to build the sub-blocks. Fig. 6. (a).

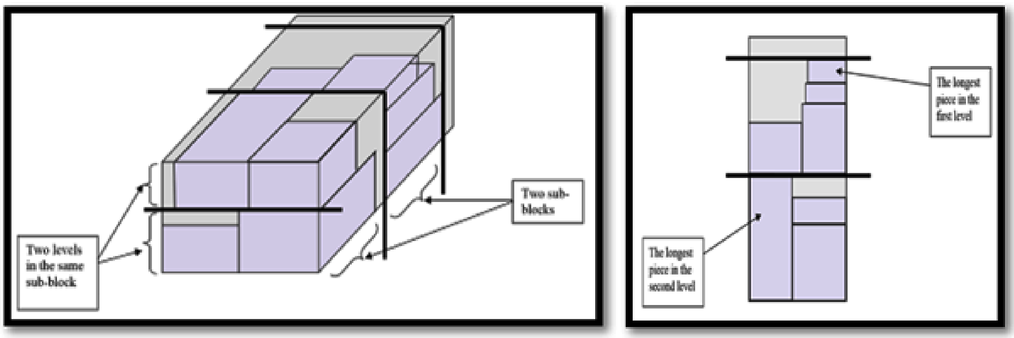
We consider a case of a single tunnel with parameters (H;W); H can take values from 1.4 meters to 1.9 meters and W can take values from1.6 meters to 2.1 meters.

Taking into account the guillotine constraint (in our case: the cut at each step from edge to edge) and the cutting of sub-blocks, we propose to build these sub-blocks.

In fact, the length of the sub-block corresponds to the longest piece placed in its levels. Fig.6. (b). These levels are constructed according to the method described hereafter.

The Fig.6. (a) represents an example of placement (cutting pattern). In fact, it is composed of two sub-blocks; the first sub-block is composed of two levels and the second is composed of three

Figure 6. Building sub-blocks. (a) Example of the composition of the sub-blocks. (b)The top view of the two sub-blocks.



levels. The length of each sub-block corresponds to the longest piece (foam mattress) placed in its levels. The Fig.6. (b) shows a top view of the Fig.6.(a). It clarifies the composition of sub-blocks.

After clustering, we place the pieces according to these two algorithms (algorithm 1 and algorithm 2). The first is a general algorithm that describes the choice of (H_j; W_j) with minimum trim loss, and the building of different blocks constituting the tunnel. The second algorithm represents a description of the level placement and the composition of sub-blocks.

Algorithm 1. The general algorithm

```
Input: pieces i=1,.., N of clusters k=1,.., K
         R [l]: residue in each level l (l=1,.., L)
Output: (H_j; W_j) with minimum Total Residue
   For each (H_j ; W_j) do
                        Total Residue ¬ 0
        For each level l do
                      Residue ¬ R [1]
                      Sub-block¬ 1
             For each pieces i of cluster k do
                         Repeat (Function of level placement)
                               Residue ¬ Residue + R [l]
          Until (h_ikl (of the last placed piece) + sum of the
          height of all horizontal levels)>H_j ) or (w_ikl
          (of the last placed piece) + sum of the width of all
          vertical levels)>W_j)
                        Long-Sub-block ¬ the longest piece in all
                        these levels
                        Sub-block¬ Sub-block+1
                        Gather the Sub-blocks to form Blocks with
                        length between [L_MIN; L_MAX]
           End for
        End for
   End for
   Total Residue ¬ Residue
   Compare total residue of all (H_j; W_j)
```

Algorithm 2. The Function of level placement

Data:
(x_i; z_i): the coordinates of the left lower point of the last placed piece.
CP1: (x_i + w_i; z_i): the corner point according to the width
CP2: (x_i; z_i + h_i): the corner point according to the height
RH: the residue according to the horizontal level placement
RV: the residue according to the vertical level placement
list1*h*: list of pieces ordered by decreasing *h*
list2*w*: list of pieces ordered by decreasing *w*.
Input: (H_j; W_j)
Pieces *i*=1,.., *N* of clusters *k*=1,.., *K*
Output: corresponding level (vertical or horizontal) placement
For each piece *i* of cluster *k* **do**
Place the piece from list1*h*
Update (x_i, z_i)
Compute CP1: (x_i + w_i; z_i) and CP2: (x_i; z_i + h_i)
If (more than one candidate piece with same height)
we choose the piece with the longest width
End if
Compute {*RH*= *W* * *CP2* – total area of all pieces in the direction of CP1}
Place the piece from list2*w*
If (more than one candidate piece with same width)
we choose the piece with the longest height
End if
Compute {*RV*= *H* * CP1 – total area of all pieces in the direction of CP2}
If (*RV* ³ *RH)*
Return horizontal placement
Else Return vertical placement
End if
End for

During the placement, first, for the horizontal placement, we use the pieces from the list1*h* and in the case where we have more than one candidate piece (with the same height), we take the piece with the longest width. However, for the vertical placement, we use the pieces from the list2*w*, and in the case where we have more than one candidate piece (with the same width), we take the piece with the longest height.

4.1 Illustrative Example

A theoretical example including seven pieces is presented here to clarify the proposed approach of placement. Suppose that: *H*=10 and *W*=10.

The first list 'list1*h*' includes pieces ordered by non-increasing order of their height h_i piece (*h*, *w*): (7,2) (6,5), (4,5) (4,4) (3,2) (3,1) (2,2).

The second list 'list2*w*' includes pieces ordered by non-increasing order of their width w_i piece (*h*, *w*): (6,5) (4,5) (4,4) (3,2) (7,2) (2,2) (3,1).

In the first iteration of the algorithm, we place the piece (7, 2) at the first corner and we eliminate this piece from the two lists. We have two sets of candidate pieces: or horizontally from list1*h* in EP1 or vertically from list2*w* in EP2. According to the first list ordered by decreasing h_i we can place in order if possible the pieces (place from the remaining pieces in the entire list but essentially that the selected pieces have a lower or equal height than of the first piece placed in the layer. In the case when we have more than one piece with the same height, we select from the second list 'list1*hw*'). Therefore, we can place (6, 5), (3, 2), (3,1) but, we can't place (4,5) and (4,4) in EP1. Now we can calculate the residue *R1*= W* EP2 – (area of all placed pieces) = 7*10- (14+30+6+3) = 17. Then we get the horizontal placement in Fig.7. (b).

According to the second list ordered by decreasing w_i we can place in order (with w_i lower or equal than 2. In this case, we have three candidates pieces with w_i=2) according to the second list 'list2*wh*' we choose (3, 2) in EP2 so, after this placement all *H* is occupied and we get now the second possible layer with vertical cut. The residue *R2*= H* EP1 – (area of all placed pieces) = 10*2- (14+6) = 0. So we get the vertical placement in Fig.7. (c).

By comparing *R1* and *R2*, the first cut should be vertical, so that the first layer is vertical and we cut the two pieces ((7, 2) and (3, 2)) according to Fig.7. (c).

For the second step, we have these two new lists. Updated of 'list1*h*': (7,2) (6,5), (4,5) (4,4) (3,2) (3,1) (2,2).

Updated of 'ist2*w*': (6,5) (4,5) (4,4) (3,2) (7,2) (2,2) (3,1).

Moreover, *H*=10, *W*=8. In the second iteration of the algorithm, we place the piece (6, 5) at the first corner and we eliminate this piece from the two updated lists. We have two sets of candidate pieces: either horizontally in the new EP1 or vertically in the new EP2.

According to the first updated list ordered by decreasing h_i we can place (3, 1) and (2, 2) but, we can't place (4, 5) and (4, 4) because their widths (5 and 4) are greater than the total remaining *W* =8. Finally, in this case, we have *R1*=W * EP2 - (area of all placed pieces) = 8*6-(30+ 3+ 4) =11. So the second horizontal possible placement shown in (Fig.8. (a))

According to the second updated list ordered by decreasing w_i we can place the piece (4, 5) or (4, 4) so we place the piece (4, 5) because it has the longest width and. The residue *R2*= H* EP1 – (area of all placed pieces) = 10*5- (30+20) =0. By comparing *R1* and *R2*, the second cut is also vertical. Fig.8. (b).

Updated of 'list1*h*': (7,2) (6,5), (4,5) (4,4) (3,2) (3,1) (2,2).

Updated of 'ist2*w*': (6,5) (4,5) (4,4) (3,2) (7,2) (2,2) (3,1).

At the third step: H=10 and W= 3, we can't place the piece (4, 4) in this remaining plaque. Therefore, this piece needs a new plaque. We perform the same process for the other pieces: Horizontal cut according to h_l and *R1*=3*3-(3+4) = 2. Fig.9. (a).

Figure 7. The first horizontal and the vertical possible placements. (a) The coordinates of the piece. (b) The horizontal placement. (c) The vertical placement.

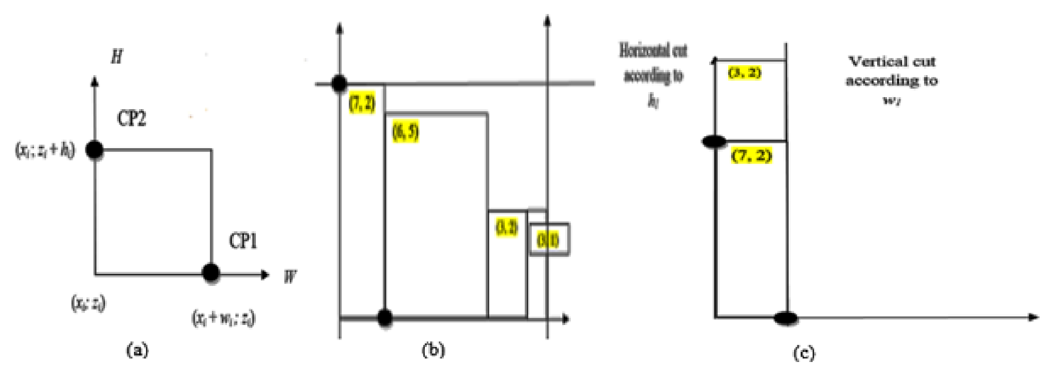


Figure 8. The second horizontal and the vertical possible placements. (a) The horizontal placement. (b) The vertical placement.

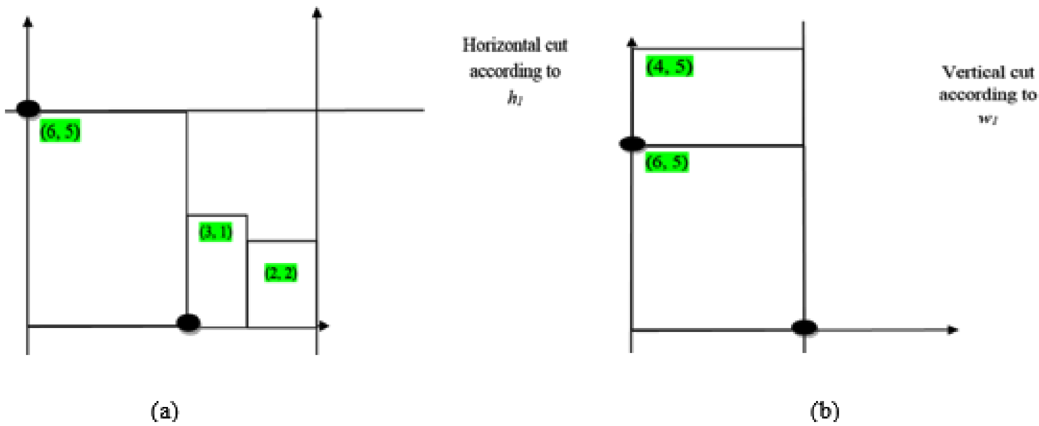
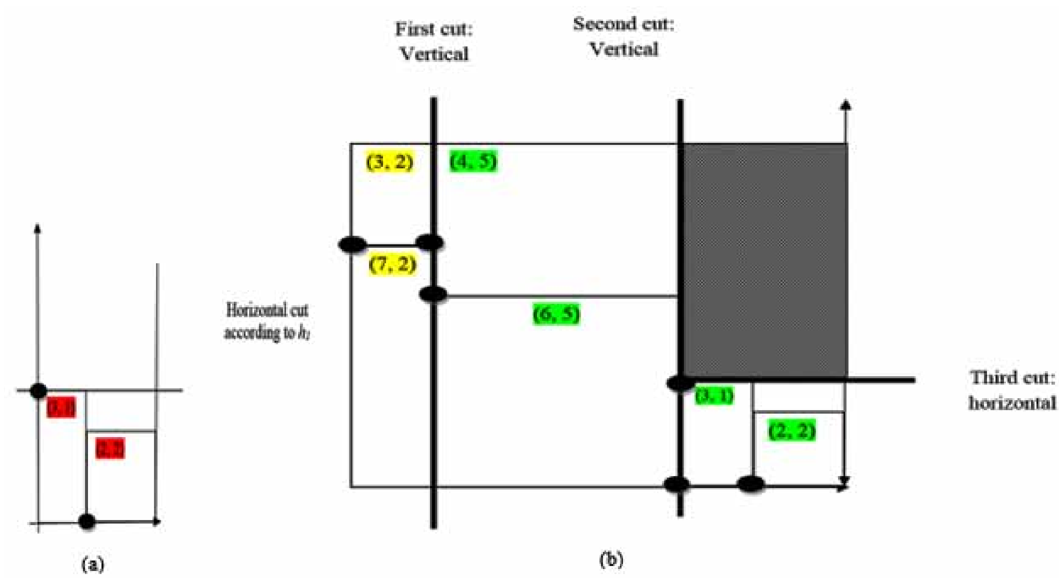


Figure 9. The last possible placement and the final cutting pattern. (a) The horizontal placement. (b) The final cutting pattern



The remaining pieces can't be placed in the vertical layer. So, we can't do a vertical cut. As result our plaque is composed of three cuts (two verticals and one horizontal Fig.9. (b)).

In the next section, we will provide results for real instances taken from the industrial case study demand.

5. COMPUTATIONAL EXPERIMENTS

Our proposed algorithm was implemented in Java. Computations were carried out on a computer with a 1.9 GHz Intel Pentium i3 CPU, 4 GB of RAM and running a Windows 7, 64 bits OS. We tested real instances given by a Tunisian industrial company. We treat a demand for one period of time; this demand consists of 450 pieces (items). By applying our method, this demand was distributed in six clusters by grouping them. Next, we introduced a table representing the different results (number and sort of blocks used, number of sub-blocks, the corresponding length of each sub-block, and the percentage of trim loss in each sub-block).

This method aims to find the corresponding (H; W; L) that able to minimize the total waste while respecting the guillotine constraint of the practical problem. We are able to obtain the cutting patterns for 450 pieces with an execution time of fewer than one minute. (Table 1).

The cutting of foam in different departments of the company is automated; it is done by programmed machines, while the cutting of mattresses and cushions is manual; it is based on the judgment and the experience of the employee who tries to minimize the trim loss of foam and satisfy the demand.

In fact, we take some different instances from the industrial demand. Based on the volume defined as *(LT* H*W)*. We compare the results of our heuristic application (LA) level algorithm to the industry results.

In Table 2, the different instances taken from real world applications correspond to the number of pieces (foam mattresses) demanded in each instance. The proposed algorithms were able to reduce the loss material by up to 9.6% on average, and to automatize the cut operations in this department.

To evaluate the performance of our proposed level algorithm (LA), we performed a comparative study and we re-implemented our heuristic with a few number of mattresses taken randomly from a part of the company demand with the proposed upper bound carried out by CPLEX in Baazaoui *et al.,*(2017).

Table 3 presents the results of the LA and the UB. For each instance, the table shows the value of the total length used and the execution time (in seconds). In the last column, the gap between the LA and the UB is given.

For these small instances taken randomly, the proposed heuristic gives results within a small lapse (fewer than 2 seconds) and for all the instances, it is less than the UB. For the first instance, the gap is equal to zero and conforms to the optimal solution. The results of the UB are better than those of our algorithm LA with a maximum gap of 36%. Except one instance gives better results than those of the UB.

CONCLUSION

In this work, we deal with one of non-classical practical problems in a local foam company in the area of cutting and packing. The described problem can be conceived as an integration problem of strip packing and multiple stock size cutting problem, which are defined, in the literature, as ones of the NP-hard problems. The treated problem presented some specifications related to the production process and the cutting ways. In this work, we extend the results of the upper bound, which is limited to small instances.

The UB represents a feasible solution to the treated problem. The results from the UB show that we are able to find the solution for 50 items (pieces) in lapse fewer than 2 minutes. Besides, it provides an optimal solution for up to 10 pieces with different types. To extend the resolution for larger instances, this study proposed a constructive heuristic based on levels placement to meet the guillotine constraint for the industrial problem. Results from the heuristic shows that we can have a placement for 450 pieces in duration fewer than one minute. The proposed level heuristic can reduce the trim loss by up to 9.6%.

In future work, we plan to adapt our proposed algorithms to solve this industrial problem for the case of more than one tunnel.

Table 1. Computational results for 450

Number and type of blocks used Block (H;W;L)	Number of sub-blocks used	L_{jk} :length of each sub-block (cm)	Percentage of trim loss (Waste)
(140;160;300)	3	100 100 100	11.87 13.03 7.678
(140;160;300)	3	100 100 100	12.85 12.85 12.85
(140;160;300)	3	100 100 100	12.85 12.85 12.85
(140;160;300)	3	100 100 100	12.85 12.85 70.14
(140;160;290)	3	90 100 100	28.57 37.50 9.375
(140;160;300)	3	100 100 100	19.42 4.710 81.25
(140;160;300)	3	100 100 100	58.25 15.55 7.125
(140;160;310)	3	100 100 110	5.151 48.40 12.36
(140;160;310)	3	100 100 110	51.69 19.50 18.891
(140;160;330)	3	110 110 110	21.46 15.62 15.62
(140;160;330)	3	110 110 110	36.36 31.25 31.25
(140;160;330)	3	110 110 110	31.25 31.25 31.25
(140;160;330)	3	110 110 110	31.25 31.25 31.25
(140;160;330)	3	110 110 110	31.25 31.25 31.25
(140;160;320)	3	110 100 110	31.25 5.951 14.57
(140;160;350)	2	100 250	28.43 14.05
(140;160;342)	2	192 150	7.270 42.70
(140;160;320)	2	150 170	24.11 20.68
(140;160;302)	2	152 150	14.69 3.154
The total length of the tunnel used (*LT*) for the production of these blocks and the cutting of 450 mattresses is 5994 cm.			

Table 2. Comparative results with the industry method

Different demands	Tunnel's Length (Employee's method) *LT* cm	Total volume of composed blocks *H*W*LT* Cm³	Tunnel's Length (LA heuristic) *LT* cm	Total volume of composed blocks *H*W*LT* Cm³	Improve (%)
100	1666	160*170*1666	1740	140*160*1740	13.9%
200	2000	150*180*2000	2200	140*160*2200	8.7%
300	4666	140*160*4666	4500	140*160*5400	3.5%
450	6116	150*160*6116	5994	140*160*5994	8.5%
500	6765	150*190*6765	7454	140*160*7454	13.4%

Table 3. Gap between the proposed method and the upper bound

Instances	Length (cm)		Time (s)		Gap = (LA-UB)/UB
	UB	LA	UB	LA	
5	200	200	0.56	0.29	0
10	250	340	0.7	0.62	0.36
20	331	320	0.95	0.91	-0.03
30	431	520	2.42	0.86	0.21
40	511	555	15	0.88	0.086
50	596	740	22	1.2	0.242
60	637	1085	103	1.64	0.70

ACKNOWLEDGMENT

We would like to express our sincere thanks to all the staff from the industry who provided the necessary data used in this paper.

FUNDING AGENCY

The publisher has waived the Open Access Processing fee for this article.

REFERENCES

Alvarez-Valdes, R., Parreño, F., & Tamarit, J. M. (2015). Lower bounds for three-dimensional multiple-bin-size bin packing problems. *OR-Spektrum*, *37*(1), 49–74. doi:10.1007/s00291-013-0347-2

Baazaoui, M., Hanafi, S., & Kamoun, H. (2014). A Mathematical formulation and a lower bound for the three-dimensional multiple-bin-size bin packing problem (MBSBPP): A Tunisian industrial case. *Control, Decision and Information Technologies (CODIT), International Conference on,* 219-224. doi:10.1109/CoDIT.2014.6996896

Baazaoui, M., Hanafi, S., & Kamoun, H. (2017). Three-Dimensional Multiple-Bin-Size Bin Packing: A Case Study with a New MILP-Based Upper Bound. In E. Grigoroudis & M. Doumpos (Eds.), *Operational Research in Business and Economics. Springer Proceedings in Business and Economics*. Springer. doi:10.1007/978-3-319-33003-7_11

Bang-Jensen, J., & Larsen, R. (2012). Efficient algorithms for real-life instances of the variable size bin packing problem. *Computers & Operations Research*, *39*(11), 2848–2857. doi:10.1016/j.cor.2012.02.018

Beasley, J. (1985). An exact two-dimensional non-guillotine cutting tree search procedure. *Operations Research*, *33*(1), 49–64. doi:10.1287/opre.33.1.49

Bortfeldt, A., & Wascher, G. (2012). *Container Loading Problems –A State- of-the- Art- Review.* Working Paper 7.

Chen, C. S., Lee, S. M., & Shen, Q. S. (1995). An analytical model for the container loading problem. *European Journal of Operational Research*, *80*(1), 68–76. doi:10.1016/0377-2217(94)00002-T

Christofoletti, M. M., de Araujo, A. S., & Cherri, A. C. (2020). Integrated lot-sizing and cutting stock problem applied to the mattress industry. *The Journal of the Operational Research Society*. Advance online publication. doi:10.1080/01605682.2020.1718013

Coffman, E. G. Jr, Garey, M. R., Johnson, D. S., & Tarjan, R. E. (1980). Performance bounds for level oriented two-dimensional packing algorithms. *SIAM Journal on Computing*, *9*(4), 808–826. doi:10.1137/0209062

Correcher, J. F., Alonso, M. T., Parreño, F., & Alvarez-Valdes, R. (2017). Solving a large multicontainer loading problem in the car manufacturing industry. *Computers & Operations Research*, *82*, 139–152. doi:10.1016/j.cor.2017.01.012

Crainic, T. G., Perboli, G., Rei, W., & Tadei, R. (2011). Efficient lower bounds and heuristics for the variable cost and size bin packing problem. *Computers & Operations Research*, *38*(11), 1474–1482. Advance online publication. doi:10.1016/j.cor.2011.01.001

Crainic, T. G., Perboli, G., & Tadei, R. (2008). Extreme point-based heuristics for three dimensional bin packing. *INFORMS Journal on Computing*, *20*(3), 368–384. doi:10.1287/ijoc.1070.0250

Crainic, T. G., Perboli, G., & Tadei, R. (2009). TS2 PACK: A two-level tabu search for the three-dimensional bin packing problem. *European Journal of Operational Research*, *195*(3), 744–760. doi:10.1016/j.ejor.2007.06.063

De Carvalho, J. M. (1999). Exact solution of bin-packing problems using column generation and branch-and-bound. *Annals of Operations Research*, *86*, 629–659. doi:10.1023/A:1018952112615

deQueiroz, A. T., Miyazawa, F., Wakabayashi, Y., & Xavier, E. (2012). Algorithms for 3D guillotine cutting problems: Unbounded knapsack, cutting stock and strip packing. Computer and Operations Research, 39, 200-212. doi:10.1016/j.cor.2011.03.011

Fekete, S. P., & Schepers, J. (1997). *A new exact algorithm for general orthogonal dimensional knapsack problems. Algorithms - ESA '97* (Vol. 1284). Springer Lecture Notes in Computer Science. doi:10.1007/3-540-63397-9_12

Fekete, S. P., & Schepers, J. (2001). New classes of lower bounds for bin packing problems. *Mathematical Programming*, *91*(1), 11–31. doi:10.1007/s101070100243

Fekete, S. P., & Schepers, J. (2004). A combinatorial characterization of higher-dimensional orthogonal packing. *Mathematics of Operations Research*, *29*(2), 353–368. doi:10.1287/moor.1030.0079

Fekete, S. P., Schepers, J., & Veen, J. (2006). An Exact Algorithm for Higher-Dimensional Orthogonal Packing. *ESA*, *97*. Advance online publication. doi:10.1287/opre.1060.0369

Fekete, S. P., & van der Veen, J. C. (2007). PackLib2: An integrated library of multidimensional packing problems. *European Journal of Operational Research*, *183*(3), 1131–1135. doi:10.1016/j.ejor.2006.04.023

George, J. A., & Robinson, D. F. (1980). A Heuristic for Packing Boxes into a Container. *Computers & Operations Research*, *7*(3), 147–156. doi:10.1016/0305-0548(80)90001-5

Gilmore, P., & Gomory, R. (1965). Multistage cutting stock problems of two and more dimensions. *Operations Research*, *13*(1), 94–120. doi:10.1287/opre.13.1.94

Gilmore, P. C., & Gomory, R. E. (1961). A linear programming approach to the cutting stock problem. *Operations Research, 9*(6), 849–859. doi:.6.84910.1287/opre.9

Ha, C. T., Nguyen, T. T., Bui, L. T., & Wang, R. (2017). An Online Packing Heuristic for the Three-Dimensional Container Loading Problem in Dynamic Environments and the Physical Internet. In G. Squillero & K. Sim (Eds.), Lecture Notes in Computer Science: Vol. 10200. *Applications of Evolutionary Computation. EvoApplications 2017*. Springer. doi:10.1007/978-3-319-55792-2_10

Hifi, M., Negre, S., & Wu, L. (2014). Hybrid greedy heuristics based on linear programming for the three-dimensional single bin-size bin packing problem. *International Transactions in Operational Research*, *21*(1), 59–79. doi:10.1111/itor.12035

Hongteng, W., Stephen, L., Yain-whar, S., Defu, Z., & Adi, L. (2017). Three-stage heuristic algorithm for three dimensional irregular packing problem. *Applied Mathematical Modelling*, *41*, 431–444. Advance online publication. doi:10.1016/j.apm.2016.09.018

Kantorovich, L. V. (1960). Mathematical methods of organizing and planning production. *Management Science*, *6*(4), 363–422. doi:10.1287/mnsc.6.4.366

Lodi, A., Martello, S., & Monaci, M. (2002). Two-dimensional packing problems: A survey. *European Journal of Operational Research*, *141*(2), 241–252. doi:10.1016/S0377-2217(02)00123-6

Martello, S., Pisinger, D., & Vigo, D. (2000). The three-dimensional bin packing problem. *Operations Research*, *48*(2), 256–267. doi:10.1287/opre.48.2.256.12386

Martin, M., Birgin, E. G., Lobato, R. D., Morabito, R., & Munari, P. (2020). Models for the Two-Dimensional Rectangular Single Large Placement Problem with Guillotine Cuts and Constrained Pattern. *International Transactions in Operational Research*, *27*(2), 767–793. doi:10.1111/itor.12703

Martin, M., Morabito, R., & Munari, P. (2021). Two-stage and one-group two-dimensional guillotine cutting problems with defects: A CP-based algorithm and ILP formulations. *International Journal of Production Research*, 1–20. Advance online publication. doi:10.1080/00207543.2021.1876270

Paquay, S., Limbourg, S., Schyns, M., & Oliveira, J. F. (2018). MIP-based constructive heuristics for the three-dimensional Bin Packing Problem with transportation constraints. *International Journal of Production Research*, *56*(4), 1581–1592. doi:10.1080/00207543.2017.1355577

Park, K. T., Ryu, J. H., Lee, H. K., & Lee, I.-B. (2013). Developing a heuristics for glass cutting process optimization: A case of two-dimensional two-stage guillotine cutting with multiple stock sizes. *Korean Journal of Chemical Engineering*, *30*(2), 278–285. doi:10.1007/s11814-012-0125-x

Parreño, F., Alvarez-Valdes, R., Oliveira, J. F., & Tamarit, J. M. (2008). A hybrid GRASP/VND algorithm for two- and three-dimensional bin packing. *Annals of Operations Research*, *179*(1), 203–220. doi:10.1007/s10479-008-0449-4

Saraiva, R., Nepomuceno, N., & Pinheiro, P. R. (2015). A layer-building algorithm for the three-dimensional multiple bin packing problem: a case study in an automotive company. International Federation of Automatic Control, 48(3), 490-495. doi:10.1016/j.ifacol.2015.06.129

Silveira, M. E., Vieira, S. M., & Da Costa Sousa, J. M. (2013). An ACO Algorithm for the 3D Bin Packing Problem in the Steel Industry. In M. Ali, T. Bosse, K. V. Hindriks, M. Hoogendoorn, C. M. Jonker, & J. Treur (Eds.), Lecture Notes in Computer Science: Vol. 7906. *Recent Trends in Applied Artificial Intelligence. IEA/AIE 2013*. Springer. doi:10.1007/978-3-642-38577-3_55

Sumetthapiwat, S., Intiyot, B., & Jeenanunta, Ch. (2020). A column generation on two-dimensional cutting stock problem with fixed-size usable leftover and multiple stock sizes, Int. J. *Logistics Systems and Management*, *35*(2), 273–288. doi:10.1504/IJLSM.2020.104781

Wäscher, G., Haußner, H., & Schumann, H. (2007). An Improved Typology of Cutting and Packing Problems. *European Journal of Operational Research*, *183*(3), 1109–1130. doi:10.1016/j.ejor.2005.12.047

Zhao, X., Bennell, J. A., Bektas, T., & Dowsland, K. (2014). A comparative review of 3D container loading algorithms. *International Transactions in Operational Research*, *23*(1-2), 287–320. doi:10.1111/itor.12094

Zhu, W., Huang, W., & Lim, A. (2012). A prototype column generation strategy for the multiple container loading problem. *European Journal of Operational Research*, *223*(1), 27–39. doi:10.1016/j.ejor.2012.05.039

Mariem Baazaoui is an assistant professor at Imam Abdularhaman Bin Faisal University. A PhD holder in operations research, the research interests includes bin packing, mathematical modeling, and optimization.

Souhir Elleuch has PhD in Informatic Systems.

Hichem Kamoun is a professor at FSEG Sfax. Research interests include group technology nurse rostering bin packing and seaport birth allocation. Mathematical modeling and size reduction are the most used methods in solving problems encountered in the previously mentioned topics.