Window Analysis and MPI for Efficiency and Productivity Assessment Under Fuzzy Data: Window Analysis and MPI

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ABSTRACT

This research develops a procedure for DEA window analysis and MPI evaluation of a manufacturing process with fuzzy inputs and outputs. A real case study was provided to illustrate relative efficiency and MPI assessment of a blowing machine over a period of one a year. The proposed approach was implemented to measure the technical, pure technical, and scale efficiency scores for decision making unit. The results showed that the blowing process was technically inefficient due to scale inefficiency. Therefore, management should optimize the size of operations and better utilize resources. Then, the lower and upper MPI values and their corresponding technology change and efficiency change were calculated. The MPI results revealed the reasons behind MPI progress or regress in the current period measured with respect to the next period. This procedure provides great assistance to process engineering in obtaining reliable feedback on process performance and guide them to take proper actions.

KEYWORDS

Efficiency, Fuzzy Data, Malmquist Productivity Index, Window Analysis

1. INTRODUCTION

In practice, production engineers regularly assess efficiency and productivity of manufacturing processes to achieve business goals (Park et al., 2018. Typically, measurement of a production unit-performance is crucial in determining whether it has achieved its objectives or not, and it generates a phase of management process that consists of feedback motivation phases (Kumar and Gulati, 2008; Al-Refaie et al., 2015). An effective technique for measuring processes’ relative efficiency is the Data envelopment analysis (DEA) method, in which a production frontier is constructed from a set of comparable Decision making Units (DMUs) and data on their inputs and outputs. The efficiency of each DMU is defined by its relative distance from the production frontier (Al-Refaie et al., 2016a; Al-Refaie et al., 2016b; Ennen and Batool, 2018). Two common DEA models can be used for this purpose Charnes, Cooper, and Rhodes (CCR) and Banker, Chang, and Cooper (BCC) by Charnes et al. (1978) and Banker et al. (1984), respectively.

However, when using the CCR and BCC models, an important rule of thumb is that the number of DMUs is at least twice the sum of the number of inputs and outputs (Arcos-Vargas et al., 2017).
Otherwise, the model may produce numerous relatively efficient units and decrease discriminating power. To resolve this difficulty, DEA window analysis was introduced in which the performance of a DMU in any period can be compared with its own performance in other periods as well as to the performance of other DMUs (Al-Refaie et al., 2014). DEA window analysis is based on a dynamic perspective, regarding the same DMU in different period of time as entirely different DMUs (Jia and Yuan, 2017). The window analysis technique relies on the traditional CCR and BCC models for estimating technical efficiency (TE) and pure technical efficiency (PTE) scores for each DMU. DEA window analysis is usually followed by the evaluation of the Malmquist productivity index (MPI), which is a formal time-series analysis method for conducting performance comparisons of DMUs over time by solving traditional DEA type models. The MPI measures the productivity change of DMU over time. The productivity of DMU from period $p$ and $p+1$ is improved when MPI is larger than one, remained unchanged when MPI equals one, and deteriorated when MPI is less than one. The productivity change can be decomposed into two parts, namely technological change (TC) and efficiency change (TEC) component, which measures the change in relative efficiency over time (Balcerzak et al., 2017).

In the traditional DEA window analysis and MPI, the main assumption is that the inputs and outputs are measured by exact values on a ratio scale (Kao and Liu, 2000). In some real applications, however, the inputs and outputs may be expressed by fuzzy values, and hence the use of the traditional window analysis makes the analysis unreliable, which may lead to obtain erroneous decisions. Consequently, this research contributes to ongoing research by proposing a procedure for DEA window analysis and MPI evaluation that is used to assess the relative efficiency and productivity of a production process under fuzzy input and output data. This procedure can more realistically represents real problems than the traditional DEA models and provides reliable analysis and enables taking accurate decisions with proper actions on production inputs and outputs that lead to enhance production performance and quality, and better utilize input resources. The assessment of the efficiency and productivity of a blowing machine, which is used in manufacturing plastic products, under fuzzy input and output data over a period of one year will be presented to illustrate the proposed procedure. The remaining of this paper including the introduction is organized as follows. Section 2 presents background and literature review. Section 3 develops research methodology and application. Section 4 discusses research results. Section 5 summarizes research conclusions.

2. DEA BACKGROUND AND LITERATURE REVIEW

2.1 DEA Relevant Background

Generally, in DEA the CCR efficiency score measures technical efficiency (TE), which reflects the firm’s ability to maximize output from a given set of inputs assuming that the size of operation of DMU is optimal. Consider a set of $n$ DMUs. For a specific DMU$_j$ ($j = 1, ..., n$), DMU$_k$, let $y_{rk}$ denote the level of $r$th ($r = 1, ..., s$) output and $x_{ik}$ the level of the $i$th ($i = 1, ..., m$) input. The efficiency score, $\theta_k$, of DMU$_k$ is then calculated as by solving the dual input-oriented CCR model as follows (Charnes et al., 1978):

\[
\min \theta_k \tag{1a}
\]

Subject to:

\[
\sum_{j=1}^{n} \lambda_{kj} x_{ij} - \theta_k x_{ik} \leq 0, \forall i \tag{1b}
\]
where \( \theta_k \) unrestricted in sign. The optimal \( \theta_k^* \) satisfies \( 0 \leq \theta_k^* \leq 1 \). If \( \theta_k^* \) equals to one, the DMU under measurement is then technically efficient. The CCR model assumes constant return to scale (CRS) where an increase in the input results an increase in the output result. Whereas, the BCC model assumes that the DMU operates under variable returns to scale (VRS) if it is suspected that an increase in inputs does not result in a proportional change in the outputs. The BCC model measures the Pure Technical Efficiency (PTE), which ignores the impact of the scale size by only comparing a DMU to a unit of similar scale. The PTE measures how a DMU utilizes its sources under exogenous environments; a low value of PTE implies that the DMU inefficiently manages its resources.

To calculate the PTE score, the following dual input-oriented BCC model is used (Banker et al., 1984):

\[
\begin{align*}
\text{Min} & \quad \theta_k \\
\text{Subject to:} & \\
\sum_{j=1}^{n} \lambda_{kj} x_{ij} - \theta_k x_{ik} & \leq 0, \quad \forall \ j \\
\sum_{j=1}^{n} \lambda_{kj} y_{ij} & \geq y_{ik}, \quad \forall \ r \\
\sum_{j=1}^{n} \lambda_{kj} & = 1 \\
\lambda_{kj} & \geq 0, \quad \forall k, j
\end{align*}
\]

Typically, the BCC model divides the TE into two parts: (i) PTE which ignores the impact of scale size by only comparing a DMU to a unit of similar scale and measures how a DMU utilizes its sources under exogenous environment and (ii) Scale Efficiency (SE) which measures how the scale size affects efficiency. The SE measures how the scale size affects efficiency. The SE provides the ability of the management to choose the optimal size of resources and is calculated using Eq. (3).

\[
SE = \frac{TE}{PTE}
\]
Given the input and output data for a manufacturing process over a period of time, the traditional window analysis in DEA divides the time period into time windows. Each window is treated as a DMU. Then, the DMU’s input and output data are utilized to measure the TE and PTE scores using the CCR (model 1) and BCC (Model 2), respectively, at each time unit in this DMU. Finally, the DMU’s TE and PTE averages are calculated and then used to estimate the corresponding SE averages using Eq. (3). Based on the window’s TE, PTE and SE scores, proper improvement actions are suggested to enhance production performance. The MPI follows to determine both the efficiency change (catch-up) and technological change (frontier-shift). Review of relevant previous studies is presented in the following section.

2.2 Literature Review
Window analysis in DEA has been applied in several studies. For example, Kumar and Gulati (2008) measured the extent of technical, pure technical, and scale efficiencies in 27 public sector banks operating in India in the year 2004/2005 using DEA. Mahajan et al. (2012) measured technical efficiencies, slacks and input/output targets for 50 large Indian pharmaceutical firms. This study uses DEA approach. Mugera (2013) measured technical efficiency of dairy farms with imprecise data using a fuzzy data envelopment analysis approach. Azadeh et al. (2014) developed an integrated fuzzy simulation fuzzy data envelopment analysis approach for optimum maintenance planning. Al-Refaie, et al. (2015) analyzed the growth potentials of five production machines in a plastic industry by employing window analysis and Malmquist productivity index. Campos et al. (2016) used DEA to evaluate the efficiency of public resource usage in health systems of autonomous communities in Spain. Jia and Yuan (2017) evaluated and compared operational efficiencies of different hospitals before and after their establishment of branched hospitals using DEA. Balcerzak, et al. (2017) proposed a methodology for a comprehensive evaluation of operational efficiency of the banking sectors in EU countries using DEA. Aye et al. (2018) used a two-stage fuzzy approach efficiency in South African agriculture. Ennen and Batool (2018) investigated 12 major airports in Pakistan for potential cost inefficiencies using DEA. Barak and Dahooei (2018) proposed fuzzy DEA for airlines safety evaluation. Park et al. (2018) suggested a new DEA-based efficiency evaluation model and conducted efficiency evaluations and benchmarking for 13 Korean national university hospitals. Anouze and Bou-Hamad (2019) employed DEA and data mining to efficiency estimation and evaluation. Zhou and Xu (2020) overviewed fuzzy DEA models in the presence of undesirable outputs. The input/output data were represented by triangular fuzzy numbers. Then, two virtual fuzzy DMUs called fuzzy ideal DMU and fuzzy anti-ideal DMU were introduced into proposed fuzzy DEA framework. Nandy and Singh (2021) employed a combination of fuzzy data envelopment analysis approach to yield crisp DEA efficiency values by converting the fuzzy DEA model into a linear programming problem and machine learning algorithms for better evaluation and prediction of the variables affecting the farm efficiency. Sánchez-Ortiz et al. (2021) employed DEA window analysis and Malmquist index to assess efficiency and productivity in the Spanish electricity sector. The study defined a model that showed how the efficiency problems associated with electricity distribution companies such as productive overcapacity or tariff deficit can be measured based on the theory of constraints and theory of economic regulation.

In most previous studies, the window analysis and MPI were performed without consideration of the impreciseness in the input and output data which gives wrong efficient frontier (Wanke et al., 2016). Hence, research contributions are necessary to develop and efficient procedure for assessing efficiency and productivity of a manufacturing process under fuzzy input and output data. Nevertheless, few studies were reported on the use window analysis in DEA and MPI for assessing the efficiency of a manufacturing process under fuzzy input and output data.
3. RESEARCH METHODOLOGY AND APPLICATION

3.1 Research Methodology

The methodology for conducting window analysis in DEA and Malmquist analysis with fuzzy inputs and outputs is outlined as follows:

Step 1: Select the production process for which the efficiency scores is to be evaluated and then measure the process’s low, middle, and high values of the inputs and outputs over a time horizon of $T$. Let $p$ denotes any period in time horizon $T$. Collect the data for each period $t = 1, ..., T$. Then, determine the fuzzy inputs and the outputs that will be used for data envelopment window analysis.

Assume each period $t$, where $t = 1, ..., T$, has fuzzy inputs $\tilde{x}_t = (x_{t,1}^{L}, x_{t,1}^{M}, x_{t,1}^{H})$ and fuzzy outputs $\tilde{y}_t = (y_{t,1}^{L}, y_{t,1}^{M}, y_{t,1}^{H})$. Divide the time horizon, $T$, into $n$ time windows $w_j$; where $w_1: t_1 = p_1 \rightarrow p_n$, $w_2: t_2 = p_2 \rightarrow p_{n+1}$, ..., $w_n: t_k = p_k \rightarrow p_{k+w-1}$, and so on. Treat each time window as a decision making unit ($DMU$); $j = 1, ..., n$. Consequently, the matrices of fuzzy inputs and outputs, $\tilde{X}_k$ and $\tilde{Y}_k$, of a specific $DMU$, $DMU_k$, can be expressed respectively as:

\[
\tilde{X}_k = \begin{bmatrix}
\tilde{x}_{1,p} & \tilde{x}_{1,p+1} & \cdots & \tilde{x}_{1,p+w-1} \\
\tilde{x}_{2,p} & \tilde{x}_{2,p+1} & \cdots & \tilde{x}_{2,p+w-1} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{x}_{m,p} & \tilde{x}_{m,p+1} & \cdots & \tilde{x}_{m,p+w-1}
\end{bmatrix},
\]

and

\[
\tilde{Y}_k = \begin{bmatrix}
\tilde{y}_{1,p} & \tilde{y}_{1,p+1} & \cdots & \tilde{y}_{1,p+w-1} \\
\tilde{y}_{2,p} & \tilde{y}_{2,p+1} & \cdots & \tilde{y}_{2,p+w-1} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{y}_{s,p} & \tilde{y}_{s,p+1} & \cdots & \tilde{y}_{s,p+w-1}
\end{bmatrix},
\]

Calculate the technical efficiency, pure technical efficiency, and scale efficiency scores for each $DMU_j; j = 1, ..., J$, as follows. That is, for a specific $DMU, DMU_k$, which covers the period $p_k$ till $p_k + w - 1$, the optimal technical efficiency score, $\theta^*_{k,p_k}$, for $DMU_k$ at time $p_k$ is calculated from the inputs and outputs of $p_k$ till $p_k + w - 1$ including the inputs and outputs of $p_k$ by solving the following dual model:

\[
\text{Min } \theta_{k,p_k}
\]

Subject to:

\[
\sum_{t \leftarrow p_k} \lambda_i x_{i,t}^L - \theta_{k,p_k} x_{i,p_k}^L \leq 0 \quad , \forall i
\]
\[
\sum_{t=p_k}^{p_k+w-1} \lambda_i x^M_{i,t} - \theta_{k,p_k} x^M_{i,p_k} \leq 0 , \quad \forall i
\] (6c)

\[
\sum_{t=p_k}^{p_k+w-1} \lambda_i x^H_{i,t} - \theta_{k,p_k} x^H_{i,p_k} \leq 0 , \quad \forall i
\] (6d)

\[
\sum_{t=p_k}^{p_k+w-1} \lambda_j y^L_{r,t} - y^L_{r,p_k} \geq 0 , \quad \forall r
\] (6e)

\[
\sum_{t=p_k}^{p_k+w-1} \lambda_j y^M_{r,t} - y^M_{r,p_k} \geq 0 , \quad \forall r
\] (6f)

\[
\sum_{t=p_k}^{p_k+w-1} \lambda_j y^H_{r,t} - y^H_{r,p_k} \geq 0 , \quad \forall r
\] (6g)

\[
\lambda_i \geq 0 , \quad \forall t \in [p_k, p_k+w-1]
\] (6h)

Then, estimate the corresponding optimal pure technical efficiency score, \(\gamma^*_{k,p_k}\), of \(DMU_k\) at time \(p_k\) is estimated as follows:

\[
Min \, \gamma_{k,p_k}
\] (7a)

Subject to:

\[
\sum_{t=p_k}^{p_k+w-1} \lambda_i x^L_{i,t} - \gamma_{k,p_k} x^L_{i,p_k} \leq 0 , \quad \forall i
\] (7b)

\[
\sum_{t=p_k}^{p_k+w-1} \lambda_i x^M_{i,t} - \gamma_{k,p_k} x^M_{i,p_k} \leq 0 , \quad \forall i
\] (7c)
\[ \lambda_i x^H_{i,t} - \gamma_{k,p} x_{i,t}^H \leq 0, \quad \forall i \quad (7d) \]

\[ \sum_{t=p_k}^{p_k+w-1} \lambda_i y^L_{r,t} - y_{r,p_t}^L \geq 0, \quad \forall r \quad (7e) \]

\[ \sum_{t=p_k}^{p_k+w-1} \lambda_i y^M_{r,t} - y_{r,p_t}^M \geq 0, \quad \forall r \quad (7f) \]

\[ \sum_{t=p_k}^{p_k+w-1} \lambda_i y^H_{r,t} - y_{r,p_t}^H \geq 0, \quad \forall r \quad (7g) \]

\[ \sum_{t=p_k}^{p_k+w-1} \lambda_i = 1, \quad (7h) \]

\[ \lambda_i \geq 0, \quad \forall t \in [p_r, p_i + w-1] \quad (7i) \]

Finally, the optimal scale efficiency score, \( \eta_{k,p}, \), is calculated by dividing the optimal TE score, \( \theta_{k,p}, \), by its corresponding optimal PTE score, \( \gamma_{k,p}, \). Mathematically,

\[ \eta_{k,p} = \frac{\omega_{k,p}}{\gamma_{k,p}} \quad (8) \]

In a similar manner, the other values of the \( \theta_t^*, \gamma_t^*, \) and \( \eta_t^* ; \forall t \in [p_r, p_i + w-1] \) and \( t \neq p_k \) of DMU are estimated. Similarly, calculate the optimal efficiency scores of \( \theta_j^*, \gamma_j^*, \) and \( \eta_j^* ; j \neq k \).

Calculate the averages of the optimal technical, pure technical, and scale efficiency scores, \( \theta_j^*, \gamma_j^*, \) and \( \eta_j^* \), respectively, of DMU using Eqs. (9) to (11), respectively.

\[ \theta_j^* = \frac{1}{w} \sum_{t=p_j}^{p_j + w-1} \theta_{j,t}^* \quad (9) \]
Estimate the Malmquist Productivity Index (MPI). In practice, the production line is evaluated at three-month periods. This results in four evaluation periods each of three months. Let \( e \) denotes the category; \( e = 1,\ldots, E \). Let \( \theta_{e^l}^{p,L}(E) \) and \( \theta_{e^u}^{p,U}(E) \) denote the optimal lower and upper efficiency of month \( p \) in a specific evaluation period \( e' \), which are calculated from the inputs and outputs of the month \( p \) in all \( E \) evaluation periods, respectively by solving the formulas (10) and (11), respectively (Ebrahimnejad and Amani, 2021; Jahanshahloo et al., 2006).

\[
\theta_{e^l}^{p,L}(E) = \min_{i} \delta_{i} \tag{13a}
\]

Subject to:

\[
\delta_{i} x_{i e}^{pU} - \sum_{e' = e, e'' = 1}^{E} \lambda_{e' e''} x_{i e''}^{pL} - \lambda_{e'} x_{i e'}^{pU} \geq 0, \quad \forall i \tag{13b}
\]

\[
y_{re}^{pL} - \sum_{e' = e, e'' = 1}^{E} \lambda_{e' e''} y_{re}^{pU} - \lambda_{e'} y_{re}^{pL} \geq 0, \quad \forall r \tag{13c}
\]

\[
\lambda_{e} \geq 0, \quad \forall e \tag{13d}
\]

and

\[
\theta_{e^u}^{p,U}(E) = \min_{z} \delta_{z} \tag{14a}
\]

Subject to:
\[
\delta x^{p+1, L}_{i e'} - \sum_{e'=1}^{E} \lambda_e x^{p, U}_{i e'} - \lambda_e x^{p+1, L}_{i e'} \geq 0, \quad \forall i \tag{14b}
\]

\[
y^{p+1, U}_{r e'} - \sum_{e'=1}^{E} \lambda_e y^{p, L}_{r e'} - \lambda_e y^{p+1, U}_{r e'} \geq 0, \quad \forall r \tag{14c}
\]

\[
\lambda_e \geq 0, \quad \forall e \tag{14d}
\]

Using \(p+1\) instead of \(p\), the \(\theta^{p+1, L}_{e'}(E^{p+1})\) and \(\theta^{p+1, U}_{e'}(E^{p+1})\) are calculated for the lower and upper bounds, respectively. Furthermore, let \(\theta^{p+1, L}_{e'}(E^{p})\) and \(\theta^{p+1, U}_{e'}(E^{p})\) denote the optimal lower and upper efficiency of month \(p+1\) in evaluation period \(e'\), which are estimated from the inputs and outputs of month \(p\) in all \(E\) evaluation periods by solving formulas (15) and (16), respectively.

\[
\theta^{p+1, L}_{e'}(E^{p}) = \text{Min } \omega_1 \tag{15a}
\]

Subject to:

\[
\omega x^{p+1, U}_{i e'} - \sum_{e'=1}^{E} \lambda_e x^{p+1, L}_{i e'} \geq 0, \quad \forall i \tag{15b}
\]

\[
y^{p+1, L}_{r e'} - \sum_{e'=1}^{E} \lambda_e y^{p+1, U}_{r e'} \geq 0, \quad \forall r \tag{15c}
\]

\[
\lambda_e \geq 0, \quad \forall e \tag{15d}
\]

and

\[
\theta^{p+1, U}_{e'}(E^{p}) = \text{Min } \omega_2 \tag{16a}
\]

Subject to:

\[
\omega x^{p+1, L}_{i e'} - \sum_{e'=1}^{E} \lambda_e x^{p+1, U}_{i e'} \geq 0, \quad \forall i \tag{16b}
\]
\[
\sum_{e=1}^{E} \lambda_y y_{re}^{p+1L} \geq 0, \quad \forall r
\]  
\[
\lambda_y \geq 0, \quad \forall e
\]

Using the \( p+1 \) instead of \( p \) and vice versa, for the above models, the \( \theta^p \) \((E^{p+1}) \) and \( \theta^{p+1} \) \((E^{p+1}) \) values can be obtained, respectively, for the lower and upper bounds. Consequently, the lower and upper \( MPI; MPI^U_{e,p} \) and \( MPI^L_{e,p} \), respectively, are calculated using Eqs. (17) and (18), respectively.

\[
MPI^L_{e,p} = \left[ \frac{\theta^U_{e,p}(p)}{\theta^L_{e,p}(p)} \times \frac{\theta^U_{e,p}(p+1)}{\theta^L_{e,p}(p+1)} \right]^{1/2} \times \left[ \frac{\theta^L_{e,p}(p+1)}{\theta^U_{e,p}(p)} \right]^{1/2} \]

\[
MPI^U_{e,p} = \left[ \frac{\theta^U_{e,p}(p)}{\theta^L_{e,p}(p)} \times \frac{\theta^U_{e,p}(p+1)}{\theta^L_{e,p}(p+1)} \right]^{1/2} \times \left[ \frac{\theta^L_{e,p}(p+1)}{\theta^U_{e,p}(p)} \right]^{1/2} \]

where \( MPI^L_{e,p} \) larger than one indicates a progress; \( MPI^U_{e,p} \) smaller than one indicates a regress in productivity from time \( p \) to \( p+1 \). Otherwise, nothing can be said.

Step 6. Analyze and discuss the obtained optimal technical, pure technical, and scale efficiency scores. Then, identify the reasons for inefficiency and suggest proper actions. Further, analyze and discuss the results of Malmquist analysis.

3.2 Application

The data is obtained from production reports for a blowing machine over year 2019 as displayed in Table 1, where three inputs; planned production quantity \((PP, x_1^t)\), defect quantity \((DQ, x_2^t)\), and idle time \((IT, x_3^t)\), and a single output actual production quantity \((PQ, y^t)\) were identified for each month, \( t \).

Table 1. The collected data for blowing machine.
(1) Window analysis

The window length is set a period of six months. This results in seven windows or DMUs; \( t_1-t_6, t_2-t_7, \ldots \), and \( t_7-t_{12} \) treated as DMU1 to DMU7, respectively. Table 2 displays the optimal technical for all periods and DMUj s. That is, the optimal technical efficiency, \( \hat{\theta}_{j,p_1} \), scores for each period of DMUj are obtained by solving Eq. (6). Similarly, the optimal pure technical efficiency, \( \gamma_{j,p_1}^\ast \), scores are estimated for all periods of DMUj by solving Eq. (7). Then, the optimal technical efficiency, \( \hat{\theta}_j \), and pure technical efficiency, \( \gamma_j \), of DMUj are using Eqs. (9) and (10), respectively. Finally, the optimal scale efficiency, \( \eta_j^\ast \), of DMUj is determined using Eqs. (11). For example, the optimal TE score, \( \hat{\theta}_{1,1} \), of DMU1 at period \( p_1 \) is estimated by solving Eq. (6) and found to be 0.9575. Similarly the \( \hat{\theta}_{1,2} \) to \( \hat{\theta}_{1,6} \) are obtained. The optimal TE score, \( \hat{\theta}_1 = 0.9493 \), of DMU1 is calculated as the average of \( \hat{\theta}_{1,1} \) to \( \hat{\theta}_{1,6} \) values. In a similar manner, the \( \hat{\theta}_j \) values are estimated. Furthermore, the coefficient of variation (CV\%) of DMUj is calculated as the standard deviation of the \( \hat{\theta}_{j,p_1} \) values divided by \( \hat{\theta}_j \). Finally, the \( \hat{\theta}_{j,1} \) for the first period and the coefficient of variation (CV\%) are calculated. In Table 2, the estimated TE scores listed in each period \( t \) (column) show stable performance, because the differences between the efficiency values are (CV\%) values in column are smaller than 0.05). However, the coefficient of variance (CV\%) listed in rows are relatively large for DMU1 and DMU2, which means that the dispersion is significant and hence there exists a trend in their corresponding \( \hat{\theta}_{j,p_1} \) scores. Moreover, it is found that all the \( \hat{\theta}_{j} \) scores are less than one, while the smallest and largest \( \hat{\theta}_{j} \) values are 0.9488 and 0.9853, which correspond to \( \hat{\theta}_2 \) and \( \hat{\theta}_7 \), respectively. Theoretically, all DMUj s are, therefore, concluded inefficient. For each period \( t \), the average TE, \( \hat{\theta}_t \), it found that the \( \hat{\theta}_t \) is equal to one at periods 6, 9, and 12, whereas the \( \hat{\theta}_t \) scores are less than one at each of the remaining periods. In other words, the blowing machine was technically- efficient in two out of twelve months.

<table>
<thead>
<tr>
<th>Month (t)</th>
<th>Input ( \bar{x}_1^t ) (PP, unit)</th>
<th>Input ( \bar{x}_2^t ) (DQ, unit)</th>
<th>Input ( \bar{x}_3^t ) (IT, unit)</th>
<th>Output ( \bar{y}_1^t ) (PQ, unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>20113, 20736, 21565</td>
<td>51, 53, 55</td>
<td>2018, 2081, 2164</td>
<td>19012, 19616, 20000</td>
</tr>
<tr>
<td>9</td>
<td>13409, 13824, 14376</td>
<td>31, 32, 33</td>
<td>657, 678, 705</td>
<td>12984, 13500, 13561</td>
</tr>
<tr>
<td>10</td>
<td>10056, 10368, 10782</td>
<td>70, 73, 75</td>
<td>2769, 2855, 2969</td>
<td>8195, 8318, 9939</td>
</tr>
<tr>
<td>11</td>
<td>15085, 15552, 16174</td>
<td>111, 115, 119</td>
<td>2873, 2962, 3080</td>
<td>13294, 13350, 15230</td>
</tr>
<tr>
<td>12</td>
<td>21790, 22464, 23362</td>
<td>237, 245, 254</td>
<td>2268, 2339, 2432</td>
<td>20137, 20750, 22252</td>
</tr>
</tbody>
</table>
The pure technical efficiency reflects the managerial performance to organize inputs of the blowing machine. For each $DMU_j$, the estimated optimal $PTE$ scores; $\gamma_j^*$ and $\theta_j^*$, are calculated using Eq. (7) and (10), respectively. The $CV_j$ % and $CV_t$ % values reveal the lack of existence of less dispersion and trend in the $PTE$ scores for all $DMU_j$s and periods. In Table 3, it is noted that two ($DMU_2$ and $DMU_3$) out of the seven $DMU_j$s are found pure technically-efficient. On the other hand, it is found that eight out of the twelve periods are concluded pure technically-efficient. Compared with the technical efficiency values in Table 2, it is found that the $\gamma_j^*$ and $\theta_t^*$ are larger than their corresponding $\gamma_j^{*}$ and $\theta_t^{*}$, respectively. Further, the $\gamma_j^* (= 0.9998)$ for $DMU_j$ indicates the same level of output could be produced by 99.98% of the recourses taking into consideration that the scale size is ignored, in addition 0.02% of all recourses could be saved by raising the performance of the machine to the highest level.

Table 2. The estimated optimal TE scores.

<table>
<thead>
<tr>
<th>Time period $t$</th>
<th>$DMU_j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>$\theta_j^*$</th>
<th>CV%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DMU_1$</td>
<td>0.9575</td>
<td>0.7851</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.9533</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9493</td>
<td>8.78%</td>
</tr>
<tr>
<td>$DMU_2$</td>
<td>0.7851</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.9533</td>
<td>1.0000</td>
<td>0.9544</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9488</td>
<td>8.78%</td>
</tr>
<tr>
<td>$DMU_3$</td>
<td>0.9528</td>
<td>1.0000</td>
<td>0.9533</td>
<td>1.0000</td>
<td>0.9510</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9762</td>
<td>2.67%</td>
</tr>
<tr>
<td>$DMU_4$</td>
<td>0.9944</td>
<td>0.9533</td>
<td>1.0000</td>
<td>0.9497</td>
<td>0.9817</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9799</td>
<td>2.34%</td>
</tr>
<tr>
<td>$DMU_5$</td>
<td>0.9638</td>
<td>1.0000</td>
<td>0.9497</td>
<td>0.9817</td>
<td>1.0000</td>
<td>0.9990</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9740</td>
<td>2.37%</td>
</tr>
<tr>
<td>$DMU_6$</td>
<td>0.9638</td>
<td>0.9830</td>
<td>1.0000</td>
<td>0.9721</td>
<td>0.9926</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9763</td>
<td>2.12%</td>
</tr>
<tr>
<td>$DMU_7$</td>
<td>0.9575</td>
<td>0.9843</td>
<td>0.9986</td>
<td>0.9533</td>
<td>1.0000</td>
<td>0.9531</td>
<td>0.9856</td>
<td>1.0000</td>
<td>0.9634</td>
<td>0.9861</td>
<td>1.0000</td>
<td></td>
<td></td>
<td>0.9853</td>
<td>1.52%</td>
</tr>
</tbody>
</table>

| $\gamma_j^*$ | 0.9990 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |       |   |   |   |     |     |   | 0.9998  | 0.04% |
| $\theta_t^*$ | 0.9990 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |       |   |   |   |     |     |   | 0.9998  | 0.04% |

Table 3. The estimated optimal PTE scores.
Table 4 lists the optimal SE efficiency scores; $\eta_{j,pj}^*$. The scare efficiency score, $\eta_j^*$, and the corresponding $CV_j$% are then calculated. It is found that all $\eta_j^*$ are less than one for all DMU$^j$s. However, the $\eta_j^*$ scores are equal to one for eight periods, and hence the size is optimal at these periods.

Table 4. The estimated optimal SE scores.

<table>
<thead>
<tr>
<th>DMU$^j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>$\eta_j^*$</th>
<th>CV%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU$^1$</td>
<td>0.9584</td>
<td>0.7851</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.9533</td>
<td>1.0000</td>
<td>0.9495</td>
<td>8.78%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DMU$^2$</td>
<td>0.7851</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.9533</td>
<td>1.0000</td>
<td>0.9544</td>
<td>0.9488</td>
<td>8.78%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DMU$^3$</td>
<td>0.9528</td>
<td>1.0000</td>
<td>0.9533</td>
<td>1.0000</td>
<td>0.9510</td>
<td>1.0000</td>
<td>0.9762</td>
<td>2.67%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DMU$^4$</td>
<td>0.9944</td>
<td>0.9927</td>
<td>1.0000</td>
<td>0.9975</td>
<td>0.9917</td>
<td>1.0000</td>
<td>0.9944</td>
<td>0.69%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DMU$^5$</td>
<td>0.9927</td>
<td>1.0000</td>
<td>0.9975</td>
<td>0.9917</td>
<td>1.0000</td>
<td>0.9590</td>
<td>0.9485</td>
<td>1.62%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DMU$^6$</td>
<td>1.0000</td>
<td>0.9975</td>
<td>0.9950</td>
<td>1.0000</td>
<td>0.9590</td>
<td>0.9544</td>
<td>0.9872</td>
<td>1.61%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DMU$^7$</td>
<td>0.9638</td>
<td>0.9830</td>
<td>1.0000</td>
<td>0.9769</td>
<td>0.9955</td>
<td>1.0000</td>
<td>0.9634</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Generally, the PTE and SE scores provide an indication for the reason behind the inefficiency in the TE, $\tilde{\eta}_j^*$, values for each DMU. For DMU$^j$, if the PTE inefficiency, $\eta_j^*$, value is larger than its corresponding SE inefficiency score, $\tilde{\eta}_j^*$, then the reason behind the $\tilde{\eta}_j^*$ is managerial. However, if the $\eta_j^*$ value is smaller than its corresponding $\tilde{\eta}_j^*$, then the reason of the $\tilde{\eta}_j^*$ is the size of operation. Finally, the size of operation is optimal when $\eta_j^*$ and $\tilde{\eta}_j^*$ scores are equal.

(2) MPI Analysis for Blowing Machine

The Malmquist productivity index (MPI) is calculated and displayed in Table 5, in which four evaluation periods each of a planning horizon of three months are considered. For illustration, the $\theta_1^{1L}(E^1)$ value of 0.5557 is calculated as follows Eq. (13):

$$\theta_1^{1L}(E^1) = \min \delta_1$$

Subject to:

$$\delta_1 x_{1,1}^{1U} - \sum_{e=1}^{4} \lambda_{1,e} x_{1,1,e}^{1L} - \lambda_{1} x_{1,1}^{1U} \geq 0,$$

$$\delta_1 x_{2,1}^{1U} - \sum_{e=1}^{4} \lambda_{2,e} x_{2,1,e}^{1L} - \lambda_{2} x_{2,1}^{1U} \geq 0.$$
\[ \delta x_{1,3,1}^U - \sum_{e=1, e \neq 2}^4 \lambda x_{1,3,e}^L - \lambda x_{1,3,1}^U \geq 0. \]

\[ y_{1,1}^L - \sum_{e=1, e \neq 2}^4 \lambda y_{1,1,e}^L - \lambda y_{1,1,1}^L \geq 0. \]

\[ \lambda_e \geq 0, \quad e = 1, \ldots, 4 \]

The \( \theta_{1,U}^L (E^1) \) of 0.5557 is estimated by solving Eq. (14). The other \( \theta_{e,L}^L (E^p) \) and \( \theta_{e,U}^L (E^p) \) are estimated similarly. Next, the \( \theta_{e,L}^{p+1,L} (E^{p+1}) \) and \( \theta_{e,U}^{p+1,U} (E^{p+1}) \) are estimated by replacing \( p \) by \( p+1 \). On the other hand, the efficiency of period \( p+1 (=2) \) in evaluation period \( e' = 1 \), \( \theta_{1,L}^{p+1,L} (E^1) = 0.6474 \), is calculated by solving Eq. (15):

\[ \theta_{1,L}^{2,L} (E^1) = \min \omega \]

Subject to:

\[ \omega \cdot x_{1,1,1}^U - \sum_{e=1}^4 \lambda x_{2,1,e}^L \geq 0, \]

\[ \omega \cdot x_{2,1,1}^U - \sum_{e=1}^4 \lambda x_{2,2,e}^L \geq 0, \]

\[ \omega \cdot x_{3,1,1}^U - \sum_{e=1}^4 \lambda x_{3,3,e}^L \geq 0, \]

\[ y_{1,1,1}^L - \sum_{e=1}^4 \lambda y_{1,1,e}^L \geq 0, \]

\[ \lambda_e \geq 0, \quad e = 1, \ldots, 4 \]

The \( \theta_{1,U}^{2,U} (E^1) = 1.0749 \) are calculated by solving Eq. (16). The other \( \theta_{e,L}^{p+1,L} (E^p) \) and \( \theta_{e,U}^{p+1,U} (E^p) \) are calculated in a similar manner. Next, the \( \theta_{e,L}^{p+1,L} (E^{p+1}) \) and \( \theta_{e,U}^{p+1,U} (E^{p+1}) \) are estimated similarly. Finally, the \( MPI_{e,L}^L \) and \( MPI_{e,U}^U \) are calculated using Eqs. (17) and (18), respectively. Table 6 displays the estimated values of \( MPI_{e,L}^L \) and \( MPI_{e,U}^U \) with the corresponding technological and efficiency changes.

Table 5. The results of the MPI components.

<table>
<thead>
<tr>
<th>Evaluation period</th>
<th>( p )</th>
<th>( \theta_{e,L}^{1,L} (E^p) )</th>
<th>( \theta_{e,L}^{1+1,L} (E^{p+1}) )</th>
<th>( \theta_{e,L}^{p,L} (E^p) )</th>
<th>( \theta_{e,U}^{p+1,L} (E^{p+1}) )</th>
<th>( \theta_{e,U}^{p,L} (E^p) )</th>
<th>( \theta_{e,U}^{p+1,U} (E^{p+1}) )</th>
<th>( \theta_{e,U}^{2,U} (E^p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e )</td>
<td>1</td>
<td>0.5557</td>
<td>0.4088</td>
<td>0.6474</td>
<td>0.2684</td>
<td>1.0000</td>
<td>0.8393</td>
<td>1.0749</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.4088</td>
<td>0.6616</td>
<td>0.2109</td>
<td>0.3901</td>
<td>0.8393</td>
<td>1.0000</td>
<td>1.0548</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.6616</td>
<td>0.9218</td>
<td>0.7141</td>
<td>1.2967</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0743</td>
</tr>
</tbody>
</table>

Table 5 continued on next page
In order to determine the sources of \(MPI\) regress, the components of the \(MPI\); efficiency change and technology change, were calculated as also shown in Table 6.

In Table 6, it is noted that \(MPI^{L,e,p}\) (worst) is only larger than one in three periods; 3, 6, and 9, which indicates a progress. However, the \(MPI^{U,e,p}\) (best) is only smaller than one in period 5, which indicates a regress in productivity from time \(p\) to \(p+1\).
4. RESEARCH RESULTS

For window analysis, the inefficiency scores for all seven DMUs were calculated and then displayed in Table 7 and Fig. 1. It is clear in Fig. 1 that the main reason behind the $\tilde{\theta}_j^*$ for five DMUs is contributed by the size of operations; scale inefficiency. However, the source of $\tilde{\theta}_j^*$ for two DMUs is contributed by management. Further, the inefficiency scores for all months were estimated and then shown in Table 8 and Fig. 2, where it is found that the scale is optimal in only three months; $t = 6, 9,$ and 12, out of the twelve months. Moreover, the technical inefficiency, $\tilde{\gamma}_t^*$, is attributed by scale inefficiency, $\tilde{\eta}_t^*$, in eight months. Finally, the pure technical efficiency, $\tilde{\gamma}_j^*$, is the reason behind the $\tilde{\theta}_j^*$ in one month, $t = 7$.

Table 7. The estimated inefficiency scores for DMUs.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$\theta_j^*$</th>
<th>$\gamma_j^*$</th>
<th>$\eta_j^*$</th>
<th>$\tilde{\theta}_j^*$</th>
<th>$\tilde{\gamma}_j^*$</th>
<th>$\tilde{\eta}_j^*$</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DMU_1$</td>
<td>0.9493</td>
<td>0.9998</td>
<td>0.9495</td>
<td>0.0507</td>
<td>0.0002</td>
<td>0.0505</td>
<td>Scale</td>
<td></td>
</tr>
<tr>
<td>$DMU_2$</td>
<td>0.9488</td>
<td>1.0000</td>
<td>0.9488</td>
<td>0.0512</td>
<td>0.0000</td>
<td>0.0512</td>
<td>Scale</td>
<td></td>
</tr>
<tr>
<td>$DMU_3$</td>
<td>0.9762</td>
<td>1.0000</td>
<td>0.9762</td>
<td>0.0238</td>
<td>0.0000</td>
<td>0.0238</td>
<td>Scale</td>
<td></td>
</tr>
<tr>
<td>$DMU_4$</td>
<td>0.9799</td>
<td>0.9854</td>
<td>0.9944</td>
<td>0.0201</td>
<td>0.0146</td>
<td>0.0056</td>
<td>Management</td>
<td></td>
</tr>
<tr>
<td>$DMU_5$</td>
<td>0.9740</td>
<td>0.9854</td>
<td>0.9885</td>
<td>0.0260</td>
<td>0.0146</td>
<td>0.0115</td>
<td>Management</td>
<td></td>
</tr>
<tr>
<td>$DMU_6$</td>
<td>0.9783</td>
<td>0.9911</td>
<td>0.9872</td>
<td>0.0217</td>
<td>0.0089</td>
<td>0.0128</td>
<td>Scale</td>
<td></td>
</tr>
<tr>
<td>$DMU_7$</td>
<td>0.9853</td>
<td>0.9993</td>
<td>0.9860</td>
<td>0.0147</td>
<td>0.0007</td>
<td>0.0140</td>
<td>Scale</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. The inefficiency values for all DMUs.
In practice, to resolve the scale inefficiency due to size of operations, planning engineers may reschedule the allocation of the customer orders, size of inventory, or purchase new production machines. Further, pure technical inefficiency was caused because some DMUs resources were managed inefficiently. This problem can be addressed by increasing the actual production quantity by utilizing the available resources, adopting an effective quality control system to reduce defects quantity, and improving planning and machine reliability to reduce idle time.

For $MPI$ analysis, on the other hand, the change ($= MPI_{t',p}^U - MPI_{t',p}^L$) in $MPI$ values are depicted in Fig. 3. Clearly, there is large change (say; greater than 1.5) in $MPI$ values for periods 1, 2, 6, and

![Figure 2. The inefficiency values for all months.](image-url)
11 when each is compared to its following period. Fig. 4 displays the comparison between the lower and upper values for each of technology change and efficiency change. It is noted that differences between the upper and lower values does not exceed one in most at most time periods in both figures of technology change and efficiency change. This figure can provide useful information about the worst to best changes in technology and efficiency change. Further, Fig. 5 depicts the MPI and its components at the upper and lower bound, where it is found that whether the regress or progress in MPI was due to technological change or efficiency change. For example, at the lower bound of MPI at period 4, the reason behind MPI progress was the technological change. Similarly, the reason behind the MPI regress at the upper bound of period 5 was due to technological change. Such information provides valuable feedback about worst to best efficiency changes and technology changes from period \( p \) to \( p+1 \), and support decision makers in identifying the proper actions to enhance efficiency and/or introduce new technology to enhance MPI.

Figure 3. The lower and upper MPI values.

Figure 4. The comparison between lower and upper values for each of technology and efficiency changes.
5. CONCLUSION

This study proposed a procedure for window analysis followed by the Malmquist productivity index in DEA to assess the efficiency and productivity of manufacturing processes under fuzzy inputs and outputs. The proposed procedure was illustrated to measure efficiency and productivity of a blowing machine. For this process, the production quantity was the output, whereas the planned production quantity, defect quantity, and idle time in units were the inputs for all windows. Then, the technical, pure technical, and scale efficiencies were calculated using the proposed optimization models in window analysis. The sources for technical inefficiency were identified for each decision making unit and each period. Next, the lower and upper MPI values with the corresponding technology change and efficiency change were calculated to identify the reason behind productivity progress or
recess in each period. The results of DEA window analysis showed that the main cause of technical inefficiency in blowing machine was the scale inefficiency. Hence, there is a need to optimize the size of operations. From productivity analysis, the $MPI$ values indicated progress in productivity at three periods; 3, 6, and 9, whereas a recess in productivity was indicated at period 5. Such analysis provides valuable guidance on which $MPI$ component to be enhanced. In conclusion, the proposed procedure can provide great assistance to decision maker when evaluating the efficiency and productivity of the manufacturing process and guide them to the proper actions to enhance its performance.

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REFERENCES


Abbas Al-Refaie received his Ph.D. degree in Industrial Engineering and Systems Management in June, 2008. He started his academic career as assistant professor of industrial engineering in Sep. 2008. Thereafter, he was promoted to the rank of Associate Professor in Sept. 2012 and to the rank of Full Professor of Industrial Engineering in Sept. 2016. In 2014, he received the UJ’s Distinguished Researcher Award in Science for years 2012 and 2013. He also received the 2018- Ali Mango award for distinguished researcher in science in Jordan. His students won the first rank in the competition of applied projects in Jordan industry for years 2014, 2016, 2017, and 2018. His research focuses on experimental design, quality engineering, optimization, and engineering management.