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Prediction of Nurses Allotment to Patient in Hospital through Game Theory

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ABSTRACT

Allotment of nurses to patients is a critical task in terms of better treatment. Nurses should be appointed according to a patient's health condition, type of disease, and financial condition. Again, understaffing of nurses may hamper patient health and condition. Similarly, overstaffing of nurses is a waste of manpower. Adequate staffing of nurses is crucial. The authors propose a technique using game theory to meet overstaffing and understaffing of nurses. Game theory plays a vital role to meet the exact requirement. Nash equilibrium can be used for taking all possible decisions, like appointment of nurses in different categories for smooth treatment of patients. However, the final and most suitable decision can be taken using perfect Nash equilibrium. This technique is a perfect technique to implement in case of vital and critical decision points.

KEYWORDS

Nash Equilibrium, Perfect Nash Equilibrium, Prediction, Sub Game Perfect Equilibrium

1. INTRODUCTION

Patient's level of satisfaction with hospital care is to be considered if they were to visit the hospital. Nurses play a vital role in taking care of patients by providing services such as on-time-injection, cleaning, blood pressure check-up and sort out different problems faced by patients. So, sincere nurses are important to the hospital and patients. Since different types of the patient come, so each patient may not require the same number of nurses or the same type of nurses. Different type of nurses may be required for different types of diseases e.g., Cardiac patients require nurses, an expert in dealing with cardiac-related ailments. Again, according to the availability of doctors and patient's conditions, different types of nurses can be assigned. It is unnecessary to assign nurses with no experience in a specific department and overqualified (Rafii, F., Hajinezhad, M. E., & Haghani, H., 2008). It is also unnecessary to assign nurses with over experience to normal patients. Thus, nurses are assigned to patients according to their educational qualification & experience together with suitability match for patients.

Generally, the main aim of the patient is to get effective care and satisfactory services. The factors for a patient emphasise for visiting the hospital is treatment and proper nursing care (Azizi-Fini, I., Mousavi, M. S., Mazroui-Sabdani, A., & Adib-Hajbaghery, M., 2012). There are various grades of

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nurses ranging from registered nurses to junior nurses. Because of varied training and specializations, specific types of nurses are to be staffed for a word, requiring specific skills. The salary of the nurses is considered according to their qualification, experience and performance. From a patient's point of view, the cost of the nurses will be considered according to the type of service they provide and how much time they serve. Time may be hourly or per day or monthly (McGivern, S., 1999).

According to Robert J. Aumann, a Game theory is an optimal decision-making process in the presence of others with different objectives (Hart, S., 2006). Game theory is a mathematical theory of interactive decision-making process based on situations. These situations are characterized by a group of agents where each agent has to make a decision, an outcome that results as a function of the decisions of all agents and each agent has his preferences on the set of possible outcomes. A mathematical decision theory (on the basis to develop game theory) is to be developed for situations in which several decision-makers interact. Decision theory deals with problems and in a decision problem, there is a decision-maker who has to choose one or more alternatives out of a set A (say) (Edwards, W., 1954). The decision-maker has to consider over A, which are usually modelled through a binary relation RÌA×A referred to as "alternative relation" in our context. Here aRb is interpreted as the decision-maker either prefer an over b or is indifferent between a and b. Requirements are imposed on R as follows:

R is symmetric i.e. a R b & b R a
R is transitive i.e. a R b, b R c implies a R c

The Nash Equilibrium is a concept of game theory, where the optimal outcome of a game is no player has an incentive to deviate from his chosen strategy after considering an opponent's choice. Overall an individual can receive no incremental benefit from changing actions assuming other players remain constant in their strategies. A game may have multiple Nash Equilibrium or none at all (Myerson R. B., 1978). In the Nash Equilibrium, we may not reach an equilibrium point. So perfect equilibrium may be used to reach an equilibrium point. Here sub-game is found and of that sub-game, Nash equilibrium is to be found called subgame perfect equilibrium (Fudenberg, D., & Levine, D., 1983).

- **Strategies game:** An n-player strategic game with a set of players N is a pair G = (A, u) whose elements are defined in the following ways.
- **Sets of strategies:** For each i∈N, A_i is to the nonempty set of strategies of player i and A = $\prod_{i=1}^{n} A_i$ is the strategy profiles.
- **Payoff functions:** For each i ∈ N, u_i: A → R is the payoff function of player i and u:= $\prod_{i=1}^{n} u_i$; u_i assigns to each strategy profile a ∈ A, the payoff that player i gets if a is played.
- **Nash Equilibrium:** A Nash equilibrium of G is a strategy profile a* ∈ A such that, for each i ∈ N and each $\hat{a}_i \in A_i$, u_i (a*) ³ u_i (a_{-i}*, $\hat{a}_i$).
- **Mixed strategies of the game:** G (A, u) is a finite game if for each i ∈ N, $|A_i| < \infty$ which may not have Nash equilibrium. By enlarging the strategic possibilities of the players and allow to choose from the strategies, there is a guarantee of the existence of Nash equilibrium. This extension of the original game is called mixed extension and the strategies called mixed strategies.
- **Perfect Nash Equilibrium:** Perfect equilibrium is one of the most important refinements of Nash equilibrium. The idea underlying is to select those Nash equilibria which are still in equilibrium even in the case that player might make mistakes when choosing their strategies. Perfect equilibrium is also known as trembling hand perfect equilibrium.

Let G be a finite game and E(G) be its mixed extension. A strategy profile $s \in S$ is a perfect equilibrium of E(G) if there are two sequences $\{\eta^k\} \subset T(G)$ where T(G) stands for trembles in G, with $\{\eta^k\} \to 0$ and $\{s^k\} \subset S$ with $\{s^k\} \to s$ such that, for each $k \in N$, s^k is a Nash equilibrium of (G, η^k).

Derivation of Perfect Nash Equilibrium from Nash Equilibrium: As mention above, the definitions are concluded that we must find Nash equilibrium in a mixed-strategy game if it is not an in-game strategy. Again mixed strategy game has at least one perfect equilibrium.

(G, u) is a mixed game strategy where u is defined as follows.

u: $(N \times N \times N) \to R$, where N is a set of natural numbers and R is a set of real numbers. $u(n_1, n_2, n_3) = r_1 n_1 + r_2 n_2 + r_3 n_3 \leq b$, r_1, r_2, r_3 & b are some constants, $n_1, n_2, n_3 \geq 0$ & belongs to N. $n_1 \leq C_1$, $n_2 \leq C_2$, $n_3 \leq C_3$ for some constants C_1, C_2 & C_3. Assume $u = \{u_1, u_2, u_3, u_4, \ldots, u_k\}$ is a set of real values for which $u_i(n_1, n_2, n_3) \leq b$.

Thus, we have k numbers of Nash equilibrium. It is possible to find the values of n1, n2, n3 for which ui (n1, n2, n3) = b and meet the Perfect equilibrium.

2. LITERATURE SURVEY

In this section, we review related literature for the project "Perfect Services of Nurses Provided by Hospital using Game Theory" and Table 1 illustrates the summary.

Availability of medicines and health care in low-income and lower-middle-income countries is an important factor for determining to access medicines and health care. This classification is as

Table 1. Literature survey of papers on perfect services of nurses provided by the hospital using game theory

Study	Objective
Patient access to health care and medicines across low-income countries Srivastava, (D., & McGuire, A., 2015).	Discrete choice model is used to access of medicines by patients, healthcare and also focus on estimating price responsiveness.
Challenges in researching violence affecting health service delivery in complex security Environments (Foghammar, L.,2016)	In complex security environments, violence is a major factor that affects healthcare services i.e. attacks on doctors, nurses, administration, security guards, ambulance drivers and discrimination.
The relationship between perceived leadership style and perceived stress on hospital employees (Baysak, B., & Yener, M. İ., 2015).	Stress has a major impact on hospital employees and so to select an appropriate leadership style which has fewer negative effects of stress on employees is very important.
Providing healthcare services on-the-fly using multiplayer cooperation game theory on the Internet of Vehicles (IoV) environment (Kumar, N., Kaur, K., Jindal, A., & Rodrigues, J. J.,2015).	It is an E-healthcare model based on game theory for providing priority to the patient. The cloud computing platform is used to give services in IoV environment on-the-fly.
A solution to the nurse scheduling problem in Hospital Management using linear programming is discussed (Patidar, M., & Choudhary, S., 2016).	A linear programming technique is used for scheduling nurses and avoid non-essential cost for the hospital.
Using game theory for medical resources scheduling in the emergency department is discussed (Wu, C. K., 2014).	The security risk values are derived for each type of emergency event by using the Nash Equilibrium. Then Shapley value is derived based on three threat levels using these values. This Shapley value helps in managing medical resources.
Service capacity planning of the community hospital under the hierarchical medical system (Zhou, Y., Zhou, W., Zhang, S., & Wan, Q., 2016).	A patient can select any hospital for treatment. A hierarchical medical system has community and Class 3-A hospital. Class 3-A hospital is better and most visited than a community hospital. Here considers delay –sensitivity of the patient.
Co-operative games in an integrated system with multiple hospitals (Luo, M., & Cai, X., 2016).	In case of emergencies, a cooperative game between the hospitals depicts that there can be an allocation which is measured by using linear programming duality theory.
Assessment of patient-physician assignment criteria in the emergency department using Discrete-Event-Simulation (Cildoz, M., Mallor, F., Azcarate, C., & Ibarra, A., 2016).	By developing a discrete-event-simulation model to assess different PPARs (Patient-Physician assignment rule) and optimizing wait time of patients and physician's workload variability (WV).

per the calculation of price-elasticity (Srivastava, D., & McGuire, A.,2015. The key challenges in dealing with health care services delivery in complex security environments may include lack of health-specific, accessible and comparable, gender-disaggregated data (Foghammar, L., 2016). Birol Baysak explains how to prevent other problems caused by stress and reduce its negative impact on an individual's emotional and physical condition (Baysak, B., & Yener, M. İ., 2015).

Neeraj Kumar proposed a model based on game theory. It is presented as a game amongst the vehicles to prioritize patients who can access the services. The formation of coalition and splitting algorithms are designed for each player in the game. A Stochastic reward net (SRN)-based model is used for representing different states and transitions of the players. These players make adaptive decisions based on proposed learning automata (LA)-based game model. Finally, a virtual machine (VM) scheduling algorithm is designed for efficient resource allocation to serve vehicles request at cloud level (Kumar, N., Kaur, K., Jindal, A., & Rodrigues, J. J., 2015). The allocation of nurses is a major factor in hospitals. The complexity of a problem is high when other components such as hospital administration related issues are added. Some other factors that are taken into consideration in case of nurses are night shift, consecutive rest days. Nurse roistering is done to some extent using linear programming tool (Patidar, M., & Choudhary, S., 2016).

Length of stay (LOS) is a major component of patient feedback. Some issues may arise if LOS is high (inefficient use of emergency resources causes a delay in the treatment of some patients or they leave without treatment, thereby putting patient safety in danger). So, to avoid this, two of the game theory models are designed. In the first model of the non-cooperative game, there is a communication between the patient and administrator of the emergency department. By using Nash Equilibrium, security risk value is derived for each emergency event (surgical, paediatric, independent emergency, medical). In a second cooperative game, Shapley value is derived based on three threat levels by collecting all security risk values of those four emergency events, based on the Shapley values, the administrator can manage medical resources (bed, doctor, nurses) allocation thereby improving patient welfare in emergency ward (Wu, C. K., 2014). Due to the lack of resource utilization, the medical system has a major problem between supply and demand. There are fewer patients in community hospitals since the Class 3-A hospital is trusted by the public because of better treatment facilities. To optimize resource utilization and distribute patients rationally, social cost should be minimized. Social cost includes patients waiting for time cost and the hospital's service cost (Zhou, Y., Zhou, W., Zhang, S., & Wan, Q., 2016).

Waiting issues can be handled using the concept of inter-hospital collaborations. There is a possibility of a collaboration of hospitals that can be developed. This can help in utilizing the resources available in other hospitals. The hospitals can help each other and reach a service level that can maximize their revenue (Luo, M., & Cai, X., 2016). The discrete event simulation model to verify doctor-patient alignment policies with multiple queues structure. Here the objective is to optimize arrival to provider time (ATPT) and to optimize physician's workload balance. ATPT measures the quality of service where WV refers to the quality of work and balancing (Cildoz, M., Mallor, F., Azcarate, C., & Ibarra, A., 2016). The mental conditions in people may be raised due to ageing, the family's poor financial condition and inactive. To prevent the issue, develop meaningful events in their free time, increase welfare facilities, participate in events (Hosseini, S. V., & Tahrekhani, M.,2015).

Finally, we have found out that in medicine, nurses' allotment to the patients is a major issue for management of the hospital with the limited budget (Fast, O., & Rankin, J. (2018)). Attending nurses to the patients have influenced both patients and their family's satisfaction, for which scheduling processes are implemented for the nurses' allotment (Kipps, A. K., Albert, M. S., Bomher, S., Cheung, S., Feehan, S., & Kim, J. (2020)). We have proposed game theory approaches i.e., Nash Equilibrium and Perfect Nash Equilibrium for allotment of nurses to the patients considering patients justify treatments, both patients and their families' satisfaction, and the budget of hospitals.

3. PROPOSED WORK

The patient must satisfy with the services of the nurse's allotments. They can get better treatment if they are served according to the requirements of the nurses. Distribution to the nurses of a hospital to different departments is a critical task for management and our proposed work may help in this ground in the following way.

Nurses are categorized as C1, C2, C3 where:

C1: M.Sc. nursing or BSc nursing with more years' experience.
C2: B.Sc. nursing with fewer years or no experience.
C3: Experience in nursing works after 10th class or 12th class.

We have a different department to treat different types of patients like cardiac, orthopaedics etc. Basing on the above categories and departments in the hospital, we can group the nurses as follows.

Assume N as the set of all nurses, i.e. N = {N1, N2, N3, N4, ..., Nk}. Each department, three types of nurses are assigned as defined below.

1. j is denoted as the department in hospital, j = 1 to n & n is the numbers of departments.
2. Initially, C_1 [j] = Ø, C_2 [j] = Ø, C_3 [j] = Ø.
3. C_1 [j] = C_1 [j] U {N_i}. If N_i is M.Sc. nurse or B.Sc. nurse with k years' experience in department j.
4. C_2 [j] = C_2 [j] U {N_i}. If N_i is B.Sc. nurse with less than k years or no experience in department j.
5. C_3[j] = C_3[j] U {N_i}. If N_i is a nurse with some training & experience in department j after schooling.

Department contains different types of nursing works and those are performed by nurses but all nurses may not do all nursing works perfectly. So, we have defined it as follows:

- Ni (wi1, wi2, wi3, ..., wik) identifying that the nurse Ni can do works wi1, wi2, wi3, ..., wik, where i = 1 to k.
- A relation R between Nurses establishes in the following way.
- Ri= {(Nα, NB) | wαk = wBl for some works defined wαk, wBl}
- M1, M2, M3...Mk is subsets of N where Mi = {Ni| Ni can do works wx, wy, wz, ...,wk}& Ri is defined on Mi for i = 1 to k.

The above problem can be interpreted as a mixed game strategy where players are patients with strategy as the assignment of nurses. We can define a utility function based on the criteria of the players, Nash Equilibrium is generated and if required Perfect Equilibrium is pointed from the Nash Equilibrium.

3.1 The Solution Model

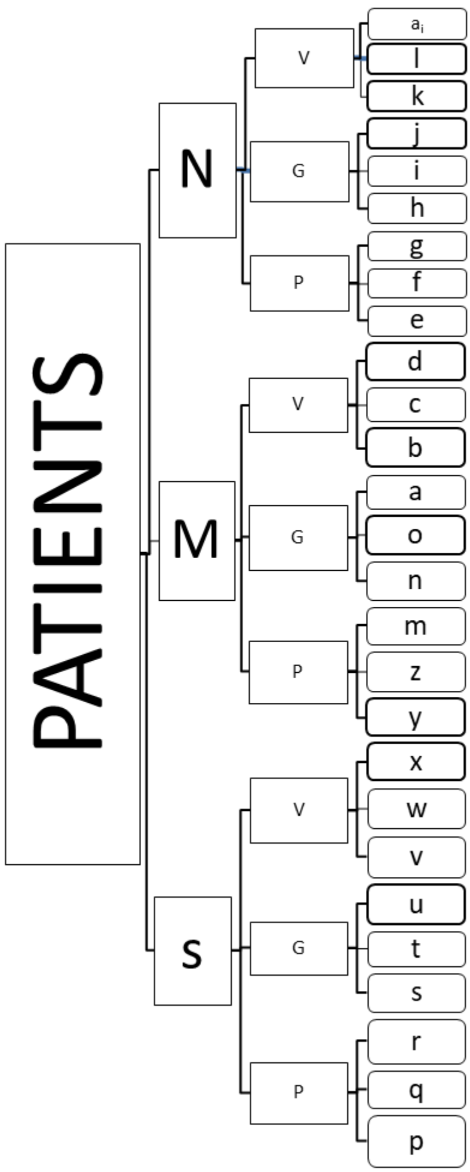
Nurses will be assigned according to the following considerations:

- Financial conditions of the patients.
- Disease type and doctors' opinions.
- Availability of nurses.

So according to the financial conditions of patients and the type & condition of disease, the patient's categories as very good(V), good(C), and poor (P) and categories as serious(S), medium (M) and normal (N) respectively. The number of nurses with different categories are p, q, r, s, t, u, v, w, x, y, z, m, n, o, a, b, c, d, e, f, g, h, i, j, k, l, ai (the natural numbers). Now we define the payoffs for all possible assignments and we have represented it through the tree as in Figure 1.

- **C1(j):** Stands for a total number of nurses (category C1) in department j.
- **C2(j):** Stands for a total number of nurses (category C2) in department j.
- **C3(j):** Stands for a total number of nurses (category C3) in department j.
- **P (i, j):** Stands for the patient with a serial number i is admitted in department j.
- **C1(i, j):** Stands for the number of nurses (category 1) of department j is assigned to a patient number i.

Figure 1. Nurses allotment to the patients according to the financial conditions of the patients' disease type and doctors' opinions



- **C2 (i, j):** Stands for the number of nurses (category 2) of department j is assigned to a patient number i.
- **C3(i, j):** Stands for the number of nurses (category 3) of department j is assigned to the patient number i.

If a new patient comes to admit in hospital, we have to check whether sufficient nurses are available to meet the requirement of the patient. From the tree structure graph (Fig 1), we can find out all possible ways that nurses can be assigned to the patient. The multi-stages situations can be represented as in Table 2.

Now, we consider the assignment of the nurses to the patient and Nash Equilibrium is applied. N represents the set of nurses. When ni = (NC1, NC2, NC3) nurses assign to P (i, j), we have:

$$u\left(n_i\right) = \begin{cases} f_i, & if\ f_i \leq budget\ of\ the\ patient\ p_i\ and\ equations\ 1,2,3\ satisfy \\ 0, & otherwise \end{cases}$$

$$f_i = R_i \times NC_1 + R_2 \times NC_2 + R_3 \times NC_3$$

where:

R1: Stands for the cost of C1 type nurses. NC1: stands for the number of C1 type nurses.
R2: Stands for the cost of C2 type nurses. NC2: stands for the number of C2 type nurses.
R3: Stands for the cost of C3 type nurses. NC3: stands for the number of C3 type nurses.

The costs R1, R2, R3 of nurses are fixed hourly or daily or monthly.

Assignment of nurses to a patient is possible if the following three conditions satisfy:

$$\sum\nolimits_{i=2,j=1} NC\left(i-1,j\right) + NC\left(i,j\right) \leq NC\left(j\right) \tag{1}$$

$$\sum\nolimits_{i=2,j=1} NC\left(i-1,j\right) + NC\left(i,j\right) \leq NC\left(j\right) \tag{2}$$

$$\sum\nolimits_{i=2,j=1} NC\left(i-1,j\right) + NC\left(i,j\right) \leq NC\left(j\right) \tag{3}$$

Table 2. Multi-Stages situations

	S	M	N
V	(p, q, r)	(s, t, u)	(v, w, x)
P	(y, z, m)	(n, o, a)	(b, c, d)
G	(e, f, g)	(h, i, j)	(k, l, a_i)

We may have more than a possible subset of nurses' assignment to the patient using Nash Equilibriums. But practically one subset of nurses is required for the patient. Hence Perfect Nash Equilibrium is applied. The following function is used for the perfect Nash equilibrium:

$$V\left(X,Y\right) = f_i, where\ X \in \left\{S,M,N\right\}, and\ \ Y \in \left\{L,C,E\right\}$$

$$f_i = R_i \times NC_1 + R_2 \times NC_2 + R_3 \times NC_3, the\ values\ of\ NC_1, NC_2, NC_3\ depends\ on\ X\ and\ Y$$

Accordingly, the value of V (X, Y), (NC_1, NC_2, NC_3) will be assigned to P (i, j).

4. CASE STUDY

In a hospital, there is a department called department j (j=2), i.e. cardiac department. In the department, the available nurses of categories C1 (2) =11, C2 (2) = 21 & C3 (2) = 36.

Suppose ten patients are suffering in the heart problems & have to admit in the cardiac department. With the treatment of doctors, nurses are required to attend the patients. Therefore, nurses will be allotted to patients. We also consider some different categories of nurses to be assigned.

Assume the ten patients are admitted with serial numbers 1, 2,3,4,5,6,7,8,9,10. When the patients or attendance of patients enter the hospital, the specific information has been collected as summarised in Table 3.

After an initial check-up of the patients and analysing the disease type, all patients are admitted in the cardiac department, i.e. j=2 department. In department 2(cardiac), doctors detect the information as in Table 4 about the patients.

S stands for very serious, M stands for serious & N stand for suffering but not serious. Doctors consider the number of nurses assigned to a patient. According to the disease & condition of the patient, different categories of nurses are to be assigned which is considered by doctors. Management has to consider the financial condition of the patient and negotiating with the doctors according to the status of the patient. The committee of management also considers the number of different categories

Table 3. Patients Information at the time of entering to Hospital

Sl No.	Name	Gender	Age	Address	Disease Type
1	Avijit Nayak	M	52	L/123, BHBC, BBSR.	Chest pain
2	Susant Panda	M	46	Plot no-18/120, Basudevpur, BHK.	Left side pain
3	Devika patra	F	39	MIG-53, CHBC, BBSR.	Breathing problems
4	Sumita Mohanty	F	42	Plot no-18/614, Siripur, BBSR.	Left hand pain& chest pain
5	Lipika Dash	F	29	Plot no-12/565, Mandarapur, Jajpur.	Chest pain
6	Aditya Roy	M	75	E/34, DHBC, BBSR.	Chest pain & breathing problems
7	Aswani Rana	M	67	Plot no-33/965, Khadar, Kujanga.	Chest pain
8	Brajeswari Saha	F	45	Plot no-99/3265, Kushpur, Cuttack.	Breathing problems & left-hand pain
9	Nitisha Ojha	F	32	HIG-72, CHBC, Kendrapada	Chest pain
10	Anup Joshi	M	44	Plot no-18/120, Tihidi, BHK.	Chest pain

Table 4. Patients' Condition according to Doctors' Opinion

Sl No.	Name	Gender	Age	Doctor Opinion
1	Avijit Nayak	M	52	S
2	Susant Panda	M	46	S
3	Devika Patra	F	39	M
4	Sumita Mohanty	F	42	N
5	Lipika Dash	F	29	M
6	Aditya Roy	M	75	S
7	Aswani Rana	M	67	N
8	Brajeswari Saha	F	45	N
9	Nitisha Ojha	F	32	M
10	Anup Joshi	M	44	S

of nurses can be assigned to a patient. Using the Nash Equilibrium, the possible number of nurses with its categories are allotted to patients are described in Table 5.

But from Table 5, it is not possible to conclude for the patients having serial numbers 2, 3, 5,6,7,8 &10, number and types of nurses to be assigned. Hence Perfect Nash Equilibrium is applied. To apply the Perfect Nash Equilibrium, the financial condition of the patients is required & after enquiry we have the information as in Table 6.

Financial condition may be categorised as very good, good & poor for the patients. By applying Perfect Nash Equilibrium, we have clarified the assignments of different types and number of nurses to patients with serial numbers 2, 3, 5, 6, 7, 8 & 10 which are summarized in Table 7.

In Table 7, we have mentioned patients by serial no, the department where patients are admitted (department 2), doctor opinion is interpreted as S for serious, M for not very serious and N for normal & financial condition is assigned as very good or good or poor. Using Nash Equilibrium, we have got all possible ways of assigning nurses and then Perfect Nash Equilibrium is used to eliminate all alternatives and find out an appropriate assignment of nurses with different categories and amounts to the patients. Finally, we have summarized in Table 8 the allotment of nurses to different patients.

Table 5. Assignment of nurses (both in considering type and number) using Nash Equilibrium are summarized for ten patients

Patient Sl No.	Department No.	Doctor Opinion	Nurses Assign (By Nash Equilibrium)
1	2	S	(2, 3, 4)
2	2	S	(3,4,5), (2,4,5), (2,3,4)
3	2	M	(1,2,4), (1,2,4)
4	2	N	(0,1,3)
5	2	M	(1,2,4), (1,2,4), (1,2,4)
6	2	S	(3,4,5), (2,4,5), (2,3,4)
7	2	N	(1,1,3), (1,1,3), (0,1,3)
8	2	N	(1,1,3), (1,1,3), (0,1,3)
9	2	M	(1,2,4)
10	2	S	(2,4,5), (2,3,4)

Table 6. Patients personal information relating to financial condition

Sl No.	Department No.	Name	Gender	Age	Financial Condition
1	2	Avijit Nayak	M	52	Poor
2	2	Susant Panda	M	46	Very good
3	2	Devika Patra	F	39	Good
4	2	Sumita Mohanty	F	42	Poor
5	2	Lipika Dash	F	29	Very good
6	2	Aditya Roy	M	75	Very good
7	2	Aswani Rana	M	67	Very good
8	2	Brajeswari Saha	F	45	Very good
9	2	Nitisha Ojha	F	32	Poor
10	2	Anup Joshi	M	44	Good

Table 7. Allotment of nurses to different patients after applying the Nash Equilibrium & Perfect Nash Equilibrium

Patient Sl No.	Department No.	Doctor Opinion	Financial Condition	Nurses Assign (By Nash Equilibrium)	Nurses Assign (By perfect Nash Equilibrium)
2	2	S	Very good	(3,4,5), (2,4,5), (2,3,4)	(3,4,5)
3	2	M	Good	(1,2,4), (0,2,4)	(1,2,4)
5	2	M	Very good	(1,2,4), (1,2,3), (1,2,4)	(1,2,4)
6	2	S	Very good	(3,4,5), (2,4,5), (2,3,4)	(3,4,5)
7	2	N	Very good	(1,2,3), (1,1,3), (0,1,3)	(1,1,3)
8	2	N	Very good	(1,2,3), (1,1,3), (0,1,3)	(1,1,3)
10	2	S	Good	(2,4,5), (2,3,4)	(2,4,5)

Table 8. Final Allotment of Nurses for different Patients

Sl No.	Department No.	Name	Gender	Age	Nurses Assignment Categories		
					C1	C2	C3
1	2	Avijit Nayak	M	52	2	3	4
2	2	Susant Panda	M	46	3	4	5
3	2	Devika Patra	F	39	1	2	4
4	2	Sumita Mohanty	F	42	0	1	3
5	2	Lipika Dash	F	29	1	2	4
6	2	Aditya Roy	M	75	3	4	5
7	2	Aswani Rana	M	67	3	4	5
8	2	Brajeswari Saha	F	45	1	1	3
9	2	Nitisha Ojha	F	32	1	2	4
10	2	Anup Joshi	M	44	2	4	5

When a new patient comes with Sl. no. 11, Name: - Ramesh Pattnaik, Gender: - M, Age: - 22, financial condition well and has a serious (S) heart problem. It is considered that the patient should be assigned 2 numbers of category C1 nurses, 4 numbers of category C2 nurses & 5 numbers of category C3 nurses:

$$NC_1(2)=\sum_{\substack{i=1\\j=2}}^{i=10} C\left(i,j\right)=17$$

$$NC_2(2)=\sum_{\substack{i=1\\j=2}}^{i=10} C\left(i,j\right)=27$$

$$NC_3(2)=\sum_{\substack{i=1\\j=2}}^{i=10} C\left(i,j\right)=42$$

Now:

$$NC_1 + 2 = 19 > 11 = C_1(2) \tag{2}$$

$$NC_2 + 4 = 31 > 21 = C_2(2) \tag{2}$$

$$NC_3 + 5 = 47 > 36 = C_3(2) \tag{2}$$

Thus, the calculations of NC1(2), NC2(2), NC3(2) shows that the nurses are not available according to the requirement of the patient. Hence it is not possible to provide the required number of nurses to Ramesh Pattnaik. But if the nurses are available in other departments having relation R with cardiac's department nurses and R satisfies symmetric and transitive relations, then it is possible to assign nurses from another department.

5. PROBLEM ANALYSIS

If ni (NC1, NC2, NC3) nurses assigned to patient P (i, j), then it must satisfy the following definitions:

$$f_i = R_i \times NC_1 + R_2 \times NC_2 + R_3 \times NC_3$$

and:

$$u\left(n_i\right)=\begin{cases} f_i, & if\ f_i \le budget\ of\ the\ patient\ P_i\ and\ the\ availability\ of\ nurses\ are\ NC_1, NC_2\ and\ NC_3 \\ 0, & otherwise \end{cases}$$

Since it is considered according to the availability of nurses and budget of patients, so we may get more than one alternative or no availability of nurses. If there is not the availability of nurses, we can appoint nurses to patients with the features as defined below.

Nurses can be exchanged or other departments nurses can serve another department if they satisfy symmetric &, transitive properties where:

Symmetric property: If $N_t R_i N_s$ then $N_s R_i N_t$.
Transitive property: If $N_t R_i N_s$ & $N_s R_i N_r$ then $N_t R_i N_r$.

Again, if there is more than one alternative, we have to choose one set of nurses for a particular patient. It is possible by using the concept of the Perfect Nash Equilibrium as defined below:

$$V\left(X,Y\right) = f_i, where \ X \in \left\{S,M,N\right\}, and \ Y \in \left\{L,C,E\right\}$$

and:

$$f_i = R_i \times NC_1 + R_2 \times NC_2 + R_3 \times NC_3, the \ values \ of \ NC_1, NC_2, NC_3 \ depends \ on \ X \ and \ Y$$

According to patient conditions i.e., serious, manageable or normal, the value of X assigned and according to the status of the patient i.e., luxurious, comfort and necessary, the value of Y assigned. Corresponding to the values of X and Y, the values of NC1, NC2, NC3 or the number of nurses of different types are assigned. Thus, a particular number and types of nurses are finalized from different alternatives.

In the case study of ten patients, using the Nash equilibrium utility function all different alternatives of assignment of nurses are defined below:

- (2, 3, 4): patient with serial number 1.
- (3,4,5), (2,4,5), (2,3,4): patient with serial number 2 has 3 alternatives.
- (1,2,4), (0,2,4): patient with serial number 3 has 2 alternatives.
- (0,1,3): patient with serial number 4.
- (1,2,4), (1,2,3), (1,2,4): patient with serial number 5 has 3 alternatives.
- (3,4,5), (2,4,5), (2,3,4): patient with serial number 6 has 3 alternatives.
- (1,2,3), (1,1,3), (0,1,3): patient with serial number 7 has 3 alternatives.
- (1,2,3), (1,1,3), (0,1,3): patient with serial number 8 has 3 alternatives.
- (1, 2, 3): patient with serial number 9.
- (2,4,5), (2,3,4): patient with serial number 10 has 2 alternatives.

For solving critical cases of choosing from alternatives, using Perfect Nash Equilibrium utility function we have found a particular set of nurses to each patient as below.

5.1 Critical Cases

- (3,4,5), (2,4,5), (2,3,4): patient with serial number 2 has 3 alternatives.
- (1,2,4), (0,2,4): patient with serial number 3 has 2 alternatives.
- (1,2,4), (1,2,3), (1,2,4): patient with serial number 5 has 3 alternatives.
- (3,4,5), (2,4,5), (2,3,4): patient with serial number 6 has 3 alternatives.
- (1,2,3), (1,1,3), (0,1,3): patient with serial number 7 has 3 alternatives.
- (1,2,3), (1,1,3), (0,1,3): patient with serial number 8 has 3 alternatives.
- (2,4,5), (2,3,4): patient with serial number 10 has 2 alternatives.

After using Perfect Nash Equilibrium, we have:

- (3,4,5): patient with serial number 2.
- (1,2,4): patient with serial number 3.
- (1,2,4): patient with serial number 5.
- (3,4,5): patient with serial number 6.
- (1,1,3): patient with serial number 7.
- (1,1,3): patient with serial number 8.
- (2,4,5): patient with serial number 10.

In the case study, the patient, Sl. no. 11, Name: - Ramesh Pattnaik, Gender: - M, Age: - 22, having financial condition well and has a serious (S) heart problem has come to the hospital for treatment. According to authority, the patient should be assigned 2 numbers of category C1 nurses, 4 numbers of category C2 nurses & 5 numbers of category C3 nurses. The availability of nurses cannot meet the patient's requirement and the issue can be solved if other department satisfies the symmetric and transitive relation as well as have the nurses available in a department.

6. CONCLUSION

In this paper, we discussed a critical problem of nurses' appointment to a patient for best treatment purpose. We have used Nash Equilibrium and Perfect Nash Equilibrium techniques of game theory to take an appropriate decision. These methods predict the number of nurses with different skill is to be appointed to different types of patients. Nash equilibrium shows all possible ways to appoint different types and number of nurses whereas Perfect Nash Equilibrium predicts the exact & appropriate number and types of nurses assign to a patient. Besides that, we have proposed the concept of equivalence relation for substitution of nurses in emergency cases. A case study is also discussed with ten patients.

The techniques can also be used in a different area for major decision & prediction especially when there are critical cases to take a decision. We may use fuzzy set theory to quantify the doctors and management opinions regarding nurse's allotment for automatizes. This technique applies to both qualitative and quantitative consideration of data.

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