

An Advanced Entropy Measure of IFSs via Similarity Measure

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ABSTRACT

The Entropy measure of an intuitionistic fuzzy set (IFS) plays a significant role in decision making sciences, for instance, medical diagnosis, pattern recognition, criminal investigation, etc. The inadequate nature of an entropy measure may lead to some invalid results. Therefore, it is significant to use an efficient entropy measure for studying various decision-making problems under IFS environment. This paper first proposes a novel similarity measure for IFS. Based on the proposed similarity measure, an advanced entropy measure is defined with a different axiomatic approach. This axiomatic approach allows us to measure an IFS's entropy with the help of a similarity measure. To show the efficiency of the proposed similarity measure, a comparative study is performed with the existing similarity measures. Some structural linguistic variables are taken as examples to show the validity and consistency of the proposed entropy measure along with the existing entropy measures. Finally, based on the proposed entropy measure, a multi-criteria decision-making problem is performed.

KEYWORDS

Entropy measure, Intuitionistic fuzzy sets, Multi criteria decision making, Similarity measure

1. INTRODUCTION

In 1986, Atanassov developed the IFS theory, which is the extension of Zadeh's fuzzy set theory. Similarity and entropy measures are two essential tools for dealing with uncertainty through the IFS theory. Different similarity and entropy measures have been proposed and applied successfully in many areas. Similarity measures defined from the well-known distance measures are depicted by Smidh et al. (2000), Wang et al. (2005), Grzegorzewski (2004), Chen (2007), Hung et al. (2007). Li and Cheng (2002), Liang and Shi (2003), Hwang et al. (2012), Xu (2007), and Xu and Yager (2009) gave several new similarity measures for IFSs. Mitchell (2003) developed a statistical method of Dengfeng and Chuntian's similarity measure by giving some counterintuitive cases. Ye (2011) has compared the existing similarity measures and proposed new and weighted similarity measures using

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the cosine function. Xu and Chen (2008) developed a series of similarity measures by generalizing the weighted Hamming distance, the weighted Euclidean distance, and the weighted Hausdorff distance. Xia and Xu (2010) and Zeng and Guo (2008) worked on distance, similarity, and entropy measures and studied their relationship. A generalization of the existing entropy measures for IFSs is proposed by Wei et al. (2011). Boran and Akay (2014) introduced a new general type of similarity measure for IFS relating two parameters norm and the level of uncertainty. Li et al. (2012) studied both the similarity measure and entropy measure for IFSs by defining an axiomatic approach to the similarity measure. Entropy is an effective measure to give a picture of the fuzziness of a fuzzy set. Many researchers have defined many entropy measures. Bhandari and Pal (1993), Luca and Termini (1972), Fan and Ma (2002), Shore and Gray (1982), Zhang and Jiang (2008), Ye (2010), Verma and Sharma (2013), pal and pal (1989), Wei et al. (2012), Wang and Wang (2012), Liu and Ren (2014), Song et al. (2014), Szmidt and Kacprzyk (2001), Vlachos and Sergiadis (2007), Burillo and Bustince (1996), Zeng and Li (2006), Zhang and Zhang (2009), Farhadinia (2013), Liu (1992), Zeng and Li (2006), Zeng and Guo (2008), Li and Deng (2012), Zhang et al. (2014), Garge et al. (2011), Hung and Yang (2006) worked on entropy measure for IFSs with a different aspect. Along with the study of entropy measures, Li et al. (2010, 2011, 2014, 2015, 2016, 2017) have worked to develop many ranking processes of IFS and Interval Valued Intuitionistic Fuzzy Sets (IVIFSs). In the field of IVIFSs, Talukdar et al. (2019) have also proposed a novel ranking method to enhance the efficiency of the process. Some novel entropy measures have been developed by Thao (2021), Verma et al. (2017), Wei et al. (2019), Zhu et al. (2016), Wei et al. (2021), Li et al. (2015), Li et al. (2002), Li et al. (2004) with a lot of different perspective.

Though different entropy measures of IFS have been developed in the literature, there are many situations where they do not incorporate the exact measure of IFSs. Therefore, sometimes it becomes challenging to select the best choice that reflects the correct nature of the IFSs. Thus, it is a fundamental and essential task for developing efficient and reliable entropy measure with different perspective.

This motivates us to study effective novel entropy measures with different approaches. This paper presents a novel similarity measure of IFSs. Based on that similarity measure, an advanced entropy measure is defined to measure the degree of fuzziness of IFSs by introducing a new axiomatic approach.

1.1 Structure of the Paper

The detailed work has been shortened as follows. Section 2 starts with some relevant preliminary definitions. The proposed intuitionistic fuzzy similarity measure and entropy measure are presented in sections 3 & in section 4, respectively. Sections 5 & 6 show the positivity and necessity of the proposed similarity and entropy measures compared to the earlier methods. Two multi-criteria decision-making problems are discussed by using the proposed entropy measure in section 7. Finally, a concrete conclusion has been drawn in section 8.

2. PRELIMINARIES

2.1 Fuzzy Set

Fuzzy set is a set in which every element has degree of membership of belonging in it. Mathematically, let X be a universal set. Then the fuzzy subset A of X is defined by its membership function $\mu_A: X \rightarrow [0,1]$ which assign a real number $\mu_A(x)$ in the interval $[0, 1]$, to each element $x \in A$, where the value of $\mu_A(x)$ at x shows the grade of membership of x in A .

2.2 Intuitionistic Fuzzy Set

A Intuitionistic fuzzy set A on a universe of discourse X is of the form $A = \{(x, \mu_A(x), \nu_A(x); x \in X)\}$ Where $\mu_A(x) \in [0,1]$ is called the “degree of membership of x in A ”, $\nu_A(x) \in [0,1]$ is called the

“degree of non-membership of x in A ”, and $\mu_A(x)$ and $\nu_A(x)$ satisfy the condition that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

The amount $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called hesitancy of x which is reflection of lack of commitment or uncertainty associated with the membership or non-membership or both in A .

2.3 Definition

For $A \in IFS(X)$ and $B \in IFS(X)$, some relation between them are defined as follows:

$$\mathbf{R1.} A \subseteq B \text{ iff } \forall x \in X, \mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x);$$

$$\mathbf{R2.} A = B \text{ iff } \forall x \in X, \mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x)$$

$$\mathbf{R3.} A^c = \left\{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \right\}, \text{ where } A^c \text{ is the complement of } A$$

$$\mathbf{R4.} \bigcap_i A_i = \left\{ \langle x, \wedge \mu_{A_i}(x), \vee \nu_{A_i}(x) : x \in X \rangle \right\}$$

$$\mathbf{R5.} \bigcup_i A_i = \left\{ \langle x, \vee \mu_{A_i}(x), \wedge \nu_{A_i}(x) : x \in X \rangle \right\}$$

2.4 Definition

Let D denote a mapping $D : IFS \times IFS \rightarrow [0, 1]$, if $D(A, B)$ satisfies the following properties, $D(A, B)$ is called distance between $A \in IFSs(X)$ and $B \in IFSs(X)$.

$$\mathbf{DP1.} \text{ Let } 0 \leq D \leq 1;$$

$$\mathbf{DP2.} D(A, B) = 0, \text{ if and only if } A = B$$

$$\mathbf{DP3.} D(A, B) = D(B, A)$$

$$\mathbf{DP4.} \text{ If } A \subset B \subset C, \text{ then } D(A, B) \leq D(A, C) \text{ and } D(B, C) \leq D(A, C)$$

2.5 Definition

A mapping $S : IFS \times IFS \rightarrow [0, 1]$ is called a degree of similarity between $A \in IFSs(X)$ and $B \in IFSs(X)$, if $S(A, B)$ satisfies the properties:

$$\mathbf{SP1.} 0 \leq S(A, B) \leq 1$$

$$\mathbf{SP2.} S(A, B) = 1 \text{ iff } A = B$$

$$\mathbf{SP3.} S(A, B) = S(B, A)$$

$$\mathbf{SP4.} \text{ If } A \subset B \subset C \text{ then } S(A, B) \geq S(A, C) \text{ and } S(B, C) \geq S(A, C)$$

3. PROPOSED INTUITIONISTIC FUZZY SIMILARITY MEASURE

In this section, a novel similarity measure is proposed for IFSs.

Let $A = \left\{ \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle \mid x_i \in X \right\}$ and $B = \left\{ \langle x_i, \mu_B(x_i), \nu_B(x_i) \rangle \mid x_i \in X \right\}$ be two IFSs in X . Then we propose the novel similarity measure as follows,

$$S_p(A, B) = \frac{1}{2n} \sum_{i=1}^n \frac{\left(1 - |\mu_A(x_i) - \mu_B(x_i)|\right) + \left(1 - |\nu_A(x_i) - \nu_B(x_i)|\right)}{\left\{1 + \sqrt{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_B(x_i)\right)^2 + \left(\mu_A(x_i)\nu_B(x_i) - \mu_B(x_i)\nu_A(x_i)\right)^2}\right\}}$$

Where $\mu_A(x_i)$ and $\nu_A(x_i)$ are membership degree and non membership degree of the element $x_i \in X$.

3.1 Theorem

$S_p(A, B)$ is the similarity measure between two IFSs A and B in X .

Proof: Let $A = \left\{ \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle : x_i \in X \right\}$ and $B = \left\{ \langle x_i, \mu_B(x_i), \nu_B(x_i) \rangle : x_i \in X \right\}$ be two IFSs on X .

SP1. Then, $0 \leq |\mu_A(x_i) - \mu_B(x_i)| \leq 1$

Hence, $0 \leq 1 - |\mu_A(x_i) - \mu_B(x_i)| \leq 1$

$$\begin{aligned} \text{Also, } & 2 \left\{ 1 + \sqrt{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_B(x_i)\right)^2 + \left(\mu_A(x_i)\nu_B(x_i) - \mu_B(x_i)\nu_A(x_i)\right)^2} \right\} \\ & \geq \left(1 - |\mu_A(x_i) - \mu_B(x_i)|\right) + \left(1 - |\nu_A(x_i) - \nu_B(x_i)|\right) \end{aligned}$$

Thus we have

$$\begin{aligned} 0 & \leq \frac{1 - |\mu_A(x_i) - \mu_B(x_i)| + 1 - |\nu_A(x_i) - \nu_B(x_i)|}{2 \left\{ 1 + \sqrt{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_B(x_i)\right)^2 + \left(\mu_A(x_i)\nu_B(x_i) - \mu_B(x_i)\nu_A(x_i)\right)^2} \right\}} \leq 1 \\ 0 & \leq \sum_{i=1}^n \frac{1 - |\mu_A(x_i) - \mu_B(x_i)| + 1 - |\nu_A(x_i) - \nu_B(x_i)|}{2 \left\{ 1 + \sqrt{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_B(x_i)\right)^2 + \left(\mu_A(x_i)\nu_B(x_i) - \mu_B(x_i)\nu_A(x_i)\right)^2} \right\}} \leq n \\ 0 & \leq \frac{1}{n} \sum_{i=1}^n \frac{1 - |\mu_A(x_i) - \mu_B(x_i)| + 1 - |\nu_A(x_i) - \nu_B(x_i)|}{2 \left\{ 1 + \sqrt{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_B(x_i)\right)^2 + \left(\mu_A(x_i)\nu_B(x_i) - \mu_B(x_i)\nu_A(x_i)\right)^2} \right\}} \leq 1 \end{aligned}$$

Thus, $0 \leq S_p(A, B) \leq 1$

SP2. If $A = B$ then $\mu_A(x_i) = \mu_B(x_i), \nu_A(x_i) = \nu_B(x_i)$ then $S_p(A, B) = 1$

Conversely, let $S_p(A, B) = 1$

$$\Rightarrow 1 - |\mu_A(x_i) - \mu_B(x_i)| + 1 - |\nu_A(x_i) - \nu_B(x_i)| = 2 \left\{ 1 + \sqrt{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_B(x_i)\right)^2 + \left(\mu_A(x_i)\nu_B(x_i) - \mu_B(x_i)\nu_A(x_i)\right)^2} \right\}$$

$$\begin{aligned}
 &\Rightarrow 2 - \left| \mu_A(x_i) - \mu_B(x_i) \right| - \left| \nu_A(x_i) - \nu_B(x_i) \right| = 2 + 2 \sqrt{\frac{\left(\mu_A(x_i) - \mu_B(x_i) \right)^2 + \left(\nu_A(x_i) - \nu_B(x_i) \right)^2}{\left(\mu_A(x_i) \nu_B(x_i) - \mu_B(x_i) \nu_A(x_i) \right)^2}} + \\
 &\Rightarrow \left| \mu_A(x_i) - \mu_B(x_i) \right| + \left| \nu_A(x_i) - \nu_B(x_i) \right| + 2 \sqrt{\frac{\left(\mu_A(x_i) - \mu_B(x_i) \right)^2 + \left(\nu_A(x_i) - \nu_B(x_i) \right)^2}{\left(\mu_A(x_i) \nu_B(x_i) - \mu_B(x_i) \nu_A(x_i) \right)^2}} = 0 \\
 &\Rightarrow \mu_A(x_i) = \mu_B(x_i) \text{ and } \nu_A(x_i) = \nu_B(x_i)
 \end{aligned}$$

SP3. clearly, $S(A, B) = S(B, A)$

SP4. Let $C = \left\{ \langle x, \mu_C(x_i), \nu_C(x_i) \rangle : x \in X \right\}$ be another IFS in X .

If $A \subset B \subset C$ then $\mu_C(x_i) \geq \mu_B(x_i) \geq \mu_A(x_i)$ and $\nu_C(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$

Then clearly $\left| \mu_A(x_i) - \mu_B(x_i) \right| \leq \left| \mu_A(x_i) - \mu_C(x_i) \right|$ & $\left| \nu_A(x_i) - \nu_B(x_i) \right| \leq \left| \nu_A(x_i) - \nu_C(x_i) \right|$

$$\Rightarrow 1 - \left| \mu_A(x_i) - \mu_C(x_i) \right| \leq 1 - \left| \mu_A(x_i) - \mu_B(x_i) \right| \quad (1)$$

&

$$1 - \left| \nu_A(x_i) - \nu_C(x_i) \right| \leq 1 - \left| \nu_A(x_i) - \nu_B(x_i) \right| \quad (2)$$

Adding (1) & (2), we have

$$1 - \left| \mu_A(x_i) - \mu_C(x_i) \right| + 1 - \left| \nu_A(x_i) - \nu_C(x_i) \right| \leq 1 - \left| \mu_A(x_i) - \mu_B(x_i) \right| + 1 - \left| \nu_A(x_i) - \nu_B(x_i) \right| \quad (3)$$

Also, it is obvious that

$$\left(\mu_A(x_i) - \mu_B(x_i) \right)^2 \leq \left(\mu_A(x_i) - \mu_C(x_i) \right)^2 \quad (4)$$

&

$$\left(\nu_A(x_i) - \nu_B(x_i) \right)^2 \leq \left(\nu_A(x_i) - \nu_C(x_i) \right)^2 \quad (5)$$

Again as $\nu_B(x_i) \geq \nu_C(x_i)$ & $\mu_B(x_i) \leq \mu_C(x_i)$

$$\therefore \left(\nu_C(x_i) - \nu_B(x_i) \right) \leq 0 \text{ \& } \left(\mu_C(x_i) - \mu_B(x_i) \right) \geq 0$$

Also, $\mu_A(x_i) \& \nu_A(x_i) \geq 0$, therefore, without lost of generality we can write

$$\begin{aligned}
 & \mu_A(x_i)(\nu_C(x_i) - \nu_B(x_i)) \leq \nu_A(x_i)(\mu_C(x_i) - \mu_B(x_i)) \\
 & \Rightarrow \mu_A(x_i)\nu_C(x_i) - \mu_A(x_i)\nu_B(x_i) \leq \nu_A(x_i)\mu_C(x_i) - \nu_A(x_i)\mu_B(x_i) \\
 & \Rightarrow \mu_B(x_i)\nu_A(x_i) - \mu_A(x_i)\nu_B(x_i) \leq \mu_C(x_i)\nu_A(x_i) - \mu_A(x_i)\nu_C(x_i) \\
 & \Rightarrow (\mu_B(x_i)\nu_A(x_i) - \mu_A(x_i)\nu_B(x_i))^2 \leq (\mu_C(x_i)\nu_A(x_i) - \mu_A(x_i)\nu_C(x_i))^2 \\
 & \text{(It holds because } (\mu_B(x_i)\nu_A(x_i)) - \mu_A(x_i)\nu_B(x_i) \geq 0 \& (\mu_C(x_i)\nu_A(x_i) - \mu_A(x_i)\nu_C(x_i) \geq 0))
 \end{aligned}$$

$$\therefore \text{It is clear that } (\mu_B(x_i)\nu_A(x_i) - \mu_A(x_i)\nu_B(x_i))^2 \leq (\mu_A(x_i)\nu_C(x_i) - \mu_C(x_i)\nu_A(x_i))^2 \quad (6)$$

Using (4), (5), and (6) we have

$$\begin{aligned}
 & 2 \left\{ 1 + \sqrt{(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\mu_A(x_i)\nu_B(x_i) - \mu_B(x_i)\nu_A(x_i))^2} \right\} \\
 & \leq 2 \left\{ 1 + \sqrt{(\mu_A(x_i) - \mu_C(x_i))^2 + (\nu_A(x_i) - \nu_C(x_i))^2 + (\mu_A(x_i)\nu_C(x_i) - \mu_C(x_i)\nu_A(x_i))^2} \right\}
 \end{aligned} \quad (7)$$

Now using (3) & (7) we have

$$\begin{aligned}
 & \frac{1 - |\mu_A(x_i) - \mu_B(x_i)| + 1 - |\nu_A(x_i) - \nu_B(x_i)|}{2 \left\{ 1 + \sqrt{(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\mu_A(x_i)\nu_B(x_i) - \mu_B(x_i)\nu_A(x_i))^2} \right\}} \\
 & \geq \frac{1 - |\mu_A(x_i) - \mu_C(x_i)| + 1 - |\nu_A(x_i) - \nu_C(x_i)|}{2 \left\{ 1 + \sqrt{(\mu_A(x_i) - \mu_C(x_i))^2 + (\nu_A(x_i) - \nu_C(x_i))^2 + (\mu_A(x_i)\nu_C(x_i) - \mu_C(x_i)\nu_A(x_i))^2} \right\}}
 \end{aligned}$$

Therefore

$$\begin{aligned}
 & \frac{1}{n} \sum_{i=1}^n \frac{1 - |\mu_A(x_i) - \mu_B(x_i)| + 1 - |\nu_A(x_i) - \nu_B(x_i)|}{2 \left\{ 1 + \sqrt{(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\mu_A(x_i)\nu_B(x_i) - \mu_B(x_i)\nu_A(x_i))^2} \right\}} \\
 & \geq \frac{1}{n} \sum_{i=1}^n \frac{1 - |\mu_A(x_i) - \mu_C(x_i)| + 1 - |\nu_A(x_i) - \nu_C(x_i)|}{2 \left\{ 1 + \sqrt{(\mu_A(x_i) - \mu_C(x_i))^2 + (\nu_A(x_i) - \nu_C(x_i))^2 + (\mu_A(x_i)\nu_C(x_i) - \mu_C(x_i)\nu_A(x_i))^2} \right\}}
 \end{aligned}$$

Thus we have, $S_p(A, B) \geq S_p(A, C)$

Hence, it is shown that the function $S_p(A, B)$ satisfy all the axioms of similarity measure.

4. ENTROPY MEASURE

Entropy measure gives the degree of fuzziness associated with a fuzzy set. The same illustration is put forward in the case of the IFS also. Eulalia Szmidt and Janusz Kacprzyk (2001) described that a non-fuzzy set is the points A and B that correspond to fully belongings and entirely non-belongings with the points $(\mu(x), \nu(x), \pi(x)) = (1, 0, 0)$ and $(\mu(x), \nu(x), \pi(x)) = (0, 1, 0)$ respectively having the entropy equal to zero. Also, if we move through the line from point A to point B, the membership values decrease, and the non membership values increase, and at the midpoint, both become equal to 0.5. So at the midpoint, the entropy measure is equal to 100%. Subsequently, it has been shown that for IFS the degree of fuzziness is highest for all points at which $\mu(x) = \nu(x)$. As the similarity measure between two IFS gives the degree of similarity of the sets, the higher the similarity degree between the IFSs A and $\langle x, 0.5, 0.5 \rangle$ will lead to a higher entropy measure of set A. This is how we motivate to define a new axiomatic approach and a new entropy measure based on similarity measure.

4.1 De Luca-Termini's Axioms for Entropy Measure of IFS

A real valued function $E : IFS(X) \rightarrow [0, 1]$ is called entropy measure on IFS(X) if it satisfies the following axiomatic requirements:

- E1.** $E(A) = 0$, if and only if A is a crisp set
- E2.** $E(A) = 1$, if and only if $\mu_A(x_i) = \nu_A(x_i)$ for all $x_i \in X$,
- E3.** $E(A) = E(A^c)$ for all $A \in IFS(X)$,
- E4.** $E(A) \leq E(B)$ if A is less fuzzy than B, that is,
 $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for $\mu_B(x_i) \leq \nu_B(x_i)$ or
 $\mu_A(x_i) \geq \mu_B(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for $\mu_B(x_i) \geq \nu_B(x_i)$

4.2 Some Existing Entropy Measure

Burillio and Bustince (1996) defined entropy function as

$$E_{BB}(A) = \frac{1}{n} \sum_{i=1}^n \pi_A(x_i)$$

Szmidt and Kacprzyk (2001) developed the ratio based entropy measure by describing the geometric interpretation of IFS as

$$E_{SK} = \frac{1}{n} \sum_{i=1}^n \frac{\max \text{count}(F_i \cap F_i^c)}{\max \text{count}(F_i \cup F_i^c)}$$

Hung and Yang (2006) defined two entropy measures as

$$E_{hc}^{\alpha}(A) = \begin{cases} \frac{1}{\alpha-1} \left[1 - \left(\mu_A^{\alpha}(x_i) + \nu_A^{\alpha}(x_i) + \pi_A^{\alpha}(x_i) \right) \right], \alpha \neq 1 (\alpha > 0) \\ - \left(\mu_A(x_i) \log \mu_A(x_i) + \nu_A(x_i) \log \nu_A(x_i) + \pi_A(x_i) \log \pi_A(x_i) \right), \alpha = 1 \end{cases}$$

$$\text{and } E_r^{\beta}(A) = \frac{1}{1-\beta} \log \left(\mu_A^{\beta}(x_i) + \nu_A^{\beta}(x_i) + \pi_A^{\beta}(x_i) \right), 0 < \beta < 1$$

Zhang and Jiang (2008) defined the entropy measure as

$$E_{ZJ}(A) = \frac{1}{n} \sum_{i=1}^n \left[\frac{\left(\mu_A(x_i) \wedge \nu_A(x_i) \right)}{\left(\mu_A(x_i) \vee \nu_A(x_i) \right)} \right]$$

4.3 New Axiomatic Approach of Entropy Measure

The new axiomatic definition is developed based on the concept that the IFS $\langle x, \frac{1}{2}, \frac{1}{2} \rangle$, whose hesitancy margin is zero is the fuzziest IFS. More similarity of an IFS with $\langle x, \frac{1}{2}, \frac{1}{2} \rangle$ indicate fuzzier it is.

Let $P = \langle x, \frac{1}{2}, \frac{1}{2} \rangle$ be an IFS. A real function $E : IFS(X) \rightarrow [0, 1]$ is known as entropy measure of IFS A , if it satisfies the following conditions:

E¹1. $E(A) = 0$ if A is crisp set.

E¹2. $E(A) = 1$ iff $\mu_A(x) = \nu_A(x) = \frac{1}{2}$

E¹3. If $S(A, P) \geq S(B, P)$ then $E(A) \geq E(B)$

E¹4. $E(A) = E(A^c)$

4.4 Proposed Intuitionistic Fuzzy Entropy Measure

Let $X = \{x_1, x_2, x_3, \dots\}$ be a universal set and $A = \left\{ \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle : x_i \in X \right\}$ be an IFS and $P = \left\{ \langle x, 0.5, 0.5 \rangle \right\}$. Then define an entropy measure as

$$E_{PP}(A) = \frac{(2 + \sqrt{3})S(A, P) - 1}{1 + \sqrt{3}}, \text{ where } S(A, P) \text{ is the similarity measure between the IFSs } A \text{ and } P.$$

4.4.1 Theorem

$E(A)$ is an entropy measure for $A \in IFS(X)$

Proof: Let $A = \langle x, \mu_A(x), \nu_A(x) \rangle$ be an IFS and $P = \langle y, 0.5, 0.5 \rangle$ be a fixed IFS. Then the entropy measure $E_{PP}(A) = \frac{(2 + \sqrt{3})S(A, P) - 1}{1 + \sqrt{3}}$ satisfies the four following properties.

E'1. If A is a crisp set then either $\mu_A(x) = 0$ and $\nu_A(x) = 1$ or $\mu_A(x) = 1$ and $\nu_A(x) = 0$. Therefore $E_{PP}(A) = 0$

E'2. Let $\mu_A(x) = \nu_A(x) = \frac{1}{2}$, then clearly $E_{PP}(A) = 1$

Conversely, if $E(A) = 1$ then

$$\begin{aligned} & \frac{(2 + \sqrt{3})S(A, P) - 1}{1 + \sqrt{3}} = 1 \\ \Rightarrow & (2 + \sqrt{3})S(A, P) - 1 = 1 + \sqrt{3} \\ \Rightarrow & (2 + \sqrt{3})S(A, P) = 2 + \sqrt{3} \\ \Rightarrow & S(A, P) = \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = 1 \\ \Rightarrow & \frac{1 - |\mu_A(x) - 0.5| + 1 - |\nu_A(x) - 0.5|}{2 \left[1 + \sqrt{(\mu_A(x) - 0.5)^2 + (\nu_A(x) - 0.5)^2 + \left(\frac{\mu_A(x) - \nu_A(x)}{2} \right)^2} \right]} = 1 \\ \Rightarrow & 1 - |\mu_A(x) - 0.5| + 1 - |\nu_A(x) - 0.5| = 2 \left[1 + \sqrt{(\mu_A(x) - 0.5)^2 + (\nu_A(x) - 0.5)^2 + \left(\frac{\mu_A(x) - \nu_A(x)}{2} \right)^2} \right] \\ \Rightarrow & 2 - |\mu_A(x) - 0.5| - |\nu_A(x) - 0.5| = 2 + 2 \sqrt{(\mu_A(x) - 0.5)^2 + (\nu_A(x) - 0.5)^2 + \left(\frac{\mu_A(x) - \nu_A(x)}{2} \right)^2} \\ \Rightarrow & |\mu_A(x) - 0.5| + |\nu_A(x) - 0.5| + 2 \sqrt{(\mu_A(x) - 0.5)^2 + (\nu_A(x) - 0.5)^2 + \left(\frac{\mu_A(x) - \nu_A(x)}{2} \right)^2} = 0 \end{aligned}$$

Since each term of left hand side is positive, therefore the equation is true only if

$$\mu_A(x) = \nu_A(x) = 0.5$$

E'3. For any $A, B \in IFS(X)$

if $S(A, P) \geq S(B, P)$

$$\Rightarrow (2 + \sqrt{3})S(A, P) \geq (2 + \sqrt{3})S(B, P)$$

$$\Rightarrow \left\{ (2 + \sqrt{3})S(A, P) \right\} - 1 \geq \left\{ (2 + \sqrt{3})S(B, P) \right\} - 1$$

$$\Rightarrow \frac{\left\{ (2 + \sqrt{3}) S(A, P) \right\} - 1}{1 + \sqrt{3}} \geq \frac{\left\{ (2 + \sqrt{3}) S(B, P) \right\} - 1}{1 + \sqrt{3}}$$

$$\Rightarrow E_{PP}(A) \geq E_{PP}(B)$$

E'4. For any $A = \left\{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \right\} \in IFS(X)$ the complement of it

$$A^C = \left\{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \right\}$$

It is obvious from the definition that $E_{PP}(A) = E_{PP}(A^C)$.

5. A COMPARATIVE STUDY OF INTUITIONISTIC FUZZY SIMILARITY MEASURE WITH THE EXISTING SIMILARITY MEASURE

This section exhibits a comparative study between the proposed similarity measure and the other existing similarity measures by considering some standard numerical examples of IFSs. The counterintuitive examples of IFSs, along with their similarity degree for various similarity measures for this study, are depicted in table-1. For the IFS $A = \langle 0.3, 0.3 \rangle$ and $B = \langle 0.4, 0.4 \rangle$, in that table it is seen that the degree of similarity for the similarity measures in Chen (1995), Hwang et al. (2012) and Ye (2011) are identical and is equal to 1. Thus, the similarity measures in Chen (1995), Hwang et al. (2012) and Ye (2011) do not satisfy the axiom (SP2). On the other hand, the similarity measures in Hong and Kim (1999), Li et al. (2002), Mitchell (2003), Liang et al. (2003) again show equal values for different pair of A, B as shown in table-1. This situation is unsuitable for application in different fields like pattern recognition, medical decision making etc. Also, the similarity measure in Mitchell (2003) take the same values for different pair of A, B . The similarity measures in Hung et al. (2004) take the same values for two different pairs of A, B in case 5 and case 6. It is seen that the similarity measure in Boran and Akay (2014) shows reasonable results for each pair of A, B . In Song et al. (2014) the similarity values are 0.936 and 0.896 for the pairs $A = \langle 0.4, 0.2 \rangle, B = \langle 0.5, 0.3 \rangle$ and $A = \langle 0.4, 0.2 \rangle, B = \langle 0.5, 0.2 \rangle$ respectively. But it is a general intuition that the similarity of the pair of IFSs $A = \langle 0.4, 0.2 \rangle$ and $B = \langle 0.5, 0.3 \rangle$ cannot be greater than the similarity degree for the pair of IFSs $A = \langle 0.4, 0.2 \rangle$ and $B = \langle 0.5, 0.2 \rangle$. Therefore, it is observed that the existing similarity measures have some limitations to describe some particular situations of IFSs. On the other hand, our proposed similarity measure can handle such situation properly and show a reasonable performance for each pair of IFSs. Furthermore, our similarity measure has the advantage of a relatively simple expression without counterintuitive examples. For instant, the similarity measure in Boran and Akay (2014) involves two parameters, L_p norm and the level of uncertainty (t). Compared to our similarity measure, the similarity measure in Boran and Akay (2014) has the difficult situation of determination of the two parameters p and t .

6. A COMPARATIVE STUDY OF THE DIFFERENT EXISTING ENTROPY MEASURE WITH PROPOSED ENTROPY MEASURE

Let us consider the IFSs $S_1 = \left\{ \left\langle x, \frac{3}{4}, \frac{1}{6} \right\rangle \right\}, S_2 = \left\{ \left\langle x, \frac{1}{2}, 0 \right\rangle \right\}$ and $S_3 = \left\{ \left\langle x, \frac{1}{2}, \frac{1}{4} \right\rangle \right\}$. It is a general intuition that if we compare these IFSs with the IFS $\left\{ \left\langle x, \frac{1}{2}, \frac{1}{2} \right\rangle \right\}$, with highest entropy and minimum

hesitancy margin then the set S_3 has the highest degree of similarity than the others. Thus from the axiom of entropy measure ($E^1 3$) S_3 should have highest entropy value.

Also the entropy values for proposed entropy measure are given by $E(S_1) = 0.2782, E(S_2) = 0.2911, E(S_3) = 0.5681$. This ordering of the entropy measure is identical with the Eulalia and Kacprzyk (2001) entropy measure in which they explained it by giving a geometrical interpretation IFS. But the entropy measure E_{BB} defined by Burillio and Bustince fails to reflect the above phenomena.

Consider $A = \{x, \mu_A(x), \nu_A(x) : x \in X\}$ be an IFS in X . De et al. (2000) define A^n for any real number n as follows:

$$A^n = \left\{ \left\langle x, [\mu_A(x)]^n, 1 - [1 - \nu_A(x)]^n \right\rangle : x \in X \right\}$$

They also define the term dilation and concentration for A as

dilation: $DIL(A) = A^{1/2}$ & concentration: $CON(A) = A^2$

The sets $DIL(A)$ and $CON(A)$ represent “more or less(A)” and “very(A)” respectively. We will consider an example of IFS as

$$A = \left\{ \langle x_1, 0.1, 0.8 \rangle, \langle x_2, 0.3, 0.5 \rangle, \langle x_3, 0.6, 0.2 \rangle, \langle x_4, 0.9, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \right\}$$

Characterising the linguistic variable as “large” for A and using the above defined operation for A^n , we compute the IFSs $A^{1/2}, A^2, A^3, A^4$ with their usual meaning as follows:

$A^{1/2}$ may be treated as “more or less large”

A^2 may be treated as “very large”

A^3 may be treated as “quite very large”

A^4 may be treated as “very very large”

The computed sets are given by

$$A^{1/2} = \left\{ \langle x_1, 0.3162, 0.5528 \rangle, \langle x_2, 0.5477, 0.2929 \rangle, \langle x_3, 0.7746, 0.1056 \rangle, \langle x_4, 0.9487, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \right\}$$

$$A^2 = \left\{ \langle x_1, 0.01, 0.96 \rangle, \langle x_2, 0.09, 0.75 \rangle, \langle x_3, 0.36, 0.36 \rangle, \langle x_4, 0.81, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \right\}$$

$$A^3 = \left\{ \langle x_1, 0.001, 0.992 \rangle, \langle x_2, 0.027, 0.875 \rangle, \langle x_3, 0.216, 0.488 \rangle, \langle x_4, 0.729, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \right\}$$

$$A^4 = \left\{ \langle x_1, 0.0001, 0.9984 \rangle, \langle x_2, 0.0081, 0.9375 \rangle, \langle x_3, 0.1296, 0.5904 \rangle, \langle x_4, 0.6561, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \right\}$$

The following table-2 shows the different entropy values for the set A .

From the table-2 it has been seen that the entropy measures E_{PP} , E_{ZJ} and E_{SK} follow an identical ordering, which is $E\left(A^{1/2}\right) > E(A) > E\left(A^2\right) > E\left(A^3\right) > E\left(A^4\right)$, whereas the other entropy

Table 1.
Counterintuitive cases are illustrated for different similarity measures

Case	1	2	3	4	5	6
A	$\langle 0.3, 0.3 \rangle$	$\langle 0.3, 0.4 \rangle$	$\langle 1, 0 \rangle$	$\langle 0.5, 0.5 \rangle$	$\langle 0.4, 0.2 \rangle$	$\langle 0.4, 0.2 \rangle$
B	$\langle 0.4, 0.4 \rangle$	$\langle 0.4, 0.3 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 0.5, 0.3 \rangle$	$\langle 0.5, 0.2 \rangle$
Chen (2007) S_C	1	0.9	0.5	1	1	0.95
Hong et la. (1999) S_H	0.9	0.9	0.5	0.5	0.9	0.95
Li & Xu (2001) S_L	0.95	0.9	0.5	0.75	0.95	0.95
Li et al. (2002) S_O	0.9	0.9	0.3	0.5	0.9	0.93
Li et al. (2002) S_{DC}	1	0.9	0.5	1	1	0.95
Mitchell (2003) S_{HB}	0.9	0.9	0.5	0.5	0.9	0.95
Liang et al. (2003) S_e^p	0.9	0.9	0.5	0.5	0.9	0.95
Liang et al. (2003) S_s^p	0.95	0.9	0.5	0.75	0.95	0.95
Liang et al. (2003) S_h^p	0.933	0.933	0.5	0.67	0.933	0.95
Hung et al. (2004) S_{HY}^1	0.9	0.9	0	0.5	0.9	0.9
Hung et al. (2004) S_{HY}^2	0.85	0.85	0	0.38	0.85	0.85
Hung et al. (2004) S_{HY}^3	0.82	0.82	0	0.33	0.82	0.82
Ye (2011) C_{IFS}	1	0.96	0	0	0.9971	0.9965
Boran et al. (2014) S_t^p	0.967	0.9	0.5	0.833	0.937	0.95
Yong (2014) S_Y	0.985	0.994	0.5	0.354	0.936	0.896
S_p	0.7885	0.777	0.25	0.2928	0.7875	0.8621

measures except the entropy E_{BB} follow the ordering as $E(A) > E(A^{\sqrt[1]{2}}) > E(A^2) > E(A^3) > E(A^4)$.

Now we will check the consistency of the proposed entropy measure for a different IFS which is obtained by reducing the hesitancy degree of the middle point x_3 . Thus the IFS A becomes

$$\text{“Large”} = A_1 = \{\langle x_1, 0.1, 0.8 \rangle, \langle x_2, 0.3, 0.5 \rangle, \langle x_3, 0.5, 0.4 \rangle, \langle x_4, 0.9, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle\}.$$

Also

Table-2.
Comparison of different entropy values for the structural linguistic variable A

Entropy Measure for IFS	$A^{1/2}$	A	A^2	A^3	A^4
$E_{hc}^{1/3} \left(\alpha = \frac{1}{3} \right)$	0.982	1.004	0.899	0.782	0.686
$E_{hc}^{1/2} \left(\alpha = \frac{1}{2} \right)$	0.840	0.865	0.754	0.642	0.557
$E_s \left(\alpha = 1 \right)$	0.404	0.414	0.348	0.299	0.264
$E_{hc}^2 \left(\alpha = 2 \right)$	0.328	0.340	0.290	0.253	0.226
$E_{hc}^3 \left(\alpha = 3 \right)$	0.228	0.236	0.203	0.179	0.163
$E_r^{1/3} \left(\beta = \frac{1}{3} \right)$	0.702	0.718	0.659	0.585	0.521
$E_r^{1/2} \left(\beta = \frac{1}{2} \right)$	0.661	0.681	0.605	0.522	0.457
E_{SK}	0.319	0.307	0.301	0.212	0.176
E_{BB}	0.462	0.600	0.660	0.672	0.680
E_{ZJ}	0.2486	0.2117	0.2261	0.0949	0.0457
E_{PP}	0.2884	0.2618	0.1925	0.1535	0.1205

$$\begin{aligned}
 A_1^{1/2} &= \left\{ \langle x_1, 0.3162, 0.5528 \rangle, \langle x_2, 0.5477, 0.2929 \rangle, \langle x_3, 0.7071, 0.2254 \rangle, \langle x_4, 0.9487, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \right\} \\
 A_1^2 &= \left\{ \langle x_1, 0.01, 0.96 \rangle, \langle x_2, 0.09, 0.75 \rangle, \langle x_3, 0.25, 0.64 \rangle, \langle x_4, 0.81, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \right\} \\
 A_1^3 &= \left\{ \langle x_1, 0.001, 0.992 \rangle, \langle x_2, 0.027, 0.875 \rangle, \langle x_3, 0.125, 0.784 \rangle, \langle x_4, 0.729, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \right\} \\
 A_1^4 &= \left\{ \langle x_1, 0.0001, 0.9984 \rangle, \langle x_2, 0.0081, 0.9375 \rangle, \langle x_3, 0.0625, 0.8704 \rangle, \langle x_4, 0.6561, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \right\}
 \end{aligned}$$

The computed entropy values for $A_1^{1/2}, A_1^2, A_1^3, A_1^4$ with their usual meaning are shown in table-3.

For a better comparison and to show the efficiency of the proposed entropy measure, we consider one more IFS “Large” defined as

Table 3.
Comparison of different entropy values for the structural linguistic variable A_1

Entropy Measure for IFS	$A_1^{1/2}$	A_1	A_1^2	A_1^3	A_1^4
$E_{hc}^{1/3} \left(\alpha = \frac{1}{3} \right)$	0.989	0.999	0.867	0.728	0.614
$E_{hc}^{1/2} \left(\alpha = \frac{1}{2} \right)$	0.850	0.860	0.715	0.576	0.471
$E_s \left(\alpha = 1 \right)$	0.433	0.431	0.327	0.253	0.208
$E_{hc}^2 \left(\alpha = 2 \right)$	0.342	0.344	0.261	0.199	0.161
$E_{hc}^3 \left(\alpha = 3 \right)$	0.238	0.241	0.187	0.146	0.120
$E_r^{1/3} \left(\beta = \frac{1}{3} \right)$	0.706	0.715	0.643	0.557	0.482
$E_r^{1/2} \left(\beta = \frac{1}{2} \right)$	0.667	0.678	0.581	0.481	0.401
E_{SK}	0.345	0.371	0.197	0.131	0.109
E_{BB}	0.409	0.500	0.490	0.467	0.467
E_{ZJ}	0.2851	0.3050	0.1042	0.0383	0.0161
E_{PP}	0.3198	0.3362	0.1597	0.0929	0.0707

$A_2 = \{ \langle x_1, 0.1, 0.8 \rangle, \langle x_2, 0.3, 0.5 \rangle, \langle x_3, 0.5, 0.5 \rangle, \langle x_4, 0.9, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \}$, which is obtained by again reducing the hesitancy of the midpoint x_3 to zero. For the IFS A_2 the other IFSs are as follows:

$$\begin{aligned}
 A_2^{1/2} &= \{ \langle x_1, 0.3162, 0.5528 \rangle, \langle x_2, 0.5477, 0.2929 \rangle, \langle x_3, 0.7071, 0.2929 \rangle, \langle x_4, 0.9487, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \} \\
 A_2^2 &= \{ \langle x_1, 0.01, 0.96 \rangle, \langle x_2, 0.09, 0.75 \rangle, \langle x_3, 0.25, 0.75 \rangle, \langle x_4, 0.81, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \} \\
 A_2^3 &= \{ \langle x_1, 0.001, 0.992 \rangle, \langle x_2, 0.027, 0.875 \rangle, \langle x_3, 0.125, 0.875 \rangle, \langle x_4, 0.729, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \} \\
 A_2^4 &= \{ \langle x_1, 0.0001, 0.9984 \rangle, \langle x_2, 0.0081, 0.9375 \rangle, \langle x_3, 0.0625, 0.9375 \rangle, \langle x_4, 0.6561, 0.0 \rangle, \langle x_5, 1.0, 0.0 \rangle \}
 \end{aligned}$$

From the viewpoint of mathematical operations and human intuition, the entropies of these IFSs should satisfy the following requirements for the structural linguistic variable:

Table 4.
Comparison of different entropy values for the structural linguistic variable A_2

Entropy Measure for IFS	$A^{\frac{1}{2}}$	A	A^2	A^3	A^4
$E_{hc}^{\frac{1}{3}}\left(\alpha = \frac{1}{3}\right)$	0.883	0.876	0.737	0.603	0.499
$E_{hc}^{\frac{1}{2}}\left(\alpha = \frac{1}{2}\right)$	0.772	0.763	0.608	0.475	0.382
$E_s\left(\alpha = 1\right)$	0.438	0.427	0.313	0.238	0.196
$E_{hc}^2\left(\alpha = 2\right)$	0.336	0.328	0.233	0.171	0.138
$E_{hc}^3\left(\alpha = 3\right)$	0.237	0.235	0.171	0.127	0.103
$E_r^{\frac{1}{3}}\left(\beta = \frac{1}{3}\right)$	0.644	0.647	0.569	0.482	0.409
$E_r^{\frac{1}{2}}\left(\beta = \frac{1}{2}\right)$	0.615	0.615	0.510	0.409	0.334
E_{SK}	0.352	0.407	0.168	0.110	0.095
E_{BB}	0.342	0.400	0.380	0.376	0.400
E_{ZJ}	0.3042	0.345	0.0927	0.0349	0.0161
E_{PP}	0.3331	0.3759	0.1393	0.0800	0.0624

$$E(A) > E\left(A^{\frac{1}{2}}\right) > E(A^2) > E(A^3) > E(A^4). \quad (8)$$

From the tables 2,3 and table 4 it has been seen that the proposed entropy measure along with the entropy measure E_{SK} and E_{ZJ} satisfies the above requirements (8). Furthermore, these results are reasonable from the view point of structural linguistic variable.

Justification: For the IFSs A, A_1 and A_2 the changes in membership and non membership values occurred only for midpoint x_3 and for the other points the membership and non membership values remained same. Thus, the entropy values for all the IFSs depend only on this midpoint x_3 only. Now, we change the IFS A into A_1 by changing the midpoint $\langle x_3, 0.6, 0.2 \rangle$ to $\langle x_3, 0.5, 0.4 \rangle$. Then the corresponding midpoint of the IFS $A_1^{\frac{1}{2}}$ becomes $\langle x_3, 0.7071, 0.2254 \rangle$. From the entropy measure and the geometrical interpretation for IFS defined by Szmidt and Kacprzyk (2001) the

entropy for the point $\langle x_3, 0.5, 0.4 \rangle$ of A_1 is higher than the point $\langle x_3, 0.7071, 0.2254 \rangle$ of $A_1^{1/2}$. As a result the entropy measure of the IFS A_1 is higher than the entropy measure of the IFS $A_1^{1/2}$, which is clear from the table-2. The same result reflects for the entropy measure defined by Zhang and Jiang (2008). From the view point of our new axiomatic definition of entropy measure if we compare the similarity of the points $\langle x_3, 0.5, 0.4 \rangle$ and $\langle x_3, 0.7071, 0.2254 \rangle$ with the highest IFS $\langle y, 0.5, 0.5 \rangle$ then by our newly defined similarity measure the similarity between the points $\langle x_3, 0.5, 0.4 \rangle$ and $\langle y, 0.5, 0.5 \rangle$ is higher than the similarity between the points $\langle x_3, 0.7071, 0.2254 \rangle$ and $\langle y, 0.5, 0.5 \rangle$. Therefore, from the axiom (E_1^3) it immediately follows that the entropy measure of the point $\langle x_3, 0.5, 0.4 \rangle$ of IFS A_1 is higher than the entropy measure of the point $\langle x_3, 0.7071, 0.2254 \rangle$ of IFS $A_1^{1/2}$. Hence the entropy measure of the IFS A_1 is higher than the entropy measure of the IFS $A_1^{1/2}$, which is seen from the table-2. Similarly we can have the proper explanation for the IFSs A_1^2, A_1^3, A_1^4 . Thus our proposed entropy measure along with the entropy measures defined by Szmidt & Kacprzyk (2001) and Zhang & Jiang (2008) follow the same ordering as

$$E(A_1) > E(A_1^{1/2}) > E(A_1^2) > E(A_1^3) > E(A_1^4)$$

Again we change the IFS A_1 into A_2 by changing $\langle x_3, 0.5, 0.4 \rangle$ to $\langle x_3, 0.5, 0.5 \rangle$. Then the corresponding midpoint of the IFS $A_2^{1/2}$ is $\langle x_3, 0.7071, 0.2929 \rangle$. Again since $\langle x_3, 0.5, 0.5 \rangle$ is the highest intuitionistic fuzzy set, therefore the entropy measure is higher than the entropy measure of the point $\langle x_3, 0.7071, 0.2929 \rangle$. Thus from the table-3 it has been seen that the entropy measures E_{SK}, E_{ZJ} and E_{pp} follow the ordering as

$$E(A_2) > E(A_2^{1/2}) > E(A_2^2) > E(A_2^3) > E(A_2^4)$$

For the IFS A entropy measures E_{pp}, E_{ZJ} and E_{SK} follow the same ordering as

$$E(A^{1/2}) > E(A) > E(A^2) > E(A^3) > E(A^4)$$

and the other entropies $E_{hc}^{1/3}, E_{hc}^{1/2}, E_s, E_{hc}^2, E_{hc}^3, E_r^{1/3}, E_r^{1/2}$, except E_{BB} follow the ordering as

$$E(A) > E(A^{1/2}) > E(A^2) > E(A^3) > E(A^4)$$

When we change the IFS A into A_1 by changing the midpoint $\langle x_3, 0.6, 0.2 \rangle$ to $\langle x_3, 0.5, 0.4 \rangle$ then the corresponding midpoint of the IFS $A_1^{1/2}$ becomes $\langle x_3, 0.7071, 0.2254 \rangle$. We have explained in above that the entropy measure of the point $\langle x_3, 0.5, 0.4 \rangle$ of IFS A_1 is higher than the entropy measure of the point $\langle x_3, 0.7071, 0.2254 \rangle$ of the IFS $A_1^{1/2}$. Hence for the IFSs A_1 and $A_1^{1/2}$ the entropy measures $E_{hc}^{1/3}, E_{hc}^{1/2}, E_s, E_{hc}^2, E_{hc}^3, E_r^{1/2}$ should satisfy the ordering as

$$E(A_1) > E(A_1^{1/2}) > E(A_1^2) > E(A_1^3) > E(A_1^4)$$

Only $E_r^{1/3}$ satisfies the requirements. Similarly since the point $\langle x_3, 0.5, 0.5 \rangle$ of the IFS A_2 has the highest fuzziness compared to $\langle x_3, 0.7071, 0.2929 \rangle$ of the IFS $A_2^{1/2}$, therefore the entropy measures should satisfy the ordering as $E(A_2) > E(A_2^{1/2}) > E(A_2^2) > E(A_2^3) > E(A_2^4)$. Thus it has been seen that the proposed axiomatic approach of entropy measure can be taken as an alternative way to defining new entropy measure with the help of similarity measure. From the comparative study in tables 2, 3, 4 of different entropy measures it can be concluded that the proposed entropy measure is a reliable entropy measure of IFS and it possesses proper order pattern in the view point of structured linguistic variables.

7. A MULTI CRITERIA INTUITIONISTIC FUZZY DECISION MAKING METHOD BASED ON PROPOSED ENTROPY MEASURE

MCDM is the most reliable approach to select the best alternative among a set of alternatives with respect to various criteria when uncertainty or ill defined information involve. In this section, based on the proposed entropy measure of IFS a multi criteria decision making method is discussed where assessments of alternatives for different criteria are specified by IFNs. Let A be the set of n alternatives, $A = \{A_1, A_2, A_3, \dots, A_n\}$, and C be the set of m criteria of the alternatives, $C = \{C_1, C_2, C_3, \dots, C_m\}$. To select the best choice under this setting the following procedures are followed. A (MCDM) problem can be expressed in the matrix format as follows:

$$D = \begin{matrix} & \begin{matrix} C_1 & C_2 & \dots & C_m \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{matrix} & \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix} \end{matrix}$$

$\tilde{x}_{ij} = \langle \mu_{ij}, \nu_{ij} \rangle$ denotes the IFNs. As there are different types of criteria exist for the alternatives, so if all the criteria are of same type then there is no necessary to normalisation of the ratings. Otherwise the benefits criteria can be transformed into cost criteria by using the following normalization formula.

$$r_{ij} = \begin{cases} x_{ij}^c & ; j \in \text{Benefits criteria} \\ x_{ij} & ; j \in \text{Cost criteria} \end{cases}$$

where x_{ij}^c is the complement of x_{ij} . Thus one can obtain the normalised intuitionistic fuzzy decision matrix $D_N = (r_{ij})_{m \times n}$. Then the aggregate entropy measures are calculated by using the proposed entropy measure. As it is general fact that, the less the uncertainty information with the alternatives

with respects to criteria, the better the alternative is. The ranking order is obtained based on this principle.

7.1 Example

Let us consider the multi criteria decision making problem adapted from Nayagam (2011). Assume there exists a panel with four possible alternatives for investment. (1) A_1 is car company, (2) A_2 is a food company, (3) A_3 is a computer company, (4) A_4 is an arms company. The investment entity must make a decision according to the following criteria: C_1 (risk), C_2 (growth), C_3 (environmental impact). Suppose the four possible alternatives are evaluated by decision maker using the intuitionistic fuzzy numbers (IFNs) over the above three criteria as the following matrix.

$$R = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \begin{bmatrix} (0.5, 0.4) & (0.6, 0.4) & (0.3, 0.6) \\ (0.7, 0.3) & (0.7, 0.3) & (0.8, 0.2) \\ (0.6, 0.4) & (0.6, 0.4) & (0.5, 0.3) \\ (0.8, 0.2) & (0.7, 0.3) & (0.4, 0.2) \end{bmatrix} \end{matrix}$$

Where each assessment (x_{ij}, y_{ij}) of the matrix is an IFN and in each entry (x_{ij}, y_{ij}) , x_{ij} and y_{ij} indicate the degree that the alternative A_i satisfies the criteria C_j and the degree that A_i does not satisfy the C_j respectively. Since the less the entropy or uncertainty information of each alternative with respect to the three criteria is, the better the investment the alternative is. Therefore, based on their entropy values we can rank the alternatives.

Using the proposed entropy measure the calculated aggregate entropy values for the alternatives are $E(A_1) = 0.67729$, $E(A_2) = 0.384828$, $E(A_3) = 0.667504$ and $E(A_4) = 0.387824$. The aggregate entropy value of the alternative A_2 is the smallest entropy value that implies that it carries less uncertainty or fuzziness. Therefore, the decision maker can assess more useful information from this alternative. Hence A_2 is the best investment place for an investor. Thus the ranking order for the alternatives is given as $A_2 > A_4 > A_3 > A_1$, which coincides with the ranking obtained by Nayagam et al. (2011).

7.2 Example

Consider the MCDM problem adopted from Garge et al. (2017). Suppose a multinational company in India is planning its financial strategy for the next year, according to group strategy objective. For this, the four alternatives are obtained after their preliminary screening and are defined as below, A_1 : to invest in the southern Asian markets; A_2 : to invest in the Eastern Asian markets; A_3 : to invest in the Northern Asian markets; and A_4 : to invest in the Local markets. This evaluation proceeds from the four aspects namely as G_1 : the growth analysis; G_2 : the risk analysis; G_3 : the social political impact analysis and G_4 : the environmental impact analysis. These four alternatives A_i ($i = 1, 2, 3, 4$) are to be evaluated by corresponding experts, by using the intuitionistic fuzzy decision matrix $D = (\alpha_{ij}) = \langle \mu_{ij}, \nu_{ij} \rangle_{4 \times 4}$, $i = 1, 2, 3, 4; j = 1, 2, 3, 4$ and their corresponding rating is shown below:

$$D = \begin{matrix} & C_1 & C_2 & C_3 & C_4 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \begin{bmatrix} (0.6, 0.2) \\ (0.4, 0.1) \\ (0.6, 0.3) \\ (0.6, 0.3) \end{bmatrix} & \begin{bmatrix} (0.8, 0.1) \\ (0.6, 0.1) \\ (0.7, 0.2) \\ (0.7, 0.1) \end{bmatrix} & \begin{bmatrix} (0.8, 0.1) \\ (0.5, 0.2) \\ (0.6, 0.3) \\ (0.8, 0.1) \end{bmatrix} & \begin{bmatrix} (0.3, 0.5) \\ (0.7, 0.2) \\ (0.4, 0.1) \\ (0.6, 0.2) \end{bmatrix} \end{matrix}$$

Using the proposed entropy measure the aggregate entropy values for the alternatives are $E(A_1) = 0.359672$, $E(A_2) = 0.380197$, $E(A_3) = 0.447422$ and $E(A_4) = 0.354718$. Since decision maker can assess more information from IFS which carries less entropy or fuzziness, therefore, based on this principle the ranking order of these alternatives are $A_4 > A_1 > A_2 > A_3$. Thus, A_4 , the Local market is the best choice for the decision maker.

8. CONCLUSION

Entropy measure of IFS plays an important role in decision making problems. Though different entropy measures have been developed in literature, many of them fail to reflect the exact nature carried out by the IFS. In this paper, based on a new axiomatic approach, a novel entropy measure is defined incorporating the concept of similarity measure of IFS. It has been proved that the proposed entropy measure satisfies all the properties of entropy measure. Consistency and advantages of the newly defined similarity measure are discussed by performing a comparative study with some existing similarity measures. Also in section 6, the comparative study reflects the efficiency and reliability of the proposed entropy measure. Finally, in section 7, the applicability of the proposed entropy measure is successfully exhibited through MCDM problems under IFS environment.

In the future, to enhance the applicability of the proposed entropy measure, it can be extended in a more sophisticated uncertainty environment, for instant, interval valued intuitionistic fuzzy sets and cubic Pythagorean fuzzy sets.

DECLARATIONS

Ethical Approval This article does not contain any studies with animals performed by any of the authors.

CONFLICT OF INTEREST

The authors declare no conflict of interests.

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