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Structure Analysis of General Type-2 Fuzzy Controller and Its Application

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ABSTRACT

In this paper, a general type-2 fuzzy controller is proposed. The proposed general type-2 fuzzy controller is based on α-plane representation, which converts the centroid of general type-2 fuzzy sets to centroids of several interval type-2 fuzzy sets. Also, the mathematical expression of general type-2 fuzzy controller is derived by fuzzy product operate and NT type reduction. By the mathematical expression, the relation between general type-2 fuzzy controller output with parameter of triangular secondary membership function is discussed. Finally, the control performances of proposed general type-2 fuzzy controller are compared with conventional PID controller, tradition type-1 fuzzy controller, and interval type-2 fuzzy controller. Furthermore, to show the effective and practical of the proposed controller, a nonlinear inverted pendulum system is tested. The simulation results show that general type-2 fuzzy controller achieves better control efforts than other compared controllers.

KEYWORDS

α Plane, Controller Structure Analysis, General Type-2 Fuzzy Logic Systems, General Type-2 Fuzzy PID Controller, General Type-2 Fuzzy Sets, Interval Type-2 Fuzzy Controller, Type Reduction

1. INTRODUCTION

PID and type-1 fuzzy controller were widely used in industry processes, but they were unable to handle uncertainties in these practical processes (Kumar, et al.2017). Many researches tried to solve these problems by a novel fuzzy controller, called type-2 fuzzy controller. The type-2 fuzzy controller was based on type-2 fuzzy sets introduced by Zadeh (Zadeh 1975). The main difference between type-1 and type-2 fuzzy sets was that type-2 fuzzy sets contained a type reduction procedure, this was a key step in type-2 fuzzy logic systems. The type reduction first reduced a type-2 fuzzy sets to a type-1 and then defuzzification was operated for type-1 fuzzy sets to get the crisp output. For many years, as the computation complexity of type reduction was very high, there were little applications about type-2 fuzzy logic systems.

Interval type-2 fuzzy sets provided a convenient condition for the developments of type-2 fuzzy logic systems. The secondary membership degree of interval type-2 fuzzy sets was set to 1, which simplified the type reduction procedure for interval type-2 fuzzy sets and the commonly applied type reduction algorithm was Karnik-Mendel (KM) algorithm (Karnik and Mendel.2001). The simulation results and some practical control problems showed that interval type-2 fuzzy controller had better control effects than type-1 fuzzy controller or PID controller. Many researchers tried to derive the analytical structure of interval type-2 fuzzy controller. The frequently used analysis method was input combination (IC) method, which derived from type-1 fuzzy controller structure analysis (Siler

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and Ying.1989; Ying et al.1990; Ying 2006). This method applied zadeh AND operator and divided the input space into several regions, the number of regions was decided by parameters of primary membership function and each region shared the same mathematical expression.

Du and Ying. (2010) analyzed a class of Mamdani interval type-2 fuzzy-PI/ PD controllers structure based on zadeh AND operator and average defuzzifier method. Ni and Tan. (2012) presented the analytical structure of a class of Mamdani interval type-2 fuzzy-PI/PD controllers that had symmetrical rule base and symmetrical consequent sets based on Zadeh AND operator and KM type reduction. El-Nagar and El-Bardini in 2014 analyzed a class of Mamdani interval type-2 fuzzy-PID controllers structure based on zadeh AND operator and a new type reduction method. Aliasghary et al. (2015) obtained the input–output relations of interval type-2 fuzzy logic systems based on Zadeh AND operator and NT type reduction for diamond-shaped primary type-2 membership functions. Raj and Mohan. (2020)described a T-S interval type-2 fuzzy-PI/PD controller structure analysis method using Zadeh AND operator and KM type reduction. Both primary membership functions in related articles were symmetrical. Zhou et al. in(2013,2017,2019) extended the primary membership function of interval type-2 fuzzy controller and obtained a more general Mamdani and T-S fuzzy controller structure analysis methods, also used Zadeh AND operator and KM type reduction.

In most situations, the primary membership functions were all linear, Lei (2016) applied a nonlinear primary membership function and analyzed the structure of interval type-2 fuzzy PI/PD controller. Long (2016) analyzed structure of a class of interval type-2 fuzzy controller using product operator, and proved that such interval type-2 fuzzy controller was equivalent to the sum of two nonlinear PI (or PD) controllers. In these literatures, KM type reduction and Zadeh AND operator were commonly applied. KM type reduction algorithm was an iterative search process, which was time consuming and lacked close-form solution. Zadeh AND operator needed to divide the input space, the number of input space was different according to controller parameters, and the space dividing process was complicated.

For interval type-2 fuzzy controller simplified secondary membership function as 1, so the uncertainties described by interval type-2 fuzzy systems were incomplete. This disadvantage promoted the researches to research general type-2 fuzzy sets for they had more membership function information than interval type-2 fuzzy sets. As the secondary membership function of general type-2 fuzzy sets was a function, so general type-2 fuzzy systems may obtain better performance in some applications with high uncertainties compared with tradition type-1 fuzzy systems and interval type-2 fuzzy systems. There existed many applications of general type-2 fuzzy logic systems, such as: stabilization of DC nanogrids (Mosayebi et al. 2019), glucose level regulation predictive control (Mohammadzadeh and Kumbasar. 2020), frequency control in an ac microgrid (Mohammadzadeh and Kayacan. 2020), medical diagnosis(Ontiveros et al.2020,2021), parrot mambo drone control(Sakalli 2021), human-machine Interfaces(He et al.2021), collaborative fuzzy clustering(Salehi et al.2021), classifier for the pulse level(Carvajal et al.2021), online frequency regulation(Mohammadzadeh et al.2021), multi-switching synchronization of fractional-order chaotic systems(Sabzalian et al.2021), induction motors control(Sedaghati et al.2021), community detection model(Mansoureh et al.2022), steam temperature at collector outlet of trough solar thermal power generation system(Shi 2022), inverted pendulum control(El-Nagar et al.2023), power-line inspection robots(Zhao et al.2020;Zhao et al.2020;Liu et al.2021) and so on.

General type-2 fuzzy sets also contained type reduction procedure, the commonly applied type reduction algorithm of general type-2 fuzzy sets is α-plane representation method (Liu 2008). This method used α-cuts to decompose general type-2 fuzzy sets into collections of interval type-2 fuzzy sets, i.e., α-planes. Thus, general type-2 fuzzy sets type reduction was converted to type reduction of several interval type-2 fuzzy sets. The main shortcoming of existed general type-2 fuzzy controller was that, in most situation, KM type reduction was applied to each α plane. In according with above problems, this paper proposes a structure analysis method for general type-2 fuzzy controller using product operator and NT type reduction.

The novelties of this paper can be described as follows:

1): Centroid of general type-2 fuzzy sets is the union of the centroids of its associated α planes. Each plane is an interval type-2 fuzzy sets whose secondary membership degree is α. NT type reduction algorithm, which uses the average of upper and lower bounds of primary membership degrees, is applied in type reduction for each α plane.
2): As NT type reduction is adopted, the close-form mathematical expression of general type-2 fuzzy controller is obtained in this article, thus it is profited to analyze the structure of general type-2 fuzzy controller.
3): On the basis of general type-2 fuzzy controller mathematical expression, the rules of how secondary membership function parameters affect general type-2 fuzzy controller output are derived.

For compared conveniently, type-1 fuzzy controller, interval type-2 fuzzy controller and general type2- fuzzy controller share the same controller parameters. The simulation results show that general type-2 fuzzy controller has better control effects than PID controller, type-1 fuzzy controller and interval type-2 fuzzy controller using KM type reduction.

2. TYPE-2 FUZZY SETS

2.1 Definition

A type-2 fuzzy sets $\tilde{\boldsymbol{A}}$ on a discrete universal sets $\boldsymbol{X}$ can be characterized by the following definition:

$$\tilde{\boldsymbol{A}} = \{(x,u), \mu_{\tilde{\boldsymbol{A}}}(x,u) \mid \forall x \in \boldsymbol{X}, \forall u \in \boldsymbol{J}_x \in [0,1]\}$$

u is the primary degree and $\mu_{\tilde{\boldsymbol{A}}}(x,u)$ is the secondary degree related to input variable x and primary degree u.

If all secondary degrees are set to 1, then interval type-2 fuzzy sets can be defined as follow:

$$\tilde{\boldsymbol{A}} = \{(x,u), 1 \mid \forall x \in \boldsymbol{X}, \forall u \in \boldsymbol{J}_x \in [0,1]\}$$

If universal set $\boldsymbol{X}$ is continuity, interval type-2 fuzzy sets can be described as follow:

$$\tilde{\boldsymbol{A}} = \int_{x \in \boldsymbol{X}} \int_{u \in \boldsymbol{J}_x} 1 / (x,u), \boldsymbol{J}_x \subseteq [0,1]$$

The secondary membership function of general type-2 fuzzy sets may be chosen as triangular, Gaussian, trapezoid, and so on.

2.2 α-Plane Representation for General Type-2 Fuzzy Sets

Liu (2008) introduced an α-plane representation for general type-2 fuzz sets, and pointed that α cut (α-plane) of general type-2 fuzzy sets, defined as $\tilde{\boldsymbol{A}}_\alpha$,is the union of its all primary degrees whose secondary degrees are greater than or equal to the special value α.

$$\tilde{\boldsymbol{A}}_\alpha = \{(x,u), \mu_{\tilde{\boldsymbol{A}}}(x,u) \geq \alpha \mid \forall x \in \boldsymbol{X}, \forall u \in \boldsymbol{J}_x \in [0,1]\}$$

A general type-2 fuzzy sets $\tilde{A}$ can be represented by the union of its associated type-2 fuzzy sets $\tilde{A}_\alpha$:

$$\tilde{A} = \bigcup_{\alpha\in[0,1]} FOU(\tilde{A}_\alpha)$$

The centroid of general type-2 fuzzy sets can be calculated by the centroids of its associated type-2 fuzzy sets $\tilde{A}_\alpha$, with $\alpha\varepsilon[0, 1]$:

$$C_{\tilde{A}(x)} = \bigcup_{\alpha\in[0,1]} \alpha / C_{\tilde{A}_\alpha(x)}$$

$$C_{\tilde{A}_\alpha(x)} = [l_{\tilde{A}_\alpha}, r_{\tilde{A}_\alpha}]$$

$l_{\tilde{A}_\alpha}$ and $r_{\tilde{A}_\alpha}$ is the left and right end point of interval type-2 fuzzy sets $\tilde{A}_\alpha$ whose secondary degree is α.

3. GENERAL TYPE-2 FUZZY CONTROLLER STRUCTURE

3.1 General Type-2 Fuzzy Control System

The structure scheme of a typical general type-2 fuzzy control system(GT2FCS) is shown as Fig 1. G_E and G_{CE} are scaling factors that transform the error and error derivative to general type-2 fuzzy controller inputs E and $\dot{E}$.

In this paper, the triangular primary membership function is adapted, which is show as Fig 2 and Fig 3. E is defined in domain $[L_1, L_m]$ and $\dot{E}$ is defined in domain $[S_1, S_n]$.

From Fig 2 and Fig 3, the triangular primary membership function is orthogonal, consistent, complete and normal, that is:

$$\overline{u}_{\tilde{A}_i}(E)+\overline{u}_{\tilde{A}_{i+1}}(E) = 1 \ \ and \ \ \overline{u}_{\tilde{A}_k}(E) = 0, i \in [1, m-1], k \neq i, i+1$$

$$\overline{u}_{\tilde{B}_j}(\dot{E})+\overline{u}_{\tilde{B}_{j+1}}(\dot{E}) = 1 \ \ and \ \ \overline{u}_{\tilde{B}_k}(\dot{E}) = 0, j \in [1, n-1] \ ,k \neq j, j+1$$

Figure 1. Structure scheme of general type-2 fuzzy logic system

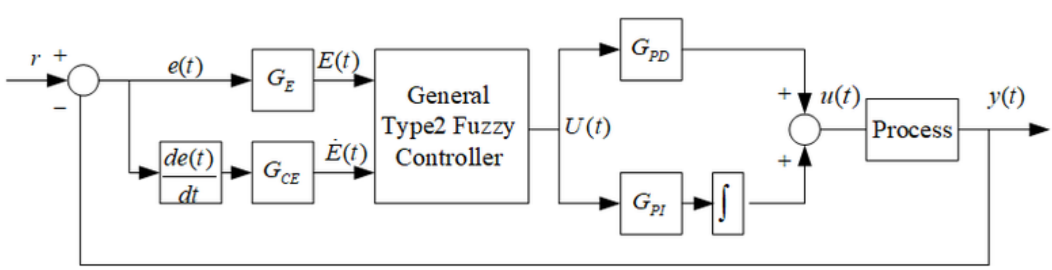


Figure 2. Primary membership function of E

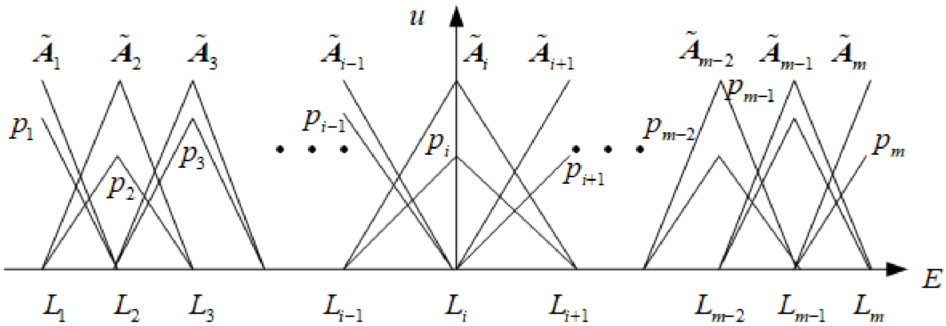
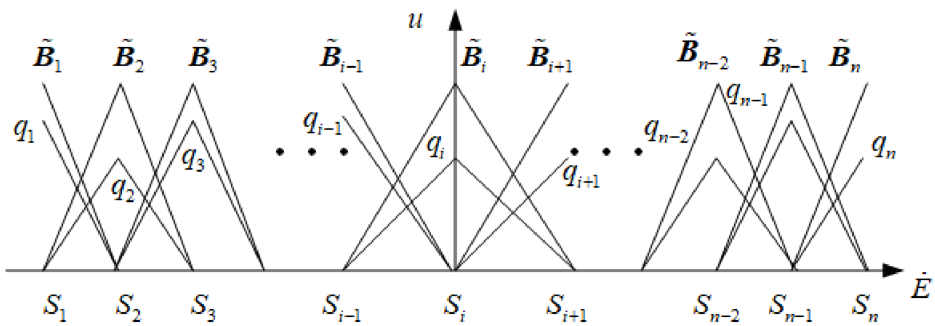


Figure 3. Primary membership function of $\dot{E}$



So, only 4 fuzzy rules will be fired simultaneously, this can be described as follow:

Rule 1:If E is $\tilde{\boldsymbol{A}}_i$ and $\dot{E}$ is $\tilde{\boldsymbol{B}}_j$, then $U=y_1$
Rule 2:If E is $\tilde{\boldsymbol{A}}_i$ and $\dot{E}$ is $\tilde{\boldsymbol{B}}_{j+1}$, then $U = y_2$
Rule 3:If E is $\tilde{\boldsymbol{A}}_{i+1}$ and $\dot{E}$ is $\tilde{\boldsymbol{B}}_j$, then $U = y_3$
Rule 4:If E is $\tilde{\boldsymbol{A}}_{i+1}$ and $\dot{E}$ is $\tilde{\boldsymbol{B}}_{j+1}$, then $U = y_4$

Consequent parameters are crisp values, footprint of uncertain of primary membership degree is decided by p_i or q_i in Fig 3 or Fig 4.

For input E, 2 adjacent fuzzy sets defined in the interval $[L_i, L_{i+1}]$ are $\tilde{\boldsymbol{A}}_i$ and $\tilde{\boldsymbol{A}}_{i+1}$, the upper and lower bounds of primary membership degrees are calculated by equations (1-4).

$$\underline{u}_{\tilde{A}_i}(E) = \frac{p_i \times (E - L_{i+1})}{L_i - L_{i+1}} = p_i \times (a_i \times E - b_i) \tag{1}$$

$$\overline{u}_{\tilde{A}_i}(E) = \frac{(E - L_{i+1})}{L_i - L_{i+1}} = a_i \times E - b_i \tag{2}$$

$$\underline{u}_{\tilde{A}_{i+1}}(E) = \frac{p_{i+1} \times (E - L_i)}{L_{i+1} - L_i} = p_{i+1} \times (a_{i+1} \times E - b_{i+1}) \tag{3}$$

$$\overline{u}_{\tilde{A}_{i+1}}(E) = \frac{(E - L_i)}{L_{i+1} - L_i} = a_{i+1} \times E - b_{i+1} \tag{4}$$

In equations (1-4), $a_i = \dfrac{1}{L_i - L_{i+1}}, b_i = \dfrac{L_{i+1}}{L_i - L_{i+1}}, a_{i+1} = \dfrac{1}{L_{i+1} - L_i}, b_{i+1} = \dfrac{L_i}{L_{i+1} - L_i}$

For input $\dot{E}$, 2 adjacent fuzzy sets defined in the interval $[S_j, S_{j+1}]$ are $\tilde{\boldsymbol{B}}_j$ and $\tilde{\boldsymbol{B}}_{j+1}$, the upper and lower bounds of primary membership degrees are calculated by equations (5-8).

$$\underline{u}_{\tilde{B}_j}(\dot{E}) = \frac{q_j \times (\dot{E} - S_{j+1})}{S_j - S_{j+1}} = q_j \times (c_j \times \dot{E} - d_j) \tag{5}$$

$$\overline{u}_{\tilde{B}_j}(\dot{E}) = \frac{(\dot{E} - S_{j+1})}{S_j - S_{j+1}} = c_j \times \dot{E} - d_j \tag{6}$$

$$\underline{u}_{\tilde{B}_{j+1}}(\dot{E}) = \frac{q_{j+1} \times (\dot{E} - S_j)}{S_{j+1} - S_j} = q_{j+1} \times (c_{j+1} \times \dot{E} - d_{j+1}) \tag{7}$$

$$\overline{u}_{\tilde{B}_{j+1}}(\dot{E}) = \frac{(\dot{E} - S_j)}{S_{j+1} - S_j} = c_{j+1} \times \dot{E} - d_{j+1} \tag{8}$$

In equations (5-8), $c_j = \dfrac{1}{S_j - S_{j+1}}, d_j = \dfrac{S_{j+1}}{S_j - S_{j+1}}, c_{j+1} = \dfrac{1}{S_{j+1} - S_j}, d_{j+1} = \dfrac{S_j}{S_{j+1} - S_j}$

In interval type-2 fuzzy control systems under product operator, the fired membership degree of *i*-th fuzzy rule is an interval and can be defined as equation (9).

$$[\underline{f}_1, \overline{f}_1] = [\underline{u}_{\tilde{A}_i}(E) \times \underline{u}_{\tilde{B}_j}(\dot{E}), \overline{u}_{\tilde{A}_i}(E) \times \overline{u}_{\tilde{B}_j}(\dot{E})] \tag{9.1}$$

$$[\underline{f}_2, \overline{f}_2] = [\underline{u}_{\tilde{A}_i}(E) \times \underline{u}_{\tilde{B}_{j+1}}(\dot{E}), \overline{u}_{\tilde{A}_i}(E) \times \overline{u}_{\tilde{B}_{j+1}}(\dot{E})] \tag{9.2}$$

$$[\underline{f}_3, \overline{f}_3] = [\underline{u}_{\tilde{A}_{i+1}}(E) \times \underline{u}_{\tilde{B}_j}(\dot{E}), \overline{u}_{\tilde{A}_{i+1}}(E) \times \overline{u}_{\tilde{B}_j}(\dot{E})] \tag{9.3}$$

$$[\underline{f}_4, \overline{f}_4] = [\underline{u}_{\tilde{A}_{i+1}}(E) \times \underline{u}_{\tilde{B}_{j+1}}(\dot{E}), \overline{u}_{\tilde{A}_{i+1}}(E) \times \overline{u}_{\tilde{B}_{j+1}}(\dot{E})] \tag{9.4}$$

3.2 NT Type Reduction Algorithm

KM type reduction algorithm was an iteration process, which utilized the switch points to determine the desired endpoints and get the final defuzzification output. At present, most interval type-2 fuzzy controller structure analysis methods were based on KM algorithm. Mendel and Liu.(2013) have proved that NT type reduction was the first order Taylor approximation to KM algorithm. And in this paper, NT type reduction algorithm will be applied in general type-2 fuzzy controller. For NT type reduction used the average of upper and lower bounds of primary membership degree, so it can directly obtain the centroid of interval type-2 fuzzy sets by equations (10-11).

$$\omega_k = \frac{\underline{f}_k + \overline{f}_k}{2} (k = 1, \ldots, M) \tag{10}$$

$$y_{\cos} = \frac{\sum_{k=1}^{M} \omega_k \times y_k}{\sum_{k=1}^{M} \omega_k} = \frac{\sum_{k=1}^{M} (\underline{f}_k + \overline{f}_k) \times y_k}{\sum_{k=1}^{M} (\underline{f}_k + \overline{f}_k)} \tag{11}$$

In (11),*M* is number of fuzzy rules, y_k is consequent parameter of *k*-th rule, $[\underline{f}_k, \overline{f}_k]$ denotes the lower and upper bounds of fired membership degree for *k*-th rule.

4. STRUCTURE ANALYSIS OF GENERAL TYPE-2 FUZZY CONTROLLER

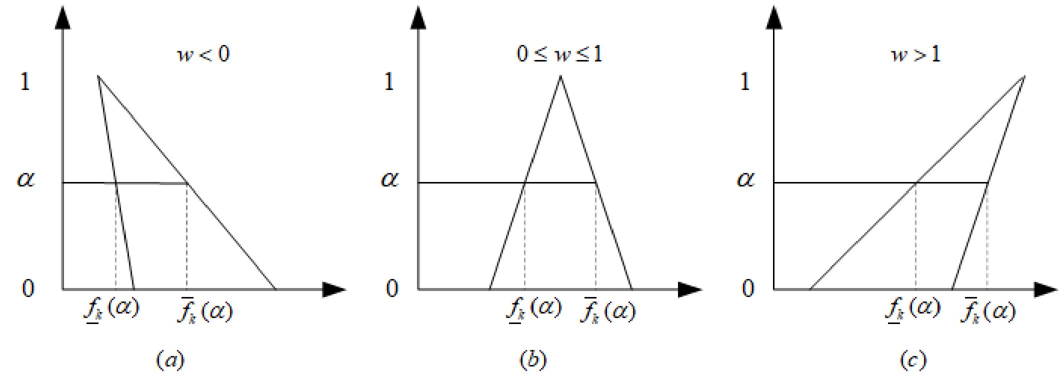
4.1 Mathematical Expression of General Type-2 Fuzzy Controller

In this paper, the triangular function is adapted as the secondary membership function, and the apex of triangular function is defined as equation (12). In (12), w_k is an adjustable parameter, for simplicity, in the following equations w_k is the same and denoted as *w*.

$$Ape(y_k) = \underline{f}_k + w_k \times (\overline{f}_k - \underline{f}_k) \tag{12}$$

Fig 4 demonstrates the triangular secondary membership function as *w* arranged in 3 ranges, that are, $w<0$, $0 \le w \le 1$ and $w>1$.

Figure 4. Triangular secondary membership function



From Fig 4, the lower and upper bounds of primary membership degree in each α plane are shown as (13)-(15).

1). w<0:

$$\underline{f}_k(\alpha) = \underline{f}_k - \alpha \times (\underline{f}_k - Ape(y_k)) \text{ (13-1)}$$
$$\overline{f}_k(\alpha) = \overline{f}_k - \alpha \times (\overline{f}_k - Ape(y_k)) \text{ (13-2)}$$

2). 0≤w≤1:

$$\underline{f}_k(\alpha) = \underline{f}_k + \alpha \times (Ape(y_k) - \underline{f}_k) \text{ (14-1)}$$
$$\overline{f}_k(\alpha) = \overline{f}_k - \alpha \times (\overline{f}_k - Ape(y_k)) \text{ (14-2)}$$

3). w >1:

$$\underline{f}_k(\alpha) = \underline{f}_k + \alpha \times (Ape(y_k) - \underline{f}_k) \text{ (15-1)}$$
$$\overline{f}_k(\alpha) = \overline{f}_k + \alpha \times (Ape(y_k) - \overline{f}_k) \text{ (15-2)}$$

Thus, each α plane can be treated as an interval type-2 fuzzy sets [$\underline{f}_k(\alpha)$, $\overline{f}_k(\alpha)$], whose secondary membership degree is α.

According to equation (13-15), the average of upper and lower bounds of primary membership degree in each α plane is shown as equation (16).

$$\frac{\underline{f}_k(\alpha)+\overline{f}_k(\alpha)}{2} = \frac{(1-\alpha)(\underline{f}_k+\overline{f}_k) + 2\alpha(\underline{f}_k + w \times (\overline{f}_k - \underline{f}_k))}{2} \tag{16}$$

In view of the definition of footprints of uncertainty, the range of w should be [0,1]. However, because of the adaption of NT type reduction, that the average of upper and lower bounds of primary membership degree will be used to calculate the centroids of α planes. Equation (16) indicates that, no matter whatever range of w, the mathematical expression of the average of upper and lower bounds of primary membership degree in each α plane is identical.

From equation (9), the lower and upper bounds of fired membership degree for k-th rule can be calculated by (17):

$$[\underline{f}_1, \overline{f}_1] = [p_i \times (a_i \times E - b_i) \times q_j \times (c_j \times \dot{E} - d_j), (a_i \times E - b_i) \times (c_j \times \dot{E} - d_j)] \tag{17.1}$$

$$[\underline{f}_2, \overline{f}_2] = [p_i \times (a_i \times E - b_i) \times q_{j+1} \times (c_{j+1} \times \dot{E} - d_{j+1}), (a_i \times E - b_i) \times (c_{j+1} \times \dot{E} - d_{j+1})] \tag{17.2}$$

$$[\underline{f}_3, \overline{f}_3] = [p_{i+1} \times (a_{i+1} \times E - b_{i+1}) \times q_j \times (c_j \times \dot{E} - d_j), (a_{i+1} \times E - b_{i+1}) \times (c_j \times \dot{E} - d_j)] \tag{17.3}$$

$$[\underline{f}_4, \overline{f}_4] = [p_{i+1} \times (a_{i+1} \times E - b_{i+1}) \times q_{j+1} \times (c_{j+1} \times \dot{E} - d_{j+1}), (a_{i+1} \times E - b_{i+1}) \times (c_{j+1} \times \dot{E} - d_{j+1})] \tag{17.4}$$

The centroid of each α plane can be calculated as (18).

$$c(\alpha) = \frac{\sum_{k=1}^{4} (\underline{f}_k(\alpha) + \overline{f}_k(\alpha)) \times y_k}{\sum_{k=1}^{4} (\underline{f}_k(\alpha) + \overline{f}_k(\alpha))} \tag{18}$$

By the centroid of each α plane $c(\alpha)$, the general type-2 fuzzy controller output can be calculated as equation (19).

$$U = \frac{\sum_{i=1}^{D} c(\alpha) \times \alpha_i}{\sum_{i=1}^{D} \alpha_i} \tag{19}$$

Where D is number of α planes, α_i is the average value in interval [0,1], which are α_1=0, α_2=1/D, α_3=2/D,…, α_{D-1}=D-1/D,α_D=1.

Define $k_i = \dfrac{\alpha_i}{\sum_{i=1}^{D} \alpha_i}$, then,

$$U = \sum_{i=1}^{D} k_i \times c(\alpha) \tag{20}$$

$$u = G_{PD}U + G_{PI}\int U \tag{21}$$

4.2 Mathematical analysis of general type-2 fuzzy controller

In this section, the mathematical expressions of general type-2 fuzzy controller and the relationship between general type-2 fuzzy controller output U and parameter w will be discussed. For simplicity, the primary membership functions of error and error derivative are identical, which is shown as Fig 5.

The consequent parameters are NB=-1, NM=-0.8, Z=0, PM=0.8, PB=1 and fuzzy rules are shown in Tab 1.

Figure 5. Triangular primary membership function of error and error derivative

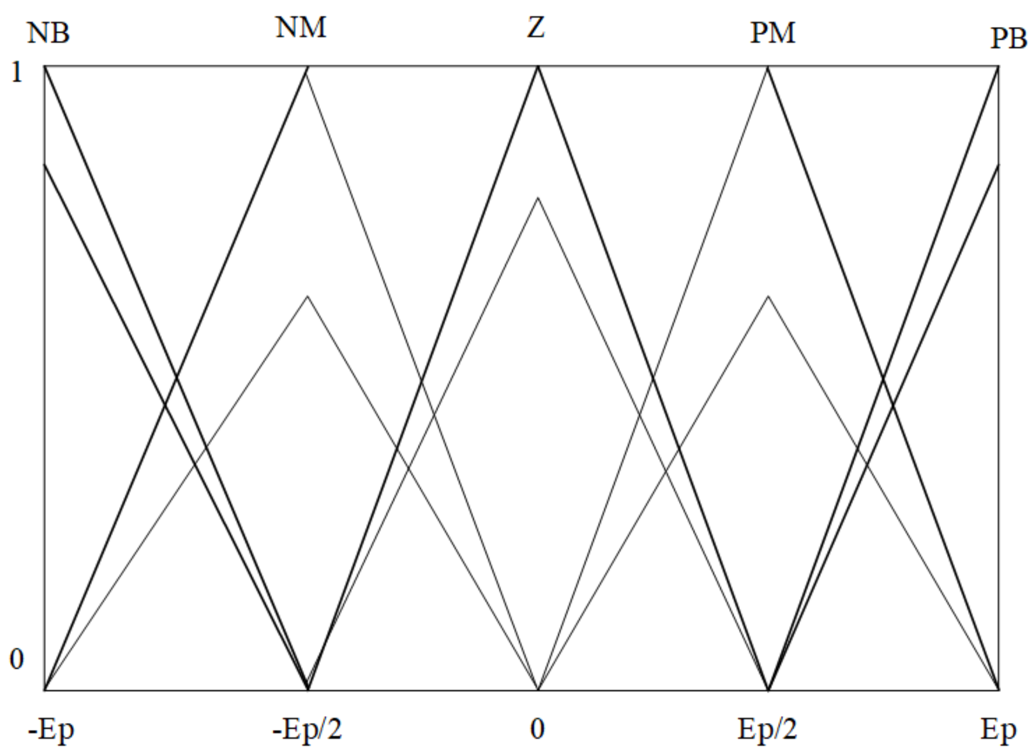


Table 1. General type-2 fuzzy controller rules

$E/\dot{E}$	NB	NM	Z	PM	PB
NB	NB	NB	NB	NM	Z
NM	NB	NB	NM	Z	PM
Z	NB	NM	Z	PM	PB
PM	NM	Z	PM	PB	PB
PB	Z	PM	PB	PB	PB

Suppose at one moment, general type-2 fuzzy controller inputs *E* is -0.8 and $\dot{E}$ is -0.3, then fuzzy variables NB and NM of *E*, NM and Z of $\dot{E}$ are fired simultaneously. So the 4 rules are listed as follows:

Rule 1:If *E* is NB and $\dot{E}$ is NM, then *U*=NB
Rule 2:If *E* is NB and $\dot{E}$ is Z, then *U* = NB
Rule 3:If *E* is NM and $\dot{E}$ is NM, then *U* =NB
Rule 4:If *E* is NM and $\dot{E}$ is Z, then *U* = NM

Firstly, define 4 α planes, the secondary membership degree of each α plane is 0,1/3,2/3,1. By NT type reduction, the centroids of each α plane are shown as (22).

$$\begin{aligned} & c(0) = -0.97 \\ & c(\frac{1}{3}) = -\frac{0.21w+0.55}{0.22w+0.57} \\ & c(\frac{2}{3}) = -\frac{0.41w+0.45}{0.43w+0.46} \\ & c(1) = -\frac{0.62w+0.34}{0.64w+0.35} \end{aligned} \tag{22}$$

According to equation (19), the final general type-2 fuzzy controller output *U* is shown as (23).

$$U = \frac{-0.96w^3 - 4.19w^2 - 4.79w - 1.55}{w^3 + 4.29w^2 + 4.94w + 1.59} \tag{23}$$

Then, calculate the derivative of *U* to *w* as (24).

$$\frac{dU}{dw} = \frac{0.0033}{(w+0.55)^2} + \frac{0.033}{(w+2.66)^2} + \frac{0.033}{(w+1.08)^2} > 0 \tag{24}$$

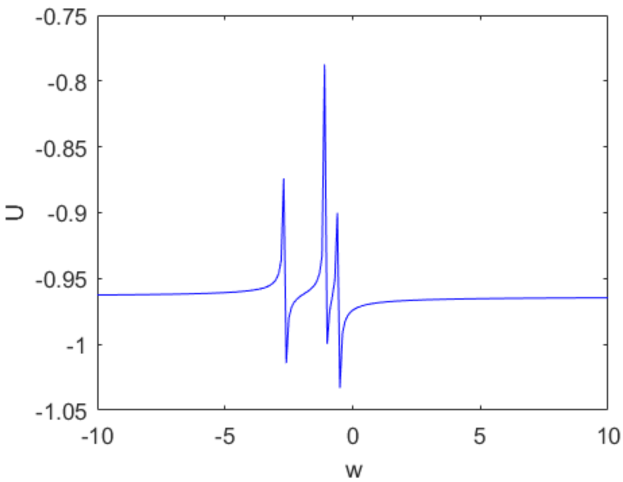
Equation (24) indicates that *U* is an increasing function of *w*, because the denominator of *U* can’t be 0, so there are 3 breakpoints of *U*, that are λ_1=-2.67, λ_2=-1.08, λ_3=-0.55. The limit of *U* is -0.96 when *w* tends to infinity.

From above analysis, the limit of *U* when parameter *w* changes from -¥ to +¥ are shown as equation (25).

$$U = \begin{cases} -0.96 \to +\infty, -\infty < w < \lambda_1 \\ -\infty \to +\infty, \quad \lambda_1 < w < \lambda_2 \\ -\infty \to +\infty, \quad \lambda_2 < w < \lambda_3 \\ -\infty \to -0.96, \lambda_3 < w < +\infty \end{cases} \tag{25}$$

This can be seen from Fig 6, where *w* changes from -10 to 10.

Figure 6. U curve as w changes from -10 to 10(4 planes)



It should be noted that, for there are 4 planes, the mathematical expression of U should be a 4 order equation, but the plane $\alpha = 0$ does not participate in the equation (19). So the order of mathematical expression of U is number of α planes minus 1, in this example is 3.

Furthermore, if define 10 α planes, the secondary membership degree of each α plane is 0,1/10,2/10,…,1. Similar to the previous analysis, U is an increasing function of w and has 9 breakpoints that are λ_1=-8.98, λ_2=-4.24, λ_3=-2.66,λ_4=-1.87,λ_5=-1.40,λ_6=-1.08,λ_7=-0.85,λ_8=-0.69,λ_9=-0.05. The limit of U when parameter w changes from -∞ to +∞ as equation (26).

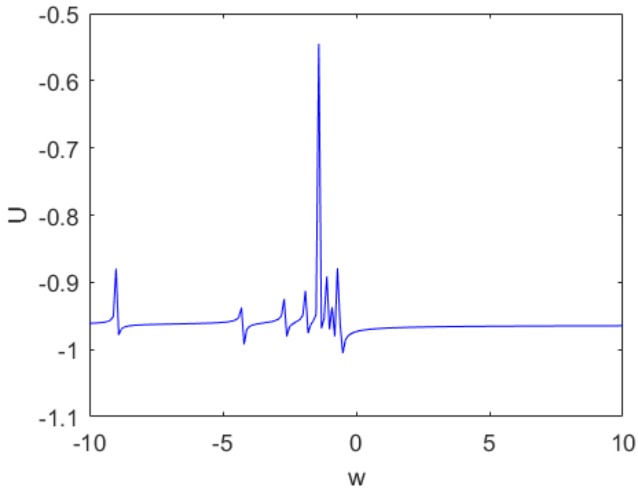
$$U = \begin{cases} -0.97 \to +\infty, & -\infty < w < \lambda_1 \\ -\infty \to +\infty, & \lambda_1 < w < \lambda_2 \\ -\infty \to +\infty, & \lambda_2 < w < \lambda_3 \\ -\infty \to +\infty, & \lambda_3 < w < \lambda_4 \\ -\infty \to +\infty, & \lambda_4 < w < \lambda_5 \\ -\infty \to +\infty, & \lambda_6 < w < \lambda_7 \\ -\infty \to +\infty, & \lambda_8 < w < \lambda_9 \\ -\infty \to -0.97, & \lambda_9 < w < +\infty \end{cases} \tag{26}$$

Fig 7 shows U curve as w changes from -10 to 10, which is consistent with equation (26).

5. SIMULATIONS

In this section, the efficacy of proposed general type-2(GT2) fuzzy controller is applied in 2 linear plants and 2 nonlinear plants. Its performance and robustness will be compared with conventional PID controller, type-1 fuzzy controller(T1) and interval type-2 fuzzy controller using KM type reduction algorithm (IT2KM). Here the 3 fuzzy controllers utilize the same parameters for all cases of per plant to display the robustness of type-1 fuzzy controller, interval type-2 fuzzy controller and

Figure 7. U curve as w changes from -10 to 10(10 planes)



general type-2 fuzzy controller. In simulation 5.1-5.3, the fuzzy universe of discourse is [-1,1] and the consequent parameters are NB=-1, NM=-0.8, Z=0, PM=0.8, PB=1.

5.1 Second Order Stable Linear Plant (P1)

$$G(s) = \frac{1}{s^2 + 2\varepsilon\omega_n s + \omega_n^2} e^{-Ls}$$

PID controller parameters are K_P=0.5837, K_I=0.116, K_D=0.0259, fuzzy controller parameters are G_E=0.7757, G_{CE}=0.7442, G_{PD}=3.5336, G_{PI}=0.6996 and ε=1.6875, ω_n=0.675, L=0.6. Fig 8 shows the system response curves of PID control system,T1 fuzzy control system,IT2 fuzzy control system and GT2 fuzzy control system for P1. The response curves contain system output curves and controller output curves.

Tab 2 summarizes some control performance comparisons of GT2 fuzzy controller with PID controller, T1 fuzzy controller and IT2KM fuzzy controller. In Tab 2, the simulation time is 20s. And control performance indexes include steady state time (t_s, that is time when the absolute value of system error reached the 2% of set value), rising time (t_r, that is time when the system output reach the 2% of set value), overshoot (OS), three error integral criterions (that is integral functions of error between the set value and system output), mainly contain ISE, ITSE, ITAE and can be calculated as follows.

$$ISE = \int_0^{ts} e(t)^2 dt$$

$$ITSE = \int_0^{ts} t \times e(t)^2 dt$$

$$ITAE = \int_0^{ts} t \times \left|e(t)\right| dt$$

Figure 8. System response curves of P1(a) System output curves of P1 (b) Controller output curves of P1

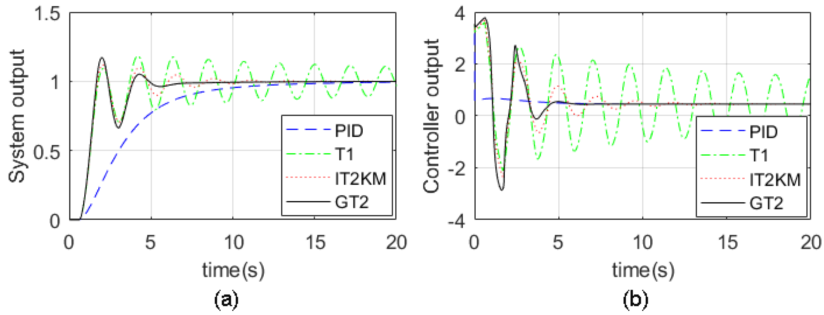


Table 2. Control performance comparisons of GT2 fuzzy controller with other controllers (P1)

P1	PID	T1	IT2KM	GT2
t_s(s)	13.87	Oscillation	8.1	6.49
t_r(s)	13.88		1.72	1.66
OS(%)	0		12	17.1
ISE	2.42		1.174	1.178
ITSE	3.92		0.897	0.893
ITAE	11.05		2.86	2.28

5.2 First Order Unstable Linear Plant (P2)

$$G(s) = \frac{K}{Ts - 1} e^{-Ls}$$

PID controller parameters are K_P=4.1479, K_I=4.0079, K_D=0.1972, fuzzy controller parameters are G_E=0.9956, G_{CE}=0.8387, G_{PD}=0.2532, G_{PI}=4.0573 and K=1, T=10, L=0.4. Fig 8 shows the system response curves of PID control system,T1 fuzzy control system,IT2 fuzzy control system and GT2 fuzzy control system for P2.

Tab 3 shows the P2 control performance comparisons of GT2 fuzzy controller with PID controller, T1 fuzzy controller and IT2KM fuzzy controller. In Tab 3, the simulation time is 30s.

5.3 Second Order Nonlinear Plant(P3)

$$\frac{d^2y(t)}{dt^2} + 2\varepsilon\sigma\frac{dy(t)}{dt} + \sigma^2 y^2(t) = \sigma^2 u(t)$$

PID controller parameters are K_P=0.8028, K_I=1.8548, K_D=0.4609, fuzzy controller parameters are G_E=0.1359, G_{CE}=0.1944, G_{PD}=20.5501, G_{PI}=20.2681 and ε=1, σ=0.7. Fig 10 shows the system response curves of PID control system, T1 fuzzy control system,IT2 fuzzy control system and GT2 fuzzy control system for P3.

Tab 4 shows the P3 control performance comparisons of GT2 fuzzy controller with PID controller, T1 fuzzy controller and IT2KM fuzzy controller. In Tab 4, the simulation time is 10s.

Figure 9. System response curves of P2 (a) System output curves of P2 (b) Controller output curves of P2

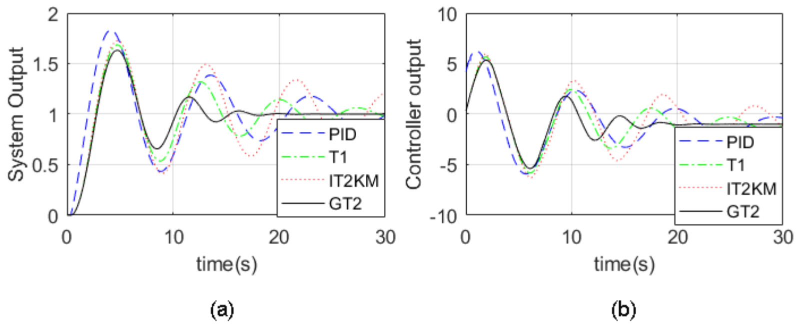


Table 3 Control performance comparisons of GT2 fuzzy controller with other controllers (P2)

P2	PID	T1	IT2KM	GT2
t_s(s)	>30	>30	>30	16.8
t_r(s)	1.94	2.68	2.64	2.75
OS(%)	83.17	68.5	72.7	63.2
ISE	3.92	3.26	4.65	2.61
ITSE	24.23	14.86	36.39	7.52
ITAE	78.33	55.01	107.70	21.65

Figure 10. System response curves of P3 (a) System output curves of P3 (b) Controller output curves of P3

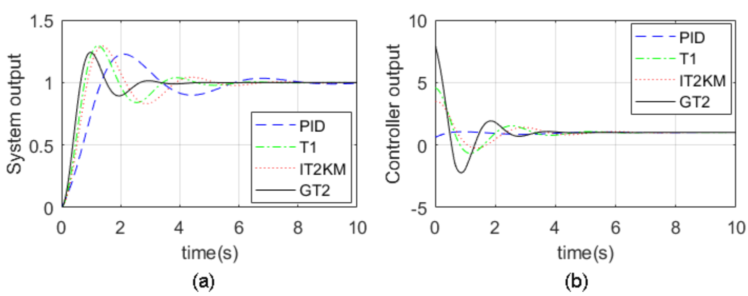


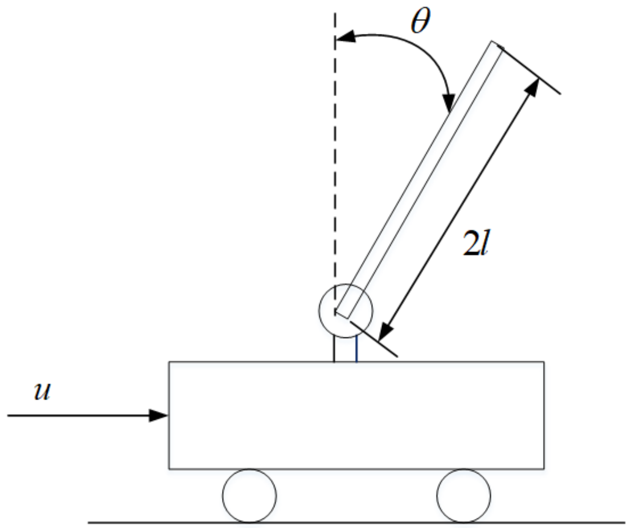
Table 4. Control performance comparisons of GT2 fuzzy controller with other controllers (P3)

P3	PID	T1	IT2KM	GT2
t_s(s)	7.52	5.35	6.13	2.5
t_r(s)	1.3	0.76	0.85	0.63
OS(%)	22.6	29.4	29	24.1
ISE	0.54	0.39	0.44	0.29
ITSE	0.33	0.18	0.23	0.08
ITAE	1.93	0.94	1.25	0.35

5.4 Nonlinear Inverted Pendulum System(P4)

In this simulation, a practical inverted pendulum system is tested to demonstrate the reliability of proposed general type-2 fuzzy controller, which is shown as Fig 11.

Figure 11. Inverted pendulum system



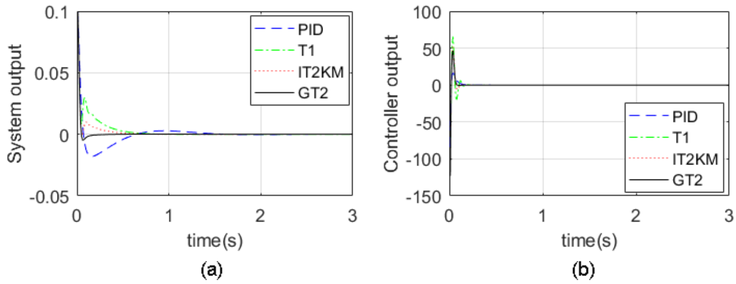
The state equations of the inverted pendulum system (El-Bardini and El-Nagar.2014) can be expressed as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{g\sin(x_1) - \dfrac{m_p l x_2^2 \sin(x_1)\cos(x_1)}{(m_p + m_c)}}{\dfrac{4l}{3} - \dfrac{m_p l \cos(x_1)^2}{(m_p + m_c)}} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{\dfrac{\cos(x_1)}{(m_p + m_c)}}{\dfrac{4l}{3} - \dfrac{m_p l \cos(x_1)^2}{(m_p + m_c)}} \end{bmatrix} u$$

Where, $x_1 = \theta$ is the angle of the pendulum and $x_2 = \dot{x}_1 = \dot{\theta}$ is angular velocity of the pendulum. u is the control force in the unit (Newton) applied horizontally to the cart. The parameters, m_c and m_p, are, respectively, the mass of the cart and the mass of the pendulum in the unit (kg), and g= 9.8m/s^2 is the gravity acceleration. The parameter l is the half length of the pendulum in the unit (m). The parameters of this example are m_c=0.5kg, m_c=0.2kg, l=0.5m.

PID controller parameters are K_P=40, K_I=100, K_D=8 and fuzzy controller parameters are G_E=0.1009, G_{CE}=0.1944, G_{PD}=30.5501, G_{PI}=30.2681. In this example, the fuzzy universe of discourse is [-0.2,0.2] and the consequent parameters are NB=-4, NM=-3.2, Z=0, PM=3.2, PB=4. The initial conditions x_1=0.1rad and x_2=0rad/s, the set value is x_1=0rad. Fig 12 shows the system response curves of PID control system,T1 fuzzy control system,IT2 fuzzy control system and GT2 fuzzy control system for P4.

Figure 12. System response curves of P4 (a) System output of curves P4 (b) Controller output curves of P4



Tab 5 shows the P4 control performance comparisons of GT2 fuzzy controller with PID controller, T1 fuzzy controller and IT2KM fuzzy controller. In Tab 5, the simulation time is 3s, and another two error integral criterions are added as follows.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} e(i)^2}$$

$$IAE = \int_0^{ts} \left|e(t)\right| dt$$

Table 5. Control performance comparisons of GT2 fuzzy controller with other controllers (P4)

P4	IT2F-PID	PID	T1	IT2KM	GT2
ISE	0.036	2.78×10^{-4}	2.31×10^{-4}	1.65×10^{-4}	1.57×10^{-4}
ITSE	-	2.34×10^{-5}	1.34×10^{-5}	3.90×10^{-6}	2.34×10^{-6}
ITAE	-	0.0036	0.0014	5.65×10^{-4}	2.82×10^{-4}
RMSE	0.0085	0.0096	0.0088	0.0074	0.0072
IAE	1.8001	0.0101	0.0074	0.0040	0.0026

From simulation results of 2 linear plants and 2 nonlinear plants, the controller output of general type-2 fuzzy control system is relatively stable. Then the system output is more smooth, thus the system output can reach the steady-state quickly, reduce steady-state time and overshoot. Meanwhile, the general type-2 fuzzy controller can ensure response time. Based on the above system response characteristics, the general type-2 fuzzy control system has smaller error integral criterions(ISE, ITSE, ITAE or RMSE), which are summarized in table 2-5.

6. CONCLUSION

This paper develops a structure derivation method for general type-2 fuzzy controller based on fuzzy product operate. There is no input space dividing process and simplifies the structure analysis procedure. Furthermore, NT type reduction is applied to get the centroid of each α plane directly and obtains the unified mathematical expression of general type-2 fuzzy controller. There have more control parameters of GT2 fuzzy controller than T1 and IT2 fuzzy controller, in this paper, the most parameters are fixed and only the effect of parameters w is discussed. w is an important parameter to determine the effect of proposed general type-2 fuzzy controller when the other parameters are fixed. And how to select a proper w is a challenging issue. That parameter w should be arranged in a certain range, otherwise, the system output will be almost the same or unstable. This can be explained by equation (25) or (26) in section 4.2, when w tends to be infinite, the output of general type-2 fuzzy controller is a fixed value, so the system output will be almost the same as w increases. When the value of w is close to the breakpoints of controller mathematical solution, the output of general type-2 fuzzy controller trends to be infinite, so the system output will be unstable.

The value of α is decided by the number of plane D, from simulations, when D is larger than a certain number, the control effort is almost the same and larger number of D will reduce real-time of GT2 fuzzy controller. In practical, proper D should be chosen to improve the control performance and ensure the real time requirements.

In the future, the following aspects will be studied:

1). Except controller parameter w, the GT2 fuzzy controller contains other parameters, such as parameters of primary membership function, consequent parameters. And how the other parameters of GT2 fuzzy controller effect the control system will be concerned.
2). In this paper, the triangular primary and secondary membership function is applied. Other primary or secondary membership function, like Gaussian or trapezoid, will be applied in GT2 fuzzy controller.

CONFLICTS OF INTEREST

The author declares there is no conflict of interest.

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