Optimizing Fresh Agricultural Product Distribution Paths Under Demand Uncertainty: A Particle Swarm Optimization-Based Algorithm

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ABSTRACT

Consumers’ demand for fresh agricultural products (FAPs) and their quality requirements are increasing in the current agricultural-product consumption market. FAPs’ unique perishability and short shelf-life features mean a high level of delivery efficiency is required to ensure their freshness and quality. However, consumers’ demand for FAPs is contingent and geographically dispersed. Therefore, the conflicting relationship between the costs associated with the logistics distribution and the level of delivery quality is important to consider. In this paper, the authors consider a fresh agricultural-product distribution path planning problem with time windows (FAPDPPPTW). To address the FAPDPPPTW under demand uncertainty, a mixed-integer linear programming model based on robust optimization is proposed. Moreover, a particle swarm optimization algorithm combined with a variable neighborhood search is designed to solve the proposed mathematical model. The numerical experiment results show the robustness and fast convergence of the algorithm.

KEYWORDS

Distribution Routing Optimization, Fresh Agricultural Product, PSO, Soft Time Windows

INTRODUCTION

With the improvement of quality of life and pursuit of a healthy diet, consumers’ demand for fresh agricultural products (FAPs) has grown steadily over the past decade. Examples of FAPs include fruits, vegetables, aquatic products, livestock, and other primary products. According to the Agrifood
Industry 2023 Outlook (Allianz Trade, 2023), the global FAPs market recorded US$8.67 trillion in 2022 and is expected to reach US$12 trillion by 2027. The market has huge potential with the increase in purchasing channels, such as agricultural e-commerce, community group buying, and live broadcast selling (Zhao et al., 2021).

However, the perishable characteristics of FAPs may pose a challenge for distribution planning because the distribution paths must be planned with explicit consideration of freshness and quality requirements. Indeed, Han et al. (2021) pointed out that the main challenges facing agricultural logistics in China are high spoilage and deterioration rate, low distribution efficiency, and high logistics cost in the distribution process. The rapid reduction in FAPs quality during transportation requires they be delivered within consumer-specified times or time windows. Early or delayed delivery will result in lower consumer satisfaction with logistics service quality (Sun et al., 2022).

In addition to the agricultural products’ quality, increasing environmental concerns regarding distribution vehicles’ high fuel consumption and carbon emissions, particularly in cold chain distribution, have become an important factor for consideration in logistics and distribution planning. The optimization of FAPs logistics distribution considering various economic and environmental aspects has been extensively examined in the literature (Bortolini et al., 2016; Chen et al., 2020; Devapriya et al., 2017; Kwon et al., 2013; Li et al., 2020; Rong et al., 2011; Sun et al., 2022; Wang et al., 2020).

The key to improving FAPs logistics distribution systems lies in effective distribution path planning. The distribution path planning problem can also be viewed as the vehicle routing problem (VRP) (Dantzig & Ramser, 1959). In the context of FAPs distribution path optimization, the vehicle routing problem with time windows (VRPTW) is typically formulated to ensure timely delivery by distribution vehicles while achieving the shortest transportation distance (time) and, thus, the lowest transportation cost (e.g., Amorim et al., 2014; Chen et al., 2009; Hsu et al., 2007; Naso et al., 2007; Ombuki et al., 2006; Osvald & Stirn, 2008; Shukla & Jharkharia, 2013; Xia & Fu, 2019).

Xia and Fu (2019) pointed out that although serving consumers’ demand for FAPs with hard time-window requirements (i.e., when the delivery is made within the specified time window) is conducive to achieving high consumer satisfaction with logistics services, this may cause low vehicle utilization and restrict the choice of distribution paths. In turn, it will result in an increased number of vehicles and higher logistics distribution costs. For this reason, soft time windows (i.e., when delivery can be made outside the specified time window) would be more advantageous in terms of gaining flexibility in distribution routing.

Solving VRP/VRPTW models can be extremely computationally challenging because the VRP/VRPTW is recognized as a combinatorial integer programming problem, which is NP-hard in general (Savelsbergh, 1985). Hence, the use of state-of-the-art heuristic/metaheuristic approaches is usually required. For example, Xia and Fu (2019) constructed a bi-objective programming model for the VRP with soft time windows and satisfaction rate. Moreover, they designed an enhanced tabu-search algorithm, numerically showing its superiority over other metaheuristic methods reported in the literature. Similarly, Gmira et al. (2021) proposed a tabu-search heuristic-based solution approach for solving the VRPWT in which the time-dependent travel time associated with each arc in the distribution network is considered. Hiermann et al. (2019) developed an integrated routing and vehicle selection model to address the VRPTW involving multiple vehicle types. For this, they proposed a solution method combining a genetic algorithm with neighborhood search. Chiang and Russell (1996) described a simulated annealing procedure for the VRPTW. Importantly, their computational results suggest the solution method has potential, in terms of solution quality and computational time, to be implemented in large-scale VRPTW environments.

This article presents a solution method based on particle swarm optimization (PSO). PSO, essentially a random search algorithm proposed by Kennedy and Eberhart (1995), seeks to iteratively improve the candidate solutions obtained (called “particles”) by traversing the search space (called a “swarm”) until the best-known solution is attained. PSO has the characteristics of strong robustness
and fast convergence; therefore, it has been successfully applied to solve NP-hard optimization problems, including the VRP and its variants. The results demonstrate that PSO can be very effective for large VRP instances (e.g., Ai & Kachitvichyanukul, 2009; Gong et al., 2011; Kachitvichyanukul et al., 2015; Khouadjia et al., 2012; Li et al., 2019). To enhance the performance of PSO, many hybrid metaheuristic approaches have been designed in which PSO serves as a main search engine. In addition, another heuristic is introduced in the local search procedure (e.g., Guo et al., 2017; Hannan et al., 2018).

The current study looked at a fresh agricultural product distribution path planning problem with time windows (FAPDPPPTW) in which soft time windows are assumed. The distribution network under study consists of a single distribution center and multiple demand locations. The distribution vehicles should be carefully planned to deliver FAPs from the distribution center to all demand locations; however, the distribution routing needs to be determined by considering the costs associated with vehicle distribution, freshness degradation, and penalty of time-window violation. The freshness degradation of agricultural products occurring during the distribution process can be characterized by the quality deterioration (Cai et al., 2013; Chen et al., 2009; Chen et al., 2018) and physical quantity deterioration (or quantity loss) (Qin et al., 2014; Wang & Chen, 2017), which reflects the portion of FAPs spoiled or damaged when being transported. In this study, the authors only considered the quantity deterioration over time, the impact of which is incorporated as part of the overall distribution cost.

The contributions of this research are threefold. First, different from the previous studies in the VRP/VRPTW literature that typically assumed deterministic or stochastic demand with known distribution, this study assumes that demand at each location point varies within an interval with known mean and deviation. Accordingly, the authors develop a robust optimization model for FAPDPPPTW (termed RO-FAPDPPPTW). To the best of the authors’ knowledge, limited research has dealt with a robust version of VRP/VRPTW in the fresh produce distribution literature (e.g., Hu et al., 2018; Liu & Zhang, 2023; Tirkolaee et al., 2020; Yan et al., 2021). Second, because of NP-hardness, the authors design a solution method based on PSO that can be used to solve large-sized problem instances. Finally, the authors perform computational studies using the benchmark problem instances to show the proposed solution method’s effectiveness and efficiency.

The remainder of this article is organized as follows. Section 2 describes FAPDPPPTW and provides the corresponding RO-FAPDPPPTW model. The PSO-based solution algorithm is introduced in Section 3. In Section 4, the computational results are presented and discussed. Finally, Section 5 concludes the article and provides directions for future research.

MODEL FORMULATION

Problem Description

In this section, the authors formulate a robust optimization model of the FAPDPPPTW. The distribution network considered consists of a single distribution center and multiple demand points with known geographic locations. The distribution center has enough vehicles undertaking distribution tasks, which depart from the distribution center and deliver one type of FAP to all the demand points according to the planned routes. These distribution vehicles have the identical maximum load capacity while their travel speeds may differ. Each demand location can be visited exactly once and served by one distribution vehicle. Each distribution vehicle can serve multiple demand points. Once the vehicles arrive at the demand points, they must provide an unloading service. The authors assume the unloading service time may vary at different demand points. After the distribution services are complete, all the vehicles return to the distribution center. This distribution network setting has been widely considered in the VRP/VRPTW literature (e.g., Ai & Kachitvichyanukul, 2009; Chen et al., 2020; Hu et al., 2018).
The authors assume a known time window at each demand location; however, the time windows may vary across locations. The time windows are soft. A vehicle may arrive at a demand point outside the time window, but a time cost will be incurred due to the early or late delivery. Specifically, if the vehicle arrives early at the demand location, it must wait until the earliest time for delivery service. Thus, an extra waiting cost will be incurred. In contrast, if the vehicle arrives late at the demand location, an overtime penalty cost will be incurred. The adoption of soft time window assumption can be also found in Fu et al. (2008), Taş et al. (2014), and Xia and Fu (2019).

Demand for the FAPs is random. Plus, all the demand must be satisfied. To ensure that each demand point can be served by only one vehicle, the authors further assume that the quantity required at any demand point shall not exceed the maximum load of the vehicle. In addition, because of the perishable nature of the FAP, the quantity deterioration aspect during the transportation process was considered. Inspired by Sana et al. (2004) and other works in which the inventory models with deteriorating items are studied, the authors characterize the distribution cost associated with the quantity deterioration by considering Weibull distributed deterioration. The goal of the FAPDPPPTW is to produce the distribution route solution yielding the lowest distribution cost and highest delivery quality.

Notations and Decision Variables

To formulate the FAPDPPPTW model, the authors use the graph \( G = (N, E) \) to represent the distribution network considered. \( N \) and \( E \) denote the set of nodes and set of paths in the graph, respectively. Let 0 denote the distribution center and \( C = \{1, 2, \ldots, n\} \) denote the set of consumer demand points. Thus, \( N = 0 \cup C = 0, 1, \ldots, n \). The set of distribution vehicles is denoted by \( V = \{1, \ldots, k\} \). The path between locations \( i \) and \( j \) is denoted as \((i, j)\); therefore, the set of paths in the network can be expressed by \( E = \{(i, j) | i, j \in N, i \neq j\} \). The travel time between locations \( i \) and \( j \) for vehicle \( k \) is denoted by \( t_{ijk} \), where \((i, j) \in E \) and \( k \in V \).

Four types of decision variables need determining in the mathematical formulation for the FAPDPPPTW. First, let \( x_{ijk} \) be the 0–1 variable corresponding to distribution routing for all \((i, j) \in E \) and \( k \in V \). If \( x_{ijk} = 1 \), vehicle \( k \) travels on path \((i, j)\) and 0 otherwise. Then, let \( y_{ik} \) be another 0–1 variable related to the delivery for all \( i \in C \) and \( k \in V \). If \( y_{ik} = 1 \), demand location \( i \) is served by vehicle \( k \) and 0 otherwise. Finally, let \( \beta_{ik} \) and \( g_{ik} \) denote the arrival and departure time of vehicle \( k \) at demand point \( i \), respectively. Other notations used for the mathematical formulation are defined in Table 1.

Distribution Cost Analysis

The distribution costs in the FAPDPPPTW model include the fixed cost of vehicle use, distribution (time) cost, quantity loss cost, and penalty cost. The mathematical expressions for these costs are described in Table 1.

**Fixed Cost \( (C_1) \)**

The fixed cost includes various types of costs of using vehicles for distribution tasks. This includes the vehicle repair and maintenance cost, labor cost of drivers, and vehicle depreciation cost. In addition, the fixed cost depends on the number of vehicles used for distribution. The fixed cost can be expressed as:

\[
C_1 = P_k \sum_{k \in V} \sum_{j \in C} r_{0jk} \tag{1}
\]
Table 1. Notations for the mathematical model

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sets</strong></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>Distribution center</td>
</tr>
<tr>
<td>$C$</td>
<td>Set of demand nodes, $C = {1, 2, \ldots, n}$</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of nodes in the distribution network, $N = {0, 1, 2, \ldots, n}$</td>
</tr>
<tr>
<td>$E$</td>
<td>Set of paths in the distribution network</td>
</tr>
<tr>
<td>$V$</td>
<td>Set of distribution vehicles owned by the distribution center</td>
</tr>
<tr>
<td><strong>Parameter</strong></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>Number of demand points</td>
</tr>
<tr>
<td>$k$</td>
<td>Number of distribution vehicles</td>
</tr>
<tr>
<td>$Q$</td>
<td>Maximum load capacity of each vehicle</td>
</tr>
<tr>
<td>$q_i$</td>
<td>Demand at location $i$</td>
</tr>
<tr>
<td>$w_i$</td>
<td>Unloading service time of each vehicle at demand point $i$</td>
</tr>
<tr>
<td>$[s_i, e_i]$</td>
<td>Time window specified by consumers at demand point $i$, where $s_i$ and $e_i$ indicate the earliest and latest delivery time, respectively</td>
</tr>
<tr>
<td>$[a_i, b_i]$</td>
<td>Time window that can be accepted by consumers at demand point $i$, where $a_i$ and $b_i$ indicate the earliest and latest delivery time consumers can accept, respectively</td>
</tr>
<tr>
<td>$\sigma_{ik}$</td>
<td>Waiting time of vehicle $k$ at demand point $i$ in case of early arrival. If $\beta_{ik} &lt; s_i$, $\sigma_{ik} = s_i - \beta_{ik}$; otherwise, $\sigma_{ik} = 0$</td>
</tr>
<tr>
<td>$P$</td>
<td>Unit cost of the fresh produce</td>
</tr>
<tr>
<td>$P_1$</td>
<td>Fixed cost per vehicle used</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Cost per unit of time for vehicle use during transportation</td>
</tr>
<tr>
<td>$t_{0jk}$</td>
<td>Travel time required from the distribution center to demand point $j$ for vehicle $k$</td>
</tr>
<tr>
<td>$t_{ijk}$</td>
<td>Travel time required on path $(i, j)$ for vehicle $k$</td>
</tr>
</tbody>
</table>
where \( x_{0,ik} = 1 \) if vehicle \( k \) leaves the distribution center (that is, the vehicle is used) and 0 otherwise.

### Distribution Time Cost \((C_2)\)

Distribution time cost refers to the time cost that varies with the paths travelled and demand points served in the distribution process. This type of cost can be derived by multiplying the total distribution time of the vehicles and the corresponding cost of vehicle use per unit of time. The distribution time is composed of travel time, waiting time, and service time (i.e., unloading service time). The authors assume travel speed may vary by vehicle; thus, the travel time of vehicles on the same path may be different (i.e., \( t_{ijk} \)). The distribution cost is calculated as:

\[
C_2 = P_2 \left( \sum_{k \in V} \sum_{j \in N} \sum_{i \in N} t_{ijk} x_{ijk} + \sum_{i \in C} \sum_{k \in V} \sigma_{ik} y_{ik} + \sum_{i \in C} \sum_{k \in V} w_{ik} y_{ik} \right)
\]

(2)

where \( \sigma_{ik} = \max \left( s_i - \beta_{ik}, 0 \right) \) for all \( i \in C \) and \( k \in V \).

### Cost of Quantity Loss \((C_3)\)

As discussed, the quantity deterioration of the FAP during the distribution process is considered in this study. Thus, the corresponding quantity loss cost is generated. In the inventory literature considering perishable items, Weibull distribution has been commonly used to describe the item deterioration (Covert & Philip, 1973; Qin et al., 2014; Skouri et al., 2009; Yang, 2012). Following these prior works, the authors assume the deterioration rate of the physical quantity of the FAP at time \( t \) follows the three-parameter Weibull distribution whose probability density function is

\[
f(t) = \alpha \theta (t - \gamma)^{-\alpha} e^{-\alpha(t-\gamma)^\theta}, \quad \alpha \geq 0, \theta \geq 1, t \geq \gamma \geq 0,
\]

where \( \alpha, \theta, \gamma \) are parameters of Weibull distribution. Notably, \( f(t) = 0 \) when \( t < \gamma \), suggesting the deterioration occurs after time \( \gamma \).

To ensure complete demand satisfaction, the actual volume of FAP being transported by vehicle \( k \) from the distribution center to demand location \( i \) is given by:

\[
Q_{ik} = \frac{q_i}{1 - F(t)} = \frac{q_i}{1 - \left(1 - e^{-\alpha(t-\gamma)^\theta}\right)} = q_i e^{\alpha(t-\gamma)^\theta}
\]
where \( F(t) \) is the cumulative density function of the Weibull distribution. The denominator \( 1 - F(t) \) in the above formula represents the non-deterioration rate of FAP during transportation.

Replacing \( t \) by arriving time at demand location \( \beta_{ik} \), the authors obtain the following expression for the quantity loss caused in the transportation from the distribution center to demand location \( i \) by vehicle \( k \):

\[
QL_{ik} = q_i \left( e^{\alpha(\beta_{ik} - \gamma)} - 1 \right)
\]  

Thus, with the assumption of Weibull distributed deterioration, the cost associated with quantity loss can be calculated as follows:

\[
C_3 = P \sum_{i \in C} \sum_{k \in V} QL_{ik} y_{ik} = P \sum_{i \in C} \sum_{k \in V} q_i \left( e^{\alpha(\beta_{ik} - \gamma)} - 1 \right) y_{ik}
\]  

Notice that the quantity loss cost is mainly affected by three factors: (1) the unit cost of the FAP \( (P) \); (2) the demand quantity \( (q_i) \); and (3) the travel time required for the delivery \( (\beta_{ik} - \gamma) \).

**Penalty Cost \( (C_4) \)**

In addition to the quantity deterioration rate, the delivery time is an important indicator to measure the service quality in the VRP for FAPs (Sun et al., 2022). An early or late delivery can result in the reduction of consumer satisfaction and, thus, incur penalty costs. In this article, the authors consider the penalty costs in five cases according to the delivery time. The corresponding unit penalty cost \( f_e \) with the delivery time is depicted in Figure 1.

As can be seen from the figure, if vehicle \( k \) arrives too early or too late at demand location \( i \) (i.e., \( \beta_{ik} \in [0, a_i] \) or \( \beta_{ik} \in (b_i, +\infty) \)), the unit waiting and late penalty costs are \( ep \) and \( lp \), respectively. If the vehicle arrives within the time interval \( [a_i, b_i] \), the unit penalty cost will decrease.
with the delivery time. In contrast, if the vehicle arrives within the time interval \( [e_i, b_i] \), the unit penalty cost will increase with the delivery time. If the time when the vehicle arrives is within the time interval \( [s_i, e_i] \), the unit penalty cost is 0. Thus, the penalty cost function with the delivery time \( \beta_{ik} \) is expressed as:

\[
C_4(\beta_{ik}) = \begin{cases} 
ep(a_i - \beta_{ik}) + \frac{ep(s_i - a_i)}{2}, & \beta_{ik} < a_i \\
ep(s_i - \beta_{ik})^2, & a_i \leq \beta_{ik} < s_i \\
0, & s_i \leq \beta_{ik} \leq s_i \\
lp(\beta_{ik} - e_i)^2, & e_i < \beta_{ik} \leq b_i \\
ep(\beta_{ik} - b_i) + \frac{lp(b_i - e_i)}{2}, & b_i < \beta_{ik}
\end{cases}
\]  

\[(5)\]

**RO-FAPDPPPTW Model**

Consumer demand for FAPs may involve a high degree of uncertainty because consumers are sensitive to freshness and logistics service quality. The presence of such uncertainty makes deterministic VRP/VRPTW models less reliable when tackling real-world FAPDPPPTW applications (Liang et al., 2021). To study the FAPDPPPTW under demand uncertainty, the authors formulate the problem under robust optimization (RO) theory, as discussed in Ben-Tal et al. (2009). Uncertain parameters are assumed to vary within intervals with known mean and deviation. The largest deviations from their mean values are restricted by the so-called budget-of-uncertainty (Bertsimas & Sim, 2004).

Following the budget uncertainty sets introduced by Agra et al. (2012) and Hu et al. (2018) for the uncertain VRPTW, the authors assume that the random demand variable \( q_i \) for all \( i \in C \) varies within the symmetric interval \( \left[ \bar{q}_i - \tilde{q}_i, \bar{q}_i + \tilde{q}_i \right] \), where \( \bar{q}_i \) and \( \tilde{q}_i \) represent the mean demand and maximum deviation from the mean value, respectively. In addition, the authors define the auxiliary variable \( \eta_i = \left( \bar{q}_i - \tilde{q}_i \right) / \tilde{q}_i \), which varies in \([-1,1]\). Then, the following demand uncertainty set \( U_q \) is considered for the robust version of the FAPDPPPTW:

\[
U_q = \times_{k \in V} U_q^k
\]

\[(6)\]

where:

\[
U_q^k = \left\{ \tilde{q} \in R^{N_k^q} \left| \tilde{q}_i = \bar{q}_i + \eta_i \tilde{q}_i, \sum_{i \in N_k^q} |\eta_i| \leq \Gamma_q^k, |\eta_i| \leq 1, \Gamma_q^k = \theta_q^k N_k^q, \forall i \in N_k^q \right. \right\}
\]

\[(7)\]

Equation (7) defines the demand budget uncertainty set \( U_q^k \) for each vehicle \( k \), where \( N_k^q \) denotes the set of consumers on the distribution route of vehicle \( k \) and \( \Gamma_q^k \) is the uncertainty budget that reflects the degree of demand uncertainty on the travel route of vehicle \( k \). The value of \( \Gamma_q^k \) is
set to $\theta_q \left| N^k \right|$, where $\theta_q$ denotes the demand uncertainty budget factor and takes a value between 0 and 1. $\omega$ represents the smallest integer that is greater than or equal to $\omega$. Together, $\theta_q$ and $\left| N^k \right|$ determine the upper limit imposed on the number of consumer locations with high demand uncertainty.

Agra et al. (2012) investigated the robust VRPTW model under demand and travel time uncertainty. According to their results, only the extreme points in the budget uncertainty set need to be considered in the resulting robust formulation. Thus, the authors formulate the RO-FAPDPPPTW with the consideration of demand vector $q \in \text{ext} \left( U_q \right)$, where $\text{ext} \left( U_q \right)$ denotes all extreme points of set $U_q$ given in Equation (6), as:

$$\min C = C_1 + C_2 + C_3 + \sum_{i \in C} \sum_{k \in V} C \left( \beta_{ik} \left( q \right) \right)$$

s.t.:

$$\sum_{j \in C} x_{0,jk} = \sum_{i \in C} x_{i0k} \leq 1, \forall k \in V$$

(9)

$$\sum_{k \in V} y_{ik} = 1, \forall i \in C$$

(10)

$$\sum_{i \in N} x_{ijk} = \sum_{j \in N} x_{ijk}, \forall i, j \in N, k \in V$$

(11)

$$\sum_{i \in N} \sum_{j \in N} x_{ijk} \leq Q, \forall k \in V, q \in \text{ext} \left( U_q \right)$$

(12)

$$\sum_{i \in N} x_{ijk} = y_{jk}, \forall j \in N, k \in V$$

(13)

$$\beta_{jk} \left( q \right) = g_{ik} \left( q \right) + t_{ij}, \forall \left( i, j \right) \in E, q \in \text{ext} \left( U_q \right)$$

(14)

$$g_{ik} \left( q \right) = \begin{cases} \beta_{ik} \left( q \right) \left( 1 - y_{ik} \right) + \left( s_i + w_i \right) y_{ik}, & \beta_{ik} \left( q \right) < s_i \\ \beta_{ik} \left( q \right) + w_i y_{ik}, & \beta_{ik} \left( q \right) \geq s_i \end{cases}$$

(15)
\[ \beta_{ik}(q) \geq 0, g_{ik}(q) \geq 0, \forall i \in N, k \in V, q \in \text{ext}(U_q) \]  

(16)

\[ x_{ijk} \in \{0,1\}, y_{ijk} \in \{0,1\}, \forall (i,j) \in E, k \in V \]  

(17)

Objective function (8) minimizes the total cost, including the fixed cost of vehicle use, vehicle distribution time cost, quantity loss cost, and penalty cost of time-window violation. Constraint (9) ensures that the vehicles departing from the distribution center will return to the distribution center after the distribution task is completed and each vehicle can be used once (at most). Constraint (10) guarantees each demand location is served by only one vehicle. Constraint (11) is the flow balance constraint, ensuring that each vehicle arrives and leaves the same demand point. Constraint (12) indicates the load on each vehicle used should not exceed the maximum load capacity of the vehicle. Constraint (13) shows a demand location can be served by a vehicle only when that vehicle travels through the location. Constraints (14) and (15) calculate the arrival and departure time of a vehicle at each location, respectively. Constraints (16) and (17) state the decision variables. Note that Constraint (15) contains a quadratic term, which can be linearized using classical Big-M method. Note also that variables \( \beta_{ik}(q) \) and \( g_{ik}(q) \) become functions of \( q \in U_q \). This is because, under the RO of Ben-Tal et al. (2009), some decision variables are allowed to adapt themselves as uncertain parameters are revealed and, thus, become functions of the uncertainty (i.e., demand).

**SOLUTION ALGORITHM**

The proposed RO-FAPDPPPTW model (8)-(17) is difficult to solve for large-scale instances. In this article, a hybrid variable neighborhood search and particle swarm optimization (VNS-PSO) algorithm is designed. As mentioned, PSO is a random search algorithm that aims to iteratively improve the candidate solutions (i.e., particles) by traversing the feasible solution space. This has been successfully applied on solving VRP/VRPTW-related problems. The heuristics-based variable neighborhood search (VNS) (Mladenović & Hansen, 1997) is used as an improvement algorithm. This is embedded to improve the search efficiency of PSO and can effectively avoid the particle swarm falling into local convergence in the search process, thus maintaining the diversity of the particles.

The structure of the VNS-PSO algorithm is identical to that of the standard PSO algorithm; however, the best solution found by PSO is further improved by VNS in each iteration. In PSO, the algorithm randomly initializes a group of particles (i.e., feasible candidate solutions). Each particle, such as \( i \)-th, has two main attributes in the search space \( d \) (namely, the current position \( x_i^d \) and velocity \( v_i^d \)). In the context of the FAPDPPPTW, the particle represents the demand nodes in the graph \( G \) separated by integer numbers (the authors discuss this natural integer number coding method later), corresponding to a feasible solution of the RO-FAPDPPPTW model. The particle \( i \) moves in the search space, moving along the following two directions: either the best position experienced by particle \( i \) in all preceding iterations (i.e., local optimum), denoted by \( pBest_i^d \); or the best position found so far among all the particles (i.e., global optimum), denoted by \( gBest_i^d \). The corresponding formulas to calculate the updated position and velocity for particle \( i \) in each iteration are given as (Shi & Eberhart, 1998):

\[ v_i^d = \omega v_i^d + c_1 r_1^d \left( pBest_i^d - x_i^d \right) + c_2 r_2^d \left( gBest_i^d - x_i^d \right) \]  

(18)
where \( c_1 \) and \( c_2 \) are fixed numbers known as particle accelerators; \( r_1^d \) and \( r_2^d \) are random numbers between 0 and 1; and \( \omega \) is the weighting factor that controls the convergence speed of the PSO algorithm.

Position \( x_i^d \), consisting of several vehicle routes, can be converted to a feasible solution of the RO-FAPDPPTW model (8)-(17), the performance of which is evaluated based on the predefined fitness function value \( f(x_i^d) \) (note the objective function \( C \) defined in Equation (8) is used as the particle fitness function in the proposed algorithm). If the current fitness function value of \( x_i^d \) is better than that of \( pBest_i^d \), then \( pBest_i^d \) is set to \( x_i^d \). Further, if the current fitness function value \( f(pBest_i^d) \) is better than that of \( gBest^d \), then \( gBest^d \) is set to \( pBest_i^d \). The algorithm continues to update the position and velocity for the particles according to Equations (18) and (19). It performs the comparisons between fitness function values until the stopping criterion is met. That is, either the number of iterations is greater than the preset maximum number of iterations or the current best fitness function value cannot be further improved. Refer to Kennedy and Eberhart (1995) for details of the PSO algorithm.

The initial group of particles considered may significantly affect the convergence speed and quality of the PSO algorithm (Gong et al., 2011; Kennedy & Eberhart, 1995). In this article, the natural integer number coding method is used for particle swarm initialization. Specifically, the natural integer numbers between two adjacent 0s are the demand locations to be served by the same vehicle. If there is no natural integer number before 0, the vehicle is considered unused. For example, suppose the distribution center has three vehicles. There are five demand locations (numbered 1, 2, 3, 4, and 5), each of which requires 1.2 tons of a particular FAP. The load capacity of each vehicle is 5 tons. Figure 2 shows a distribution plan for these randomly generated demand points in which 0s correspond to the vehicles. The figure indicates that the first vehicle is not used. The second and third vehicles are used to serve demand locations Path 1: 1 \( \rightarrow \) 5 and Path 2: 3 \( \rightarrow \) 2 \( \rightarrow \) 4, respectively. As a result, the cargo weights of the second and third vehicles are 2.4 tons and 3.6 tons, respectively. Clearly, such a distribution plan will not violate the capacity constraint. To make the position of the particle swarm more effective, two positions (\( x_v \) and \( x_r \)) are constructed in which \( x_v \) represents a vehicle and \( x_r \) represents the paths taken by the vehicle.

Once the best particle position is obtained by PSO at the end of each iteration, the neighborhood operators used in the VNS heuristics are applied, seeking to optimize the particle position further. In this article, several classic internal-route and external-route neighborhood operators are used in the VNS-PSO algorithm to explore the search space and improve the quality of the solutions. Specifically,
the following three types of neighborhood operators are considered. Note that these operators can be applied within the same route or between two different routes:

1. **Relocation**: In this type of neighborhood operator, one of the demand nodes is removed from its current position in the route. It is either inserted into the same position in another route (see Figure 3a) or inserted into a different position in the same route (see Figure 3b).
2. **Exchange**: This neighborhood operator involves exchanging the position of two demand nodes from the same route (see Figure 3c) or from the two distinct routes (see Figure 3d).
3. **Reverse**: This neighborhood operator changes the sequence of demand nodes in the route to its reverse order (see Figure 3e).

The relocation, exchange, and reverse operators are applied to particle \( i \) in a sequential manner. Each operator is performed and the fitness function value of the constructed position is evaluated until no improvement can be found. Then, the next operator is applied. When the reverse operation

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**Figure 3. Example of the neighborhood operators: Relocation (a and b), exchange (c and d), and reverse (e)**
is completed, if a newly obtained position yields a better fitness function value, then the position of the particle is updated. Otherwise, the position is kept as that before the local search procedure. The overall flowchart of the VNS-PSO algorithm is shown in Figure 4.

**COMPUTATIONAL EXPERIMENTS**

In this section, the simulation experiments are carried out to investigate the performance of the proposed VNS-PSO algorithm. The well-known C1-type, R1-type, and RC1-type instances from Solomon’s (2005) VRPTW test problem library are selected for testing purposes. These instances correspond to the VRPTW with 100 demand locations and 25 distribution vehicles, each with a maximum loading capacity of 200 kg. In solving the RO-FAPDPPTW model, the vehicle speed is set to 20 km/h, the fixed cost of a vehicle is $300/vehicle, the unit cost of the product is $12/kg, and the cost per unit time for vehicle use is $5/km. In addition, the authors set the mean demand value $q_i$ equal to the corresponding demand considered in each of Solomon’s instance. They assume the maximal deviation of $q_i = 0.2q_i$. The parameters for the Weibull distributed deterioration are $\alpha = 0.05$, $\theta = 1$, and $\gamma = 0$. Each demand location has its own demand quantity (up to four), service time, and time window. For the penalty cost calculation at each demand location, for simplicity, the authors assume that $a_i = s_i$, $b_i = e_i$, and $ep = lp = $10/min. The travel time between two demand locations is set to be the corresponding Euclidean distance. The geographical coordinates of demand locations are randomly generated in type-R1 instances, clustered in C1-type instances.

Figure 4. Flowchart of the variable neighborhood search (VNS) and particle swarm optimization algorithm
and mixed in RC1-type instances (i.e., a small number of demand locations is clustered while the rest are randomly generated). The three instances also differ in the width of the time windows and service time durations. The authors observe that most demands in C1-type instances have relatively narrower time windows (less than 1 h), whereas demands in R1-type and RC1-type instances typically have wider time windows (more than 1 h). The authors assume that the earliest departure time for all vehicles from the distribution center is 7:00 a.m. (set to time 0) and the available time for all vehicles is set to 720 minutes (i.e., 7:00 p.m. of the same day). In PSO, according to Norouzi et al. (2017), the authors set $\omega = 1$, $c_1 = 1.5$, and $c_2 = 2$. The maximum iteration is set to 100. The proposed algorithm is coded using MATLAB. Finally, the experiments are performed on a PC running Windows 10 Home (64-bit) with a 1.5 GHz Intel Core i5 Processor and 8 GB of RAM.

For comparison purposes, the optimization results obtained from solving the RO-FAPDPPPTW model using the standard PSO and proposed VNS-PSO algorithms are shown in Table 2. Specifically, the model outputs, including total travel time (or distance), quantity loss rate (QLR, in percent), number of vehicles used, number of iterations needed to converge on the optimal solution, and CPU time (in seconds), are reported in the table. Note that “QLR” refers to the average percentage of the quantity of deteriorated FAP during the transportation process. “CPU time” represents the average computational time of 10 runs. Several observations are made from the results:

1. C1-type instances lead to the lowest total travel time. The demand locations are clustered; thus, the distances between demand locations are relatively close. There are narrower consumer time windows (less than 1 h) and longer service time considered in C1-type instances. Therefore, the authors observe 10 vehicles that still needed to complete the distribution task. Thus, the higher cost from the time-window violation could be avoided.
2. The total travel time of RC1-type instances is lower than that of R1-type instances. RC1-type instances require fewer vehicles for distribution. The main reason for this is the demand locations in RC1-type instances have a mix of randomly generated and clustered demand locations. Those in R1-type instances are uniformly distributed (more geographically dispersed). The demand locations in RC1-type instances are closer to each other overall and the faster service time is assumed. Thus, the number of vehicles needed for the distribution task is reduced compared with the number needed for R1-type instances.
3. Regarding average quantity deterioration, less than 20% of the product delivered deteriorated in both C1-type and R1-type instances. Yet RC1-type instances achieved more than 20% quantity deterioration. This is mainly because fewer vehicles are used in RC1-type instances. Thus, each vehicle may have a longer driving distance.
4. The VNS-PSO yields better results than PSO in terms of the number of vehicles used for each instance. The average computational efficiency for both PSO and VNS-PSO algorithms seems stable across different problem instances, suggesting that the algorithms may achieve good

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>PSO</th>
<th>VNS-PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TRT</td>
<td>QLR (%)</td>
</tr>
<tr>
<td>C1</td>
<td>828.90</td>
<td>16.6</td>
</tr>
<tr>
<td>R1</td>
<td>1633.23</td>
<td>17.23</td>
</tr>
<tr>
<td>RC1</td>
<td>1108.34</td>
<td>22.32</td>
</tr>
</tbody>
</table>

Note: PSO = particle swarm optimization; VNS-PSO = variable neighborhood search and particle swarm optimization; TRT = total travel time; QLR = quantity loss rate; NV = number of vehicles used; I/N = iteration numbers.
computational efficiency for solving a RO-FAPDPPPTW model of similar scale. It makes intuitive sense that VNS-PSO requires a higher computation time than PSO because of the involvement of local search in the neighborhood in each iteration. Nevertheless, VNS-PSO outperforms PSO in terms of convergence speed. On average, VNS-PSO takes 14 iterations to obtain the stable solution while PSO requires 24.33 iterations.

The distribution routes obtained from VNS-PSO and the corresponding algorithm convergence diagrams are shown in Figure 5 and Figure 6, respectively. Figure 5 indicates that the vehicle routes of C1-type instances exhibit a simple distribution pattern where the neighboring demand locations are served by the same vehicle. The R1-type and RC1-type instances generate many intersecting routes.

Figure 6 demonstrates the robustness, effective neighborhood search, and fast convergence speed of the VNS-PSO algorithm. The authors see a noticeable improvement of the total travel time value at the beginning of the search process. The algorithm achieves the stable travel time value (i.e., optimal or near-optimal solution) for C1-type, R1-type, and RC1-type instances only after 15, 15, and 12 iterations, respectively. Therefore, the convergence performance of VNS-PSO is satisfactory and the algorithm has the protentional to be used for solving larger FAPDPPPTW instances.

Figure 5. Path planning diagram for C1 (a), R1 (b), and RC1 (c) instances
The detailed vehicle distribution routes obtained by VNS-PSO are shown in Table 3. Each vehicle departs from the distribution center (numbered 0) and completes its distribution tasks at different demand points before returning to the distribution center. For example, route 0-59-99-94-0 for vehicle 15 in Table 3 indicates that the vehicle leaves the distribution center to serve demand points 59, 99, and 94. Then, it returns to the distribution center.

CONCLUSION

In this article, the authors consider the FAPDPPPTW under demand uncertainty. Addressing demand uncertainty by a budget uncertainty set, a mixed-integer structured mathematical model based on the RO approach is developed to minimize the total distribution cost associated with vehicle use, vehicle distribution time, product quantity deterioration, and consumer time-window violation. To solve this model, the PSO algorithm is used. The VNS operators are embedded to prevent the particle swarm falling into local optimum, thus enhancing the overall algorithm performance. The performance of VNS-PSO is validated using well-known benchmark instances. In particular, the computational results demonstrate that VNS-PSO is superior to the standard PSO in terms of solution convergence speed. For future research, extending this problem setting by considering simultaneous quality and quantity
Table 3. Vehicle distribution routes

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>Vehicle No.</th>
<th>Vehicle Route</th>
</tr>
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<tr>
<td><strong>C1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>C1</td>
<td>0-98-96-95-94-92-93-97-100-99-0</td>
</tr>
<tr>
<td>3</td>
<td>C1</td>
<td>0-57-55-54-53-56-58-60-59-0</td>
</tr>
<tr>
<td>4</td>
<td>C1</td>
<td>0-20-24-25-27-29-30-28-26-23-22-21-0</td>
</tr>
<tr>
<td>6</td>
<td>C1</td>
<td>0-32-33-31-35-37-38-39-36-34-0</td>
</tr>
<tr>
<td>7</td>
<td>C1</td>
<td>0-81-78-76-71-70-73-77-79-80-0</td>
</tr>
<tr>
<td>8</td>
<td>C1</td>
<td>0-90-87-86-83-82-84-85-88-89-91-0</td>
</tr>
<tr>
<td>9</td>
<td>C1</td>
<td>0-13-17-18-19-15-16-14-12-0</td>
</tr>
<tr>
<td>10</td>
<td>C1</td>
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<tr>
<td><strong>R1</strong></td>
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<td></td>
</tr>
<tr>
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</tr>
<tr>
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<td>R1</td>
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</tr>
<tr>
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<td>0-39-23-67-54-24-80-0</td>
</tr>
<tr>
<td>4</td>
<td>R1</td>
<td>0-31-88-7-10-0</td>
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<td>5</td>
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<td>R1</td>
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<td>0-92-42-15-87-57-97-0</td>
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<td>11</td>
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<td>15</td>
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<tr>
<td>19</td>
<td>R1</td>
<td>0-72-75-22-56-74-58-0</td>
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<tr>
<td><strong>RC1</strong></td>
<td></td>
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<tr>
<td>1</td>
<td>RC1</td>
<td>0-65-22-23-25-77-58-75-13-11-0</td>
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<td>2</td>
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<td>3</td>
<td>RC1</td>
<td>0-96-54-41-42-44-1-3-5-45-4-55-0</td>
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<tr>
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<td>RC1</td>
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<tr>
<td>7</td>
<td>RC1</td>
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<td>8</td>
<td>RC1</td>
<td>0-90-82-99-52-86-74-59-97-87-9-10</td>
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</table>
deterioration would be of great practical value. In addition, considering various uncertainties, such as unexpected vehicle breakdown and road damage, could be an interesting direction.

COMPETING INTEREST

All authors of this article declare there is no competing interest.

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