Measurement and Policy Optimization of Regional Preschool Education Development Level Based on Generalized Orthogonal Fuzzy Sets and Prospect Theory

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ABSTRACT

Preschool education belongs to non-compulsory enlightenment education, and it is difficult to measure and analyze the development of preschool education in different regions because of its multiple attributes and diversity of influencing factors. In addition, decision makers will be limited by their own cognition when facing multi-attribute factors and uncertain factors, and there is a big gap between the decision results given and the actual situation. Therefore, this chapter introduces generalized orthogonal fuzzy sets and prospect theory into the measurement model of preschool education development level based on technique for order of preference by similarity to ideal solution (TOPSIS) method to improve the decision accuracy of decision makers. The experimental results show that the model can effectively deal with uncertain information and improve the analysis accuracy, and the results are more in line with the actual situation than other models. At the same time, it can intuitively compare and analyze the development of preschool education in different regions, and provide reliable data support for decision makers.

KEYWORDS

Generalized Orthogonal Fuzzy Set, Preschool Education, Policy Optimization, Prospect Theory, Regional Education

INTRODUCTION

Preschool education plays a guiding role in developing children’s intelligence, forming life habits, and understanding the world. Preschool education strengthens the cultivation of children’s physical and mental health in a scientific way, which plays a positive role in promoting children’s future development. China’s education is developing rapidly, and the concept and mode of preschool education are constantly improving. The enrollment rate has increased year by year, but several problems remain in the development process. Due to the unbalanced economic development in China, children in some areas have difficulty entering school. The excessively high standard tuition fees also
exert financial pressure on children’s families. According to relevant statistical data, the regions with high preschool education enrollment rates in China are Beijing, Shanghai, Guangzhou, and coastal cities. The enrollment rate in these regions is generally over 80%; by contrast, the highest enrollment rate in Xinjiang, Inner Mongolia, Yunnan, and other places is 45%. The minimum enrollment rate has fallen below 10%, seriously hindering the development of preschool education in China. Education is the driving force for the harmonious development of society, and preschool education is the premise of essential learning and the beginning of children’s systematic education.

Preschool education can provide children with a comprehensive, enlightening education and a learning environment combining education and entertainment. Exposure to knowledge in various fields in a relaxed environment helps children develop a correct understanding of the world, establish emotional communication, improve physical fitness, and promote the comprehensive development of children’s various abilities (Hu et al., 2016). Compared with compulsory education, its learning objectives are weaker, children’s achievements are more diverse, and educational attainment is not uniform. Many parents believe preschool education focuses on caring for children’s lives and guiding them to play, ignoring its comprehensive educational effect on children (Iskakova & Abdilakim, 2021). In addition, the resource allocation and teacher strength of preschool education in different regions is affected by factors such as regional economic development and geographical location. In remote areas with relatively backward economies, the development level of preschool education is relatively low. Correspondingly, in more accessible areas with relatively good economic development, the comprehensive development level of preschool education is relatively high. Moreover, many parents pay more attention to their children’s preschool education and have relatively higher requirements for it (Gershon & Pellitteri, 2018). Therefore, apparent differences exist in children’s abilities in different regions after they enter primary school. Many children cannot meet the requirements of the curriculum teaching objectives, which hinders students’ interest in learning and self-confidence.

With the improvement of the domestic economic level and the continuous development and improvement of the educational concept, preschool education has gradually moved towards specialization, standardization, and scale. The scope of inclusive public preschool education services has been expanding, providing more children with preschool education opportunities (Zhang et al., 2022). At the same time, measuring the development level of regional preschool education has gradually become a key concern for people and researchers. In the past, indicators for measuring the development of preschool education were selected based on experience, and there were many subjective and uncertain factors. The development of the preschool education industry is affected by many factors, such as the cultural and natural environment (Ge et al., 2017). Some scholars have pointed out that when decision-makers face an uncertain, multi-attribute decision-making environment, it is difficult to quantify and quantify the evaluation information, which means their language does not accurately match the evaluation information they want to express (Zhou & Tong, 2022). Based on this, some scholars have added corresponding fuzzy variables based on language term sets and used variables to characterize the fuzziness of decision-makers’ evaluation index information so that the information is closer to their description (Khan et al., 2018). However, the environment in which preschool education is developing is also constantly changing. Decision-makers’ evaluation information expressed by fuzzy variables in practical applications is relatively low in accuracy, and the range of information ambiguity is relatively small (Lan et al., 2018).

Therefore, combined with generalized orthogonal fuzzy sets and prospect theory, this paper constructs a measurement index of regional preschool education development, analyzes the development of regional preschool education through simulation experiments, and puts forward corresponding policy optimization suggestions. The research on the current situation and regional preschool education development strategies presented in this study is mainly based on a survey and analysis of the current preschool education development situation in Daiyue District. Exploring effective management mechanisms for developing preschool education in the current context is conducive to promoting the scientific development of preschool education in underdeveloped areas.
of China’s eastern region. Moreover, constructing a comprehensive management system for the development of preschool education, including diversified investment, multi-form kindergartens, multi-form teacher construction, and education quality supervision, is an operational theoretical system.

RELATED WORK

The concept of preschool education was introduced into China from European and American countries. In the early stage of the concept’s development, preschool education developed rapidly. Still, most kindergartens remain fixed, serving employees of factories and other companies. In addition, the scale of public kindergartens and their teaching qualifications and resources are uneven, with significant differences in cost (Liu & Lam, 2014). At the same time, many parents do not attach importance to preschool education or understand the concept of a systematic and comprehensive education. Furthermore, some regions have few kindergartens (Akram et al., 2020). By 2010, the state preschool education plan indicated that preschool education should be popularized for one to three years according to different regions’ objectives and economic conditions (Farkhodovna, 2021). At the same time, the government and relevant departments have increased investment in education according to the situation in different regions to promote the development of preschool education.

Accordingly, the gross kindergarten enrollment rate in different regions is increasing yearly, especially in the western region. This indicates that the preschool education enrollment problem has significantly improved. At the same time, in recent years, the quality of preschool teachers has continuously improved. Areas with higher development levels have built a relatively complete preschool education teacher quality evaluation framework to ensure teachers meet the requirements. In the relatively backward western region, the educational resources and the number of teachers have increased significantly, meeting the most basic regional coverage. However, with the promotion of preschool education, significant differences exist in the development of different regions, mainly due to variations in economic growth, educational environments and concepts, and government policy guidance (Toda et al., 2019). At the same time, in the face of educational development in different regions, many evaluation model indicators are relatively simple, do not meet actual needs, and cannot obtain scientific, reasonable, and comprehensive evaluation results (Mutlu et al., 2021).

Ceyhan et al. (2018) proposed obtaining the corresponding preschool education evaluation indicators through the expert method and analyzing the results of each indicator in combination with the analytic hierarchy process. Deng and Gao (2021) proposed principal component analysis combined with cluster analysis to construct an evaluation model to conduct a comprehensive evaluation and analysis of the development of preschool education. In addition, Xian et al. (2020) introduced the concept of geographic education, analyzed the allocation of preschool education resources, and studied the sustainable development of education from a geospatial perspective. However, the above methods have a certain degree of subjectivity. The factors affecting the growth of preschool education involve uncertain information and multiple attributes. Those choosing the evaluation indicators cannot quantify fuzzy information and provide evaluation information that matches the actual situation, thereby affecting the final evaluation results.

In response to the above, Hou et al. (2020) proposed fuzzy set theory to help with multi-attribute decision-making. Decision-makers often encounter the problem of the side in a fuzzy environment. Therefore, Senapati and Yager (2019) put forward the concept of fuzzy language to help decision-makers represent fuzzy problems through the defined subjunctive language. However, its application shows certain limitations. Chen et al. (2018) developed a calculation method for the similarity of intuitionistic fuzzy sets in describing the uncertainty problem in this method, achieving good results in medical diagnosis. The uncertain factors of any indicator evaluation are in a state of change. This change involves information that is difficult for decision-makers to obtain, which increases the risk of decision-making. Coupled with the psychological characteristics of decision-makers, a clear difference exists between theoretical expectations and actual utility (Liu et al., 2019). In response to
this situation, Aydemir and Yılmaz Gunduz (2020) proposed applying prospect theory by screening the existing prospects to determine the appropriate reference and evaluating the obtained prospects to obtain the results.

Model Construction Based on Generalized Orthogonal Fuzzy Sets and Prospect Theory

Weight of Regional Preschool Education Development Level Indicators

The government and relevant departments have also increased investment in education according to different regions’ conditions to promote the development of preschool education. Figure 1 shows the gross enrollment rate of kindergartens in different regions from 2009 to 2015, the proportion of kindergartens per 10,000 people in the various areas, and the statistical results of population changes. The results show that the gross enrollment rate of kindergartens in multiple regions is increasing yearly, especially in the western region. This indicates that the enrollment problem of preschool education has been significantly improved in different places.

The regional preschool education development level indicators are mainly measured comprehensively based on educational input, participation, efficiency, goal completion, and results (Abankina, 2018). Considering the actual situation of different regions, the indicator takes preschool education investment, opportunity, quality, equity, and educational foundation as the indicator criterion layers, obtains the indicator layer, and calculates the corresponding weights. The results are shown in Figure 2.

Among the secondary indicators, the enrollment rate reflects educational opportunities, and the investment in education funds, the proportion of the number of kindergartens of different natures, and the level of support from urban and rural governments all represent educational investment. The quality of education relies on the professional quality and number of teachers needed to achieve the quality requirements of preschool education on an economic basis. The difference in educational environment and educational philosophy between urban and rural areas affects the development of regional preschool education to varying degrees. This influencing factor has yet to be fully considered.
due to its hidden nature. In addition, the development of per capita gross domestic product (GDP) and urban and rural economic benefits will directly affect the amount of household disposable funds and the proportion of household funds invested in children’s education so that it can reflect the basic situation of education.

**Generalized Orthogonal Fuzzy Sets and Prospect Theory**

The generalized orthogonal fuzzy set can be transformed into an intuitionistic or Pythagorean fuzzy set according to the sum of the power of the membership degree and non-membership degree of the constraint condition. It can also be regarded as an extension of the two, and its scope is broader than the latter. Suppose there are two mappings in the universe, as shown in (1) and (2):

\[ \eta_D : M \rightarrow [0, 1] \]  
(1)

\[ \gamma_D : M \rightarrow [0, 1] \]  
(2)
So, $m \in M \mapsto \eta_D(m) : M \rightarrow [0,1]$, $m \in M \mapsto \gamma_D(m) : M \rightarrow [0,1]$, and, at the same time, we need to meet the conditions as shown in (3):

$$0 \leq \eta_D^0(m) + \gamma_D^0(m) \leq 1$$

(3)

A generalized orthogonal fuzzy set can be determined as the sum in the universe of discourse, which is shown in (4):

$$\tilde{D} = \{< m, \eta_D(m), \gamma_D(m) | m \in M > \}$$

(4)

and its hesitation degree for any element is shown in (5):

$$\mu_{\tilde{D}}(m) = \sqrt{1 - \eta_D^0(m) + \gamma_D^0(m)}$$

(5)

It can be seen that the range of constraints of generalized orthogonal fuzzy sets is wider, and the range of possible values is larger. Figure 3 compares the spatial ranges of the three fuzzy sets.
The application of prospect theory to problem-solving generally includes two parts, namely editing and evaluation. The former must judge the actual benefits and losses according to the set reference, and the latter calculates the prospect value according to the value obtained by the former and the weight function. The subjective value in the decision-making information is expressed by the value function, as shown in (6):

\[
J(m) = \begin{cases} 
  c(m - m_0)^\varepsilon & m \geq m_0, c > 0, \varepsilon > 0 \\
  -d(m_0 - m)^\phi & m < m_0, d > 0, \phi > 0 
\end{cases}
\]

(6)

The decision weight function is a nonlinear increasing function of probability. If we let the gain and loss probability weights be denoted as sums, their function expressions are shown in (7) and (8), respectively:

\[
w^+(g) = \frac{g^\lambda}{[g^\lambda + (1 - g^\lambda)]^{\frac{1}{\lambda}}}
\]

(7)

\[
w^-(g) = \frac{g^\alpha}{[g^\alpha + (1 - g^\alpha)]^{\frac{1}{\alpha}}}
\]

(8)

The calculation of the foreground value of the alternative indicator is shown in (9):

\[
Z(m_1, g; m_2, k) = J(m_1) \cdot w(g) + J(m_2) \cdot w(k)
\]

(9)
Figure 4 shows the prospect theory value function and weight function curves.

In this paper, we selected the technique for order preference by similarity to the ideal solution (TOPSIS) decision method as the basic fusion prospect theory and generalized orthogonal fuzzy language set. The weight of each evaluation index in the decision-making process has certain uncertainty, and the index weight can be divided into some known situations and all unknown situations. This work introduces a novel fuzzy TOPSIS approach to decision-making grounded in prospect theory. The decision makers’ various choices for the indicator set are considered first. The attribute weight is calculated using the entropy fork approach based on hesitation fuzzy entropy. Next, the hesitation fuzzy decision matrix is converted into the value matrix by defining the prospect value function of the hesitation fuzzy number. The computed profit–loss ratio takes the role of the relative proximity. Lastly, a numerical example is utilized to validate the ranking of the alternatives using the fundamental principle of TOPSIS. However, we obtained the indicator weights above, so we only analyzed some known situations. The steps of the TOPSIS multi-attribute decision-making method combining prospect theory and generalized orthogonal fuzzy language set are similar to prospect theory. The decision information is edited first, and then the corresponding evaluation is performed. The overall workflow framework is shown in Figure 5.
According to the above formula, the corresponding data of the generalized orthogonal fuzzy set and foreground theory are obtained. To better process the data, the data normalization operation will be performed on the foreground decision matrix, as shown in (10):

\[ J'_{mn} = J_{mn} / \max |J_{mn}| \] (10)

\[ \max |J_{mn}| = \max_{n=1,2,...,i} |J_{mn}| \] (11)

The TOPSIS method can divide its ideal solution into two types, positive and negative, according to \( M_1, M_2 \), and the geometric space \( L^+, L^-, H^+, \) and \( H^- \). The distance between them can be used to rank the decision-making methods. The more decision-making methods, the more difficult it is for decision-makers to describe their uncertain preferences. Fusing generalized orthogonal fuzzy sets and prospect theory can help decision-makers complete multi-attribute decision-making. After obtaining the corresponding information and data values, the positive ideal solution and the negative ideal solution can be obtained by normalizing the foreground decision matrix, as shown in (12) and (13):

\[ L^+ = \{ l_1^+, ..., l_i^+, ..., l_n^+ \} \] (12)

\[ L^- = \{ l_1^-, ..., l_i^-, ..., l_n^- \} \] (13)

\[ l_n^+ = \max_{m} (J'_{mn}), l_n^- = \min_{m} (J'_{mn}), \quad m = 1, 2, ..., i \]

The ideal solution weighted distance calculation is shown in (14) and (15):

\[ H^+ = \sqrt{\sum_{n=1}^{i} w_n (J'_{mn} - l_n^+)^2} \] (14)

\[ H^- = \sqrt{\sum_{n=1}^{i} w_n (J'_{mn} - l_n^-)^2} \] (15)

The decision-making method obtained according to the results of the above formula is relatively slow, as shown in (16):

\[ d_m = \frac{H^-}{H^+ + H^-} \] (16)

Judging the decision-making scheme according to the relative sticking progress, the larger the value, the better the ideal effect.
EXPERIMENTAL RESULTS

The main factor influencing preschool education levels throughout China’s regions is the stark disparities in preschool education across the country. Nonetheless, the primary factor affecting the development of preschool education as a whole is the degree of supply and demand for education. The amount of money the financial department invests in education depends on the economic prosperity of each region. The economies of China’s coastal areas are comparatively advanced. People’s ideologies display a modernizing trend, and the infrastructure for education is excellent. These factors can work in tandem with the relevant education agencies. We ardently advocate for school-age children’s preschool education. This paper analyzes the results of five dimensions of preschool education development in the eastern, western, and central regions and randomly selects kindergartens in the three regions for comparative analysis.

Before measuring the development level of preschool education in different regions, we first conduct a comprehensive analysis of the overall development level in China and then conduct further analysis and comparison. Figure 6 shows the calculation results of the extensive development of domestic preschool education. The results show that domestic preschool education is in a state of continuous growth. The constant improvement in recent years has promoted the development of preschool education in relatively inaccessible areas in China. It has entered a new stage of development, showing remarkable change. In 2019, the comprehensive development level of preschool education reached 0.725, changing rapidly over the past twelve years. From an overall development perspective, although the extensive development of preschool education in different regions has made significant breakthroughs, it has yet to transition from low-level development to high-level development. The comprehensive measurement results show that the development of preschool education in different regions in China has dramatically improved. However, it is still in the primary development stage and requires further improvement.

Figure 7 shows the comprehensive analysis results of the five dimensions of preschool education development in different regions in China. The calculation results in the figure show that the trend of education quality in the five dimensions of preschool education development shows a slight decline after a slow rise. The five dimensions’ overall indicators are gradually increasing. Thus, the five dimensions of preschool education development have made clear progress. Among the five-dimension indicators, the performance of the educational equity index is relatively good, which mainly involves promoting the growth of rural preschool education according to the rural conditions in different regions. In addition, the dimensions of basic education and educational opportunities have experienced relatively large growth rates. This is mainly due to the impact of economic development and educational investment. With the continuous improvement of living standards, the number of kindergartens is increasing, and the concept of parental education is also developing. The phenomenon of declining education quality stems from the fact that in the primary stage of preschool education development, significant differences exist in the degree of attention to preschool in different regions, and the educational structure and the distribution of teachers are insufficient, resulting in a mismatch between the quality of education and the speed of development.

To verify the gap between the results of the regional preschool education development level measurement model based on generalized orthogonal fuzzy sets and prospect theory and the actual results, we randomly selected five kindergartens in different regions of the east, central, and western regions to measure the development level of preschool education. We compared the calculated results with the actual results, and we compared the proposed model with the TOPSIS method. The results show that the measurement results of the model in this paper are closer to the actual results than those obtained using the TOPSIS method, and the difference between the results is relatively small. When using the TOPSIS method to measure the development of preschool education in different regions, the gap between the results and the actual results fluctuates wildly. In addition, the overall situation is constantly fluctuating, and the stability is weak. However, a gap exists in the number of parts of
the measurement results obtained by the model in this paper, but the fluctuation range is small, and it is overall more in line with the actual situation.

Figure 8 shows the development of regional preschool education based on generalized orthogonal fuzzy sets and prospect theory. It compares the results of the measurement model and TOPSIS method with the actual results. To make the comparison more intuitive, Figure 9 shows the comparison results of the ideal weighted distance and relative closeness of the positive degree of preschool education in different regions. The results show that preschool education is more developed in the eastern and some central regions than in the western region. Meanwhile, the overall development level in the western region is relatively low, and only some kindergartens reach the level of the central region. This is mainly because the economic development of the western region lags behind that of the eastern and central regions, and the educational system in the western region is relatively less complete in terms of cultural environment and educational philosophy. Therefore, the development of preschool education started late, with little investment and slow development. Although only a few kindergartens in the central region have reached the development level of the eastern region, the gap between them can be narrowed through subsequent development, and the overall comprehensive level can be improved.
Figure 7. Comprehensive analysis results of five dimensions of preschool education development in different regions in China

Figure 8. Comparison of regional preschool education development level measurement model and TOPSIS method based on generalized orthogonal fuzzy set and prospect theory with actual results
In summary, the regional preschool education development level measurement model based on generalized orthogonal fuzzy sets and prospect theory can effectively deal with the uncertain decision-making information in the indicators and improve the accuracy of the TOPSIS method’s results through prospect theory, making it more applicable to the actual situation. At the same time, the model in this paper can intuitively and effectively present the development status of preschool education in different regions by calculating relative fit and providing data with high accuracy for analysis.

Suggestions on the Development of Regional Preschool Education

Preschool education does not belong to the compulsory nine-year education sequence, but it significantly affects children’s ability, social interaction, thinking, and other aspects. Differences exist in the impact of not receiving preschool education and the length of time receiving preschool education on children’s comprehensive development (Jiang et al., 2022). These differences become apparent in the early stage of entry into primary school. Over the course of the education process, children’s abilities in all aspects develop and improve to varying degrees, reducing the gap brought about by preschool education. However, children receiving a complete preschool education have improved learning and communication abilities (Hong et al., 2013), conducive to developing their social communication ability, emotional communication, and character cultivation (Vernadakis et al., 2005). According to the measurement results of preschool education levels in different regions, the region’s economic development and the parents’ cognitive level are the most direct influences, indirectly affecting the opportunities, quality, and foundation of children’s preschool education. Based on these results, we put forward the following policy optimization suggestions for the development of regional preschool education:

First, a preschool education market supervision system should be established and improved. The preschool education market is sizable, and its openness allows it to accept various types of private
capital. To achieve maximum returns in a short period, some kindergartens ignore the purpose of preschool education and raise the threshold for children to obtain this education. At the same time, ensuring a high quality of teaching is difficult without unified supervision and quality testing standards in preschool education; a lack of such standards can even harm children’s physical and mental health. Therefore, establishing and improving the preschool education supervision system can guarantee and promote its development.

Second, China should increase investment in preschool education and optimize its related policies. Although preschool education can absorb significant private funds, government support and financial education funds represent the fundamental forces for its development. Different regions have varying degrees of economic and educational development. Based on the premise of attaching importance to preschool education, the government should classify preschool education as a government service according to the actual situation, increase the government’s supervision and support for preschool education, increase the proportion of financial investment, and optimize these factors by region. The allocation of preschool education resources will improve the utilization rate of funds and the rationality of allocation, gradually achieving an ideal balance.

Finally, the development of preschool education should be promoted according to regional urban and rural conditions. In our model analysis, the development gap between urban and rural areas is one factor affecting the growth of preschool education. The lack of rural educational resources has already caused many children to lose the opportunity to receive education. The difference in parents’ educational attitudes widens the gap between urban and rural preschool education, meaning that many rural children cannot access education. A good preschool education would be conducive to developing their future comprehensive abilities. Therefore, while increasing capital investment and support, the government should also improve the attractiveness and security of the profession of rural preschool teachers, improve teacher treatment, establish a reasonable management system for preschool teachers, expand the team of preschool teachers in rural areas, and comprehensively enhance the teaching profession. In addition, different regions are restricted by environmental and economic conditions. The government should encourage innovation in new forms of kindergartens, reasonably optimize the rigid policies and conditions of kindergartens, and reduce the cost of rural kindergartens.

The government should also modify how much it spends on early childhood education. Local governments should actively modify the notion of development, and funding for private kindergartens should be increased. They should optimize financial resources scientifically and raise the overall standard of preschool instruction. Regions should implement scientific reform of education finances in conjunction with the current state of regional development to support the growth of preschool education. For families struggling financially, the amount of education subsidies should be raised. In addition to providing specific education subsidies to certain families and vulnerable groups of school-age children experiencing financial hardship, the local government departments should step up their investigations into the family status of children in school.

CONCLUSION

The education that children receive in preschool is part of their growing process. However, because preschool education is influenced by a wide range of circumstances and diverse features, it is challenging to quantify and analyze how it develops in different places. Thus, this study builds a preschool education development level measuring model based on the TOPSIS approach and integrates the prospect theory of generalized orthogonal fuzzy sets. The experiment outcomes demonstrate that the model can assist decision-makers in evaluating ambiguous data and enhance their capacity to make sound decisions. Additionally, this method can intuitively assess preschool education progress across various locations by calculating relative fitting. According to the comparative results, the eastern area of China has the highest overall preschool education level, whereas the western region’s overall preschool education level is merely passing. This research also suggests appropriate policy optimization
based on the comparative analysis results. Subsequent investigations should concentrate on the dynamic mechanism of quality enhancement. According to this perspective, learning how to assess an institution’s quality is insufficient. We must focus more on the process of changing institutional quality and investigate how different influencing elements interact to have an overall impact.

DATA AVAILABILITY

The figures used to support the findings of this study are included in the article.

COMPETING INTERESTS

The authors of this publication declare that there are no competing interests.

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