GISwaps:
A New Method for Decision Making in Continuous Choice Models Based on Even Swaps

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ABSTRACT

This article describes how continuous GIS-MCDM problems are commonly managed by combining some weighting method based on pairwise comparisons of criteria with an aggregation method. The reliability of this approach may be questioned, though. First, assigning weights to criteria, without taking into consideration the actual consequences or values of the alternatives, is in itself controversial. Second, the value functions obtained by this approach are in most cases linear, which is seldom the case in reality. The authors present a new method for GIS-MCDM in continuous choice models based on Even Swaps. The method is intuitive and easy to use, based on value trade-offs, and thus not relying on criteria weighting. Value functions obtained when using the method may be linear or non-linear, and thereby are more sensitive to the characteristics of the decision space. The performed case study showed promising results regarding the reliability of the method in GIS-MCDM context.

KEYWORDS

Continuous Choice Models, Even Swaps, GIS-MCDM, Trade-Offs, Value Functions

1. INTRODUCTION

The ultimate goal of geographic information systems (GIS) is to provide support for spatial decision-making. For the last two or three decades there has been a growing interest in the subject of integrating multi-criteria decision analysis (MCDA) and GIS (Malczewski, 2007). The need to address the issue of applying the established concepts of multi-criteria decision making to spatial problems, and adapting them with respect to the nature and the format of GIS data, has resulted in a whole new interdisciplinary field of study, commonly referred to as GIS-based multi-criteria decision analysis (GIS-MCDA). There is a vibrant community within the field conducting research related to a number of application areas, such as environment, transportation, urban planning, waste management, hydrology, agriculture, forestry etc.

Malczewski & Rinner (2015) define GIS-MCDA as "a collection of methods and tools for transforming and combining geographic data and preferences (value judgments) to obtain information

for decision making." Greene et al. (2011) point out that, while virtually all multi-criteria decision-making (MCDM) methods can be applied to spatial problems, not all MCDM methods are suitable for all spatial problems. A list of factors which describe a given decision problem and which influence the choice of a method, presented in Greene et al. (2011), includes problem types (choice, ranking, sorting), number of decision makers, number of objectives, number of alternatives, uncertainty, risk tolerance, existence of constraints, decision phase, measurement scales and units, experience and computational capacity, among others.

Malczewski (2007) presents a survey of GIS-MCDA approaches where he classifies articles from 1990 to 2004 according to which decision making methods were used. From the classification scheme, it became obvious that the use of methods in GIS-MCDM studies has been limited to only a few approaches such as weighted summation, ideal/reference point, AHP and outranking methods, despite a considerable number of alternative decision-making methods being proposed in the MCDA literature. It is worth reminding that the survey was conducted nearly ten years ago, but our analysis of the representative recent case studies shows that the trend has not changed significantly. We performed a search on articles listed in SCOPUS and published after 2009, whereby we used the search term "GIS" combined with "MCDM", "MCDA", "multiple criteria decision" and "multi-criteria decision". Out of 533 articles, we analysed the ten most cited GIS case studies from the list with respect to the used method. The result of this analysis, presented in Table 1, shows that the majority of research within GIS-MCDM still focuses on a limited number of methods, most notably Analytic Hierarchy Process (AHP). AHP has been used in GIS-MCDM both in discrete choice models, where the number of alternatives is relatively small, and in continuous choice models, where decision problems are usually modelled using raster layers where each raster cell is an alternative, thus making pairwise comparison of the alternatives practically impossible. In the former case, AHP is used to aggregate the priority on all levels - between the criteria with respect to the main objective, between sub-criteria (if any) with respect to the parent criterion, and between the alternatives with respect to each criterion. In the latter case, AHP is only used to derive the weights of the criteria, i.e. the weights associated with attribute map layers (Malczewski, 2007). Some compensatory aggregation method is then used to obtain the score for each alternative in the set. Combining a weighting method, most commonly AHP, and an aggregation method is by far the most common approach in GIS decision making. Weighted linear combination (WLC) and boolean overlay operations are the most often deployed aggregation methods, as they are most intuitive and most straight-forward (Malczewski, 2004). In WLC, a total score for each alternative is obtained by multiplying the weight assigned to

Table 1. MCDM methods used in ten most cited GIS case studies published in SCOPUS listed journals between 2010 and 2016

Author	Method Used
(Gorsevski et al., 2012)	AHP, OWA (Ordered Weighted Averaging)
(Akgun & Türk, 2010)	AHP
(Fernández & Lutz, 2010)	AHP
(Charabi & Gastli, 2011)	OWA
(Machiwal et al., 2011)	AHP, WLC (Weighted Linear Combination)
(Al-Yahyai et al., 2012)	AHP, OWA
(Jha et al., 2010)	AHP
(Sánchez-Lozano et al., 2013)	AHP, TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution)
(Feizizadeh & Blaschke, 2013)	AHP
(Eskandari et al., 2012)	AHP

each criterion by the scaled value given to the alternative on that criterion, then summing the products over all attributes. Another popular aggregation method is ordered weighting averaging (OWA) that uses a fuzzy approach based on Yagers work on ordered weighted aggregation operator (see Yager, 1988; Jiang & Eastman, 2000).

1.1. AHP: Analytic Hierarchy Process

When working with a GIS decision problem in a continuous choice model, it is common that values for all alternatives (represented by grid cells or pixels) with respect to each criterion are normalized to an interval [0..N]. If for example criterion Z_1 represents the distance from a road network, the minimum value of Z_1 is min_1 and the maximum value of Z_1 is max_1, then an alternative a is assigned a value in terms of Z_1 according to the following:

$$f_1(z_1) = N \cdot \frac{(z_1 - min_1)}{(max_1 - min_1)} \tag{1}$$

For an MCDM problem involving n criteria, the total value for an alternative is a sum of all n values, when each value $f_i(z_i)$ is multiplied by a coefficient w_i, $1 \le i \le n$:

$$f(z) = \sum_{i=1}^{n} w_i f_i(z_i) \tag{2}$$

where:

- z_i is the descriptive value of the alternative a with respect to the criterion Z_i;
- w_i is the coefficient of the (normalized) measure f_i that represents the value with respect to criterion Z_i; and:

$$\sum_{i=1}^{n} w_i = 1 \tag{3}$$

The coefficients are often assigned in the GIS applications using the AHP method. AHP was developed by Saaty (see for example Saaty (1980, 2008)) and it is based on pairwise comparisons, both between the criteria, eventual sub-criteria, and between the alternatives with respect to each criterion. When AHP is applied, the decision maker needs to answer the following type of questions: which of two compared criteria is more important and how much more important than the other one, as well as which of two compared alternatives is better with respect to a certain criterion and how much better than the other one.

When used in the GIS context for continuous decision problems, AHP is not used as a complete method because the pairwise comparisons are not performed between different alternatives, i.e. individual cells in a map. Instead, AHP is usually only used in order to derive the weights of the criteria. In the Appendix we show examples of how weight coefficients are derived using the AHP approach.

From (3) follows that the coefficients w_i are assigned values between 0 and 1. The assumption is that the coefficients reflect the importance of the criteria. However, this assumption is not unproblematic (see for example Keeney, 1992; Choo et al., 1999; Korhonen et al., 2013). For example Keeney (2013) suggests working with trade-offs when assigning the weight coefficients.

One issue to consider regarding value functions f_i is that they are often assumed to be linear, which means that a value difference d in terms of a certain criterion C is equally important over the whole range of values of C. For example, in a decision problem where the distance to main roads is one of the criteria, a difference between 2 km and 3 km is equally valuable as the difference between 17 km and 18 km when using a linear value function f_i. In reality, the difference between 2 km and 3 km is likely to be more valuable than the difference between 17 km and 18 km. Thus, it is not certain that f_i should be linear.

1.2. Even Swaps

When making a decision that takes into account multiple criteria, one of the difficulties is to compare criteria that are measured using different scales, for example metric and monetary scale. By forcing a decision maker to think about the value of one criterion in terms of another, *Even Swaps* provides a practical way of making trade-offs among any set of criteria across a range of alternatives (Hammond et al., 1998). It provides a way to adjust the consequences of considered alternatives in order to render them equivalent in terms of a chosen criterion, thus making this criterion irrelevant for further analysis. The Even Swaps method is based on a long tradition in decision making research (see for example Keeney & Raiffa (1976)). In Hammond et al. (1999), the method is defined by the following five steps:

1. Determine the change necessary to cancel out criterion R;
2. Assess what adjustments need to be done in another criterion, M, in order to compensate for the needed change;
3. Make the even swap. An even swap is a process of increasing the value of an alternative in terms of one criterion and decreasing the value by an equivalent amount in terms of another. After the swaps are performed over the whole range of alternatives, all alternatives will have the same value on R and it can be cancelled out as irrelevant in the process of ranking the alternatives;
4. Cancel out the now-irrelevant criterion R;
5. Eliminate the dominated alternative(s). Alternative *a* is said to be dominated by alternative *b* if it is inferior to *b* on at least one criterion and not superior to *b* on any other criteria.

These steps are repeated until a single alternative remains. A step-by-step explanation of the method is given in Hammond et al. (1998) and Hammond et al. (1999). A number of enhancements of the method, as well as new methods based on Even Swaps, were proposed in recent years. Mustajoki & Hämäläinen (2007) introduce Smart-Swaps, a web-based decision support tool based on Even Swaps. The system implements their own preference programming model (Mustajoki & Hämäläinen, 2005) to suggest the decision maker how to proceed with the decision process, i.e. how to choose the next swap. Bhattacharjya & Kephart (2014) propose a Bayesian approach to guiding the even swap process. The system is assumed to have prior beliefs about decision maker's preferences. Those beliefs are updated through the interactive process by gradually learning more and more about the decision maker's preferences in order to guide him/her to a final choice. Modified Even Swaps is a framework suggested by Altun et al. (2013) to support customer co-creation in new product development. The framework is based on Even Swaps with an embedded fuzzy inference system that can bargain on several issues (criteria) simultaneously. Li & Ma (2008) propose a visualisation model to assist the decision maker in ranking alternatives by the means of visualising the decision process, in order to effectively handle inadequacies of the Even Swaps method, such as the fact that only the best alternative is found, and that the trade-off values often are inconsistent.

All mentioned studies have in common that they treat Even Swaps as a method that operates in discrete choice models, where the number of criteria and the number of alternatives are limited and reasonably small. To our knowledge, no studies have been conducted that propose an Even Swaps based method that would be applicable to a continuous choice model which is common in the GIS

context, where the number of alternatives is often limited only by the resolution of the digital elevation model representing the geographic area.

1.3. Aim and Objectives

Many MCDM methods are based on the assumption of the relative importance of the criteria, usually using pairwise comparison to determine their relative importance or weights. The methods may differ in how the relative importance of the criteria is decided, but regardless, one important issue remains, namely: what does it mean that one criterion is more important or less important than another? Assume a client wants to buy an apartment and decides that the price is more important to him/her than the size. However, the client would probably rather pay 105 000$ for a four-room apartment than 100 000$ for a two-room one. Does that mean that the size is actually more important to the client than the price? He/she might try to quantify the relative importance of the criteria and decide that the price is twice as important as size, but that would only make things even more unclear. Obviously, the rationality of comparing the importance of different objectives or criteria without considering the actual degree of variation among the consequences of the alternatives under consideration might be questioned (Hammond et al., 1998; Keeney, 2013; Korhonen et al., 2013). Another issue is that the value functions obtained by applying a method based on criteria weighting are in many cases linear. In reality, this is not always the case, and the actual importance of a decision criteria is often context-dependent, influenced both by the value of an alternative in terms of that criterion, and the value of the same alternative in terms of other criteria. In this paper we present GISwaps - a new method for GIS decision-making in continuous choice models, inspired by, and in parts based on, the Even Swaps trade-off method developed by Hammond, Keeney and Raiffa (Hammond et al., 1998, 1999). In its original form, the Even Swaps method is only suitable for decision-making in discrete choice models, when the number of alternatives is relatively small. Many decision-making situations within GIS-MCDM concern a continuous choice model though, with the number of grid cells as the only factual limitation of the number of alternatives.

The aim of this study is to develop a method that will bring the strengths and the intuitiveness of the Even Swaps method to GIS decision making in continuous choice models. The objective of the study is to automatize the trade-off process based on Even Swaps in order to make it applicable to decision problems in quasi-continuous decision space. In order to achieve this objective, following research questions are answered:

- How can the trade-off process based on Even Swaps be automatized in order to be efficient regardless of the number of alternatives?
- How can the issue of non-linearity of value functions be handled in the method?

2. GISWAPS

Consider a site location decision problem, e.g. setting up a small scale solar plant. Suppose that the region of interest is 10 000 km^2, represented by a map layer with ground resolution of 100 m, i.e. each grid cell representing an area of 10 000 m^2. If the area of 10 000 m^2 is sufficient to set up a plant, then the number of alternatives (possible locations) is only limited by the grid size, i.e. 1 000 000. Even Swaps is obviously not a feasible choice of a method in such a decision problem. In order to be able to use Even Swaps in GIS-MCDM context, the process of changing the value of each alternative to the same level in terms of one criterion (reference) and changing the value by an equivalent amount in terms of another criterion (measuring stick) needs to be automatized. The method presented here uses input from the decision maker on a number of virtual alternatives in order to calculate, for each alternative in the set, the coefficient of the value change on the measuring stick criterion. In its basic form, our method can be expressed by Algorithm 1.

Algorithm 1.

```
Repeat
    Decide the reference criterion R
    Decide the measuring stick criterion M
    Set virtual alternatives
    Obtain trade-off values for virtual alternatives
    Calculate update coefficients for virtual alternatives
    For each alternative in the set
        Calculate the update coefficient for the alternative
        Update the value of the alternative with respect to M
    Discard R
Until a single criterion remains
Rank the alternatives
```

The elimination of dominated alternatives, which is an important part of the *Even Swaps* method, is not a part of our method for obvious reasons. In geospatial decision making we rarely try to filter out all but the best possible alternative; most often we want to obtain an ordered set of all feasible alternatives. Deploying elimination of dominated alternatives would leave us with a single alternative, which, in the context of geospatial decision-making, is usually a single grid cell, or a single pixel. The calculation of trade-off values for all the alternatives is based on trade-off values set by the decision maker for a number of virtual alternatives. As the name suggests, the virtual alternatives do not need to actually exist in the set, but are hypothetical alternatives used to fine-tune the value update function. The number of virtual alternatives, as well as assigned descriptive values in terms of each criterion, is set in each step for current reference and measuring stick criteria, respectively. This makes the method flexible to meet different needs of a specific decision problem and the format of the criteria.

In order to present the method in a clear and concise way, in this chapter we explain its main features under the assumption that the number of virtual alternatives is predetermined, equal to 16 in each step, defined by four reference values in terms of the reference criterion and four reference values in terms of the measuring stick criterion. Furthermore, we assume that, for all criteria, the utility of an alternative in terms of a certain criterion increases as its descriptive value in terms of the criterion increases, i.e. that "the more, the better" principle applies to all criteria. This is obviously not always the case in reality, and any implementation of the method must also consider the other three combinations of "the more, the better" and "the less, the better" types of reference and measuring stick criteria, as each of these combinations requires minor modifications of the equations presented in this chapter. However, we have chosen not to include the details of these modifications as they are rather trivial, and we believe that they would not in any way facilitate the understanding of the method.

2.1. The Method

The set of virtual alternatives consists of 16 alternatives, each of which is assigned a pair of values: a reference value from the arrays V_R (four reference values in terms of the reference criterion R) and a reference value from the array V_M (four reference values in terms of the measuring stick criterion M). We suggest following reference values for the virtual alternatives in terms of R and M, respectively:

$$V_R = \left[R_{min}, R_{min} + R_q, R_{min} + 2R_q, R_{min} + 3R_q \right]$$

$$V_M = \left[M_{min} + M_q, M_{min} + 2M_q, M_{min} + 3M_q, M_{max} \right] \tag{4}$$

where:

- R_{min}: Minimum value with respect to R in the set of alternatives;
- R_{max}: Maximum value with respect to R in the set of alternatives:
- $R_q = (R_{max} - R_{min}) / 4$
- M_{min}: Minimum value with respect to M in the set of alternatives;
- M_{max}: Maximum value with respect to M in the set of alternatives:
- $M_q = (M_{max} - M_{min}) / 4$

At this stage, based on his/her judgement and preferences, the decision maker needs to make even swaps. He/she chooses a value $M_{u(i,j)}$ of criterion M for each virtual alternative, i.e. for each pair of reference values ($V_{R(i)}$, $V_{M(j)}$), where $i,j \in [1..4]$. Each value $M_{u(i,j)}$ is chosen so that the decision maker is indifferent between the differences ($R_{max} - V_{R(i)}$) and ($V_{M(j)} - M_{u(i,j)}$). The values are stored in a 4x4 matrix M_u:

$$M_u = \begin{bmatrix} u_{(1,1)} & u_{(1,2)} & u_{(1,3)} & u_{(1,4)} \\ u_{(2,1)} & u_{(2,2)} & u_{(2,3)} & u_{(2,4)} \\ u_{(3,1)} & u_{(3,2)} & u_{(3,3)} & u_{(3,4)} \\ u_{(4,1)} & u_{(4,2)} & u_{(4,3)} & u_{(4,4)} \end{bmatrix} \tag{5}$$

A matrix M_c containing the corresponding update coefficients for each element in M_u is then created, with each value $M_{c(i,j)}$ calculated as:

$$M_{c(i,j)} = \frac{V_{M(j)} - M_{u(i,j)}}{R_{max} - V_{R(i)}} \tag{6}$$

In order to calculate the update coefficient for an alternative a, we need to determine index i from a_R (the value of a in terms of the reference criterion R) and index j from a_M (the value of a in terms of the measuring stick criterion M). The indexes are determined as follows:

$$i = k \text{ for } V_{R[k]} \le a_R < V_{R[k+1]}, \quad k \in [1...3]$$

$$i = 4 \text{ for } a_R \ge V_{R[4]} \tag{7}$$

$$j = 1 \text{ for } a_M \le V_{M[1]}$$

$$j = k \text{ for } V_{M[k-1]} < a_M \le V_{M[k]}, \quad k \in [2...4] \tag{8}$$

For $j > 1$, the update coefficient a_c for a is then calculated as:

$$a_c = M_{c(i,j-1)} + (M_{c(i,j)} - M_{c(i,j-1)}) \cdot \frac{a_M - V_{M(j-1)}}{V_{M(j)} - V_{M(j-1)}} \tag{9}$$

For $j = 1$, the update coefficient a_c for alternative a is considered equal to the value of $M_{c(i,1)}$:

$$a_c = M_{c(i,1)} \tag{10}$$

which means that update coefficients for all alternatives with value of a_M between M_{min} and $V_{M[1]}$ are equal.

The updated value of a in terms of M, $a_M{'}$, is calculated as:

$$a_M{'} = a_M - a_c \cdot (R_{max} - a_R) \tag{11}$$

Finally, the obtained values for all alternatives with respect to M are normalized over the original value interval for M. For each alternative a in the set, the normalized value is calculated as:

$$a_M{''} = M_{min} + (M_{max} - M_{min}) \cdot \frac{a_M{'} - M_{min}{'}}{M_{max}{'} - M_{min}{'}} \tag{12}$$

where:

- a_M': Updated value of a in terms of M prior to normalization;
- M_{min}': Minimum value with respect to M in the set of alternatives prior to normalization;
- M_{max}': Maximum value with respect to M in the set of alternatives prior to normalization.

The method is demonstrated in the following office space example. Assume X is looking for a new office space, and for simplicity, assume X only considers two criteria: *Size* and *Customer access index*. Let us choose *Customer access index* as the reference criterion R and *Size* as the measuring stick criterion M. Initial values for all the alternatives with respect to each criterion are given in Table 2 and the update values for virtual alternatives chosen by the decision maker are given in Table 3.

From Table 2 we obtain the following values:

$R_{min} = 55$
$R_{max} = 78$
$R_q = 5.75$
$M_{min} = 60$
$M_{max} = 120$
$M_q = 15$

According to (4), V_R and V_M for the office space example are calculated to be:

Table 2. Initial values for alternatives a, b, c, d, e and f with respect to customer access index and size, respectively (the office space example)

	a	*b*	*c*	*d*	*e*	*f*
Customer access index	60	74	62	65	55	78
Size	90	65	80	110	120	60

$V_R = [55, 60.75, 66.5, 72.25]$
$V_M = [75, 90, 105, 120]$

In accordance with (5) and (6), we obtain the following matrices M_u and M_c from update values in Table 3:

$$M_u = \begin{bmatrix} 60 & 72 & 80 & 90 \\ 65 & 78 & 88 & 100 \\ 68 & 82 & 95 & 108 \\ 70 & 85 & 98 & 112 \end{bmatrix} \quad M_c = \begin{bmatrix} 0.65 & 0.78 & 1.09 & 1.30 \\ 0.58 & 0.70 & 0.99 & 1.16 \\ 0.61 & 0.70 & 0.87 & 1.04 \\ 0.87 & 0.87 & 1.22 & 1.39 \end{bmatrix} \tag{13}$$

A graph representation of Mc is given in Figure 1.

From (13) and Table 2, we can now calculate updated values for all the alternatives. The updated value for alternative *a* is obtained as follows:

Table 3. Compensated values with respect to size suggested by the decision maker for all virtual alternatives (the office space example)

Change V_R to R_{max}	Compensate on V_M			
from 55 to 78	from 75 to 60	from 90 to 72	from 105 to 80	from 120 to 90
from 60.75 to 78	from 75 to 65	from 90 to 78	from 105 to 88	from 120 to 100
from 66.5 to 78	from 75 to 68	from 90 to 82	from 105 to 95	from 120 to 108
from 72.25 to 78	from 75 to 70	from 90 to 85	from 105 to 98	from 120 to 112

Figure 1. A graph representation of the coefficient values M_c for the office space example (13). The steepness of the function slopes mirrors the willingness to compensate on size. The increase in slope steepness for values of 90 and higher indicates that the difference in size is not as important in the range between 90 and 120 as it is in the range between 60 and 90.
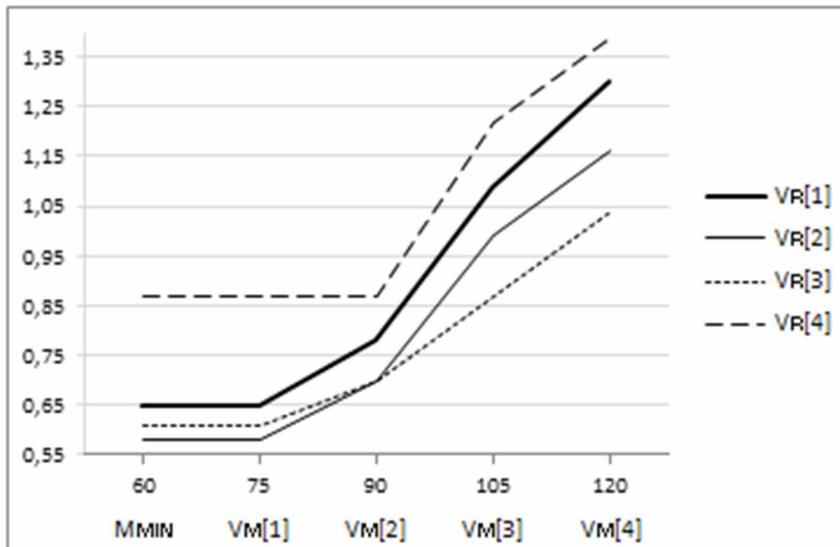
$$a_R = 60 \, , \; a_M = 90$$

In accordance with (7):

$$i = 1, \; j = 2$$

In accordance with (9):

$$a_c = 0.65 + (0.78 - 0.65) \cdot \frac{90 - 75}{90 - 75} = 0.78$$

In accordance with (11):

$$a_M{}' = 90 - 0.78 \cdot (78 - 60) = 75.96$$

Calculations for alternatives $b - f$ are performed in the same way. The result of applying the algorithm to the example is shown in Table 4.

Now that all alternatives have the same descriptive value in terms of *Customer access index*, we can ignore this criterion. Since there is only one remaining criterion, there is no need to normalize the values, i.e. to calculate $a_M{}'$ for each alternative. Assuming that a larger office (higher descriptive value of *Size*) is better than a smaller one (which is natural in this case), the preferred alternative is the alternative with the highest value in terms of the remaining criterion (*Size*), in this case d, followed by e, a, c, b and f.

3. CASE STUDY

The *GISwaps* method was initially tested on a number of simple decision problems, with 3-5 criteria and 5-10 alternatives. The problems were first handled manually, by the decision maker explicitly deciding the updated value in terms of the measuring stick criterion for each alternative. Afterwards, the same problems were handled by applying the *GISwaps* algorithm; the updated values for the virtual alternatives in terms of the measuring stick criterion were set by the decision maker. For all tests the ranking of the alternatives was the same in both cases. In order to test and evaluate the method on a continuous GIS decision problem, we developed a GIS application implementing the algorithm and used it in a case study. The objective of the study was to determine the best site location for a dam and reservoir for hydro-electrical power production on the Reventazón River in Costa Rica.

Table 4. Coefficients and new values with respect to size for alternatives a, b, c, d, e and f (the office space example)

	a	*b*	*c*	*d*	*e*	*f*
Customer access index (CAI)	60	74	62	65	55	78
CAI, evened-out	78	78	78	78	78	78
Size	90	65	80	110	120	60
Trade-off coefficient	0.78	0.87	0.62	1.05	1.30	0.87
Size, new value	75.96	61.52	70.08	96.35	90.01	60

The Reventazón River is a 145 kilometres long river located in central part of Costa Rica. The Reventazón River basin is the third largest watershed in Costa-Rica. It covers an area of 2953 km^2 and includes around 550 000 inhabitants, mostly concentrated in the upper and middle sections. The drainage area is determined by Cordillera Central mountain to the north with peak elevation of 1900 metres, Cordillera de la Talamanca with peak elevation of 3300 to the south, the water divide between the Eastern and Western Meseta Central to the west, and the water divide of the Pacuare River to the east (Brandt, 1999).

3.1. Data Description

A digital elevation model (DEM) of the area was produced from the SRTM (Shuttle Radar Topography Mission) elevation data in raster format, downloaded from CGIAR-CSI (CGIAR-CSI, 2016). The SRTM data was delivered as 90 metres linear cell size raster in WGS84 coordinate system. In order to fit the land use data for the area that was obtained from a raster file, a 120 spacing grid was produced from the original data and reprojected to *Cuadricula Lambert, Costa Rica Norte* system. Additional elevation data included height contour lines of the drainage basin and additional points describing the terrain close to the main river in higher resolution. Land information data included raster files representing the relief (flat, gently undulating, moderately undulating, strongly undulating, steep) and land use (forests, pasture, perennial agriculture, annual agriculture, urban areas). Other relevant data included rainfall information (precipitation data for stations surrounding the Reventazón) and evaporation information obtained by means of runoff coefficients for different land use classes. A number of other sources were used as a reference in order to optimize the decision process. The sources included topographic maps of the area, used to calibrate the values for stream network generation, as well as high resolution satellite photos of the area from Google Maps, used to evaluate the obtained result.

3.2. Criteria Factors

Six relevant criteria factors were considered during the decision process:

- Hydraulic head;
- Water discharge (Flow accumulation);
- Undulation;
- Distance to forests;
- Distance to agricultural areas;
- Distance to urban areas.

Hydraulic head, i.e. the elevation difference between two points on a river course, was obtained by assigning the elevation values from the DEM to the streams.

Water discharge, or a volume rate of water flow, is directly connected to the energy production. The minimum value of water discharge for this case study was set to 30 m^3/s.

Undulation (the steepness of the terrain) values were obtained using a DEM grid as input. The maximum values within a radius of 8 cells (960 meters) were obtained, and an undulation layer was produced by subtracting the DEM grid from the obtained grid.

For hydraulic head, water discharge and undulation, "the more, the better" principle applies.

Distance to forests is a "the further, the better" factor. The dam is not supposed to be built close to the large virgin forest areas above 1400 meters above sea level. In the case that a dam is to be built near such an area, there must be a buffer zone of at least 300 meters. Furthermore, even if the dam is built at lower altitudes, avoiding forest areas is preferred.

Agricultural areas are better avoided as well, which means that also here "the further, the better" principle applies.

With respect to distance to urban areas, somewhat conflicting conditions apply. Avoiding flooding urban areas, and especially large cities, is preferred. At the same time, it is critical that a dam is built close to a large city, since minimizing the costs of delivering the power is an obvious objective. A pragmatic approach was chosen where "the closer, the better" factor map was used in the decision making process, and the chosen location was analysed on the reference data in order to check for flooding risks.

Factor maps for all 6 criteria are shown in Figure 2.

3.3. Constraints

One obvious constraint to be applied in the decision making process is that a dam must be built on the river. The generated stream network grid was used "as is" as the river constraint map. Another constraint was that there must be a buffer zone of at least 300 meters if the dam was to be built in the area over 1400 meters above sea level, populated by virgin forests. However, since the forest constraint area never intersects the stream network, i.e. it falls into the area already cut off by the river constraint, it was unnecessary to define the 300-meter buffer constraint. Finally, a discharge constraint was applied, disregarding areas with accumulation flow below 30 m³/s.

3.4. Results

Regardless of the method used, decision making requires good understanding of the decision situation, as well as good knowledge regarding factors relevant for the decision. For this case study, the input, i.e. trade-off values for the virtual alternatives used in the decision process, was provided by an expert in hydrology and multi-criteria decision making within the field. The decision process was performed in five steps, presented in Table 5.

Figure 2. Factor maps for: a) Water discharge; b) Hydraulic head; c) Undulation; d) Agriculture; e) Forest; f) Urban areas
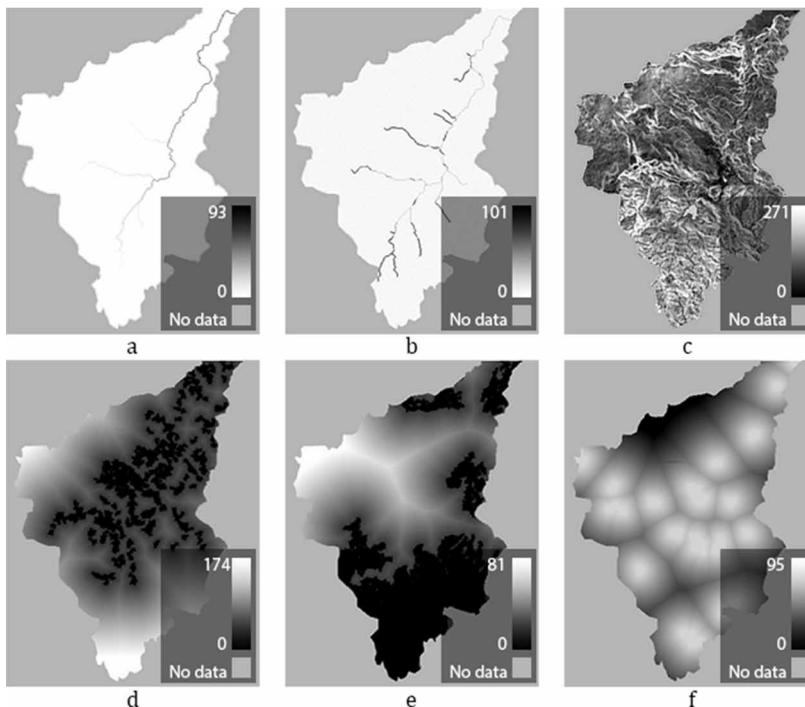
Table 5. Steps performed in the case study

	Reference Criterion	Measuring Stick Criterion
Step 1	Distance to urban areas	Distance to forests
Step 2	Distance to agricultural areas	Distance to forests
Step 3	Hydraulic head	Undulation
Step 4	Distance to forests	Discharge
Step 5	Discharge	Undulation

In the first step, *distance to urban areas* was chosen as the reference criterion and *distance to forests* as the measuring stick criterion. The decision maker's task in this step was to decide new, decreased values in terms of *distance to forests* when the distance to urban areas is reduced to the minimum descriptive value of an alternative in the set in terms of that criterion. The decision maker's trade-off choices, based on his preferences and judgement, are given in Table 6. We see that, if the distance to urban areas is decreased from 4.2 km to the minimum value of 120, the decision maker was willing to compensate on the distance to forests by decreasing it the from 960 m to 480 m, from 1.7 km to 720 m, from 2.5 km to 1.2 km and from 3.2 km to 1.7 km. If the distance to urban areas is decreased from 3.2 km to 120 m, the decision maker was willing to decrease the distance to forests from 960 m to 720 m, from 1.7 km to 1.1 km and so on. Assigned trade-off values and obtained corresponding trade-off coefficients for step 1 are given in (14).

The decision maker's trade-off choices for steps 2-5 are given in Tables 7-10, and assigned trade-off values and obtained corresponding trade-off coefficients for steps 2-5 are given in (15)-(18):

$$M_u = \begin{bmatrix} 480 & 720 & 1200 & 1700 \\ 720 & 1100 & 1600 & 1900 \\ 720 & 1200 & 1800 & 2300 \\ 840 & 1300 & 1900 & 2600 \end{bmatrix} \quad M_c = \begin{bmatrix} -0.10 & -0.23 & -0.30 & -0.37 \\ -0.06 & -0.19 & -0.29 & -0.42 \\ -0.09 & -0.23 & -0.32 & -0.46 \\ -0.06 & -0.35 & -0.52 & -0.58 \end{bmatrix} \tag{14}$$

$$M_u = \begin{bmatrix} 480 & 840 & 1300 & 1900 \\ 720 & 1200 & 1700 & 2000 \\ 720 & 1300 & 1900 & 2400 \\ 840 & 1400 & 2000 & 2600 \end{bmatrix} \quad M_c = \begin{bmatrix} 0.12 & 0.23 & 0.32 & 0.37 \\ 0.07 & 0.18 & 0.29 & 0.44 \\ 0.10 & 0.20 & 0.30 & 0.47 \\ 0.07 & 0.27 & 0.48 & 0.67 \end{bmatrix} \tag{15}$$

Table 6. Distance to urban areas compensated on distance to forests

Value Interval	From 960 m	From 1.7 km	From 2.5 km	From 3.2 km
from 4.2 km to 120 m	to 480 m	to 720 m	to 1.2 km	to 1.7 km
from 3.2 km to 120 m	to 720 m	to 1.1 km	to 1.6 km	to 1.9 km
from 2.2 km to 120 m	to 720 m	to 1.2 km	to 1.8 km	to 2.3 km
from 1.2 km to 120 m	to 840 m	to 1.3 km	to 1.9 km	to 2.6 km

Table 7. Distance to agricultural areas compensated on distance to forests

Value Interval	From 960 m	From 1.7 km	From 2.5 km	From 3.2 km
from 120 m to 3.7 km	to 480 m	to 840 m	to 1.3 km	to 1.9 km
from 1.0 km to 3.7 km	to 720 m	to 1.2 km	to 1.7 km	to 2.0 km
from 1.9 km to 3.7 km	to 720 m	to 1.3 km	to 1.9 km	to 2.4 km
from 2.8 km to 3.7 km	to 840 m	to 1.4 km	to 2.0 km	to 2.6 km

Table 8. Hydraulic head compensated on undulation

Value Interval	From 35	From 68	From 103	From 136
from 1 to 46	to 30	to 56	to 84	to 105
from 12.3 to 46	to 32	to 60	to 91	to 109
from 23.5 to 46	to 33	to 62	to 94	to 119
from 34.8 to 46	to 34	to 66	to 96	to 125

Table 9. Distance to forests compensated on discharge

Value Interval	From 53	From 66	From 80	From 93
from 120 m to 3.2 km	to 48	to 58	to 70	to 81
from 900 m to 3.2 km	to 50	to 60	to 72	to 83
from 1.7 km to 3.2 km	to 51	to 62	to 74	to 86
from 2.5 to 3.2 km	to 52	to 64	to 76	to 90

Table 10. Discharge compensated on undulation

Value Interval	From 35	From 68	From 103	From 136
from 40 to 93	to 30	to 55	to 83	to 103
from 53.3 to 93	to 31	to 58	to 88	to 111
from 66.5 to 93	to 32	to 62	to 94	to 119
from 79.8 to 93	to 33	to 65	to 94	to 121

$$M_u = \begin{bmatrix} 30 & 56 & 84 & 105 \\ 32 & 60 & 91 & 109 \\ 33 & 62 & 94 & 119 \\ 34 & 66 & 96 & 125 \end{bmatrix} \quad M_c = \begin{bmatrix} 0.11 & 0.28 & 0.41 & 0.69 \\ 0.08 & 0.25 & 0.34 & 0.80 \\ 0.08 & 0.29 & 0.38 & 0.76 \\ 0.07 & 0.22 & 0.58 & 0.99 \end{bmatrix} \tag{16}$$

$$M_u = \begin{bmatrix} 48 & 58 & 70 & 81 \\ 50 & 60 & 72 & 83 \\ 51 & 62 & 74 & 86 \\ 52 & 64 & 76 & 90 \end{bmatrix} \quad M_c = \begin{bmatrix} 0.20 & 0.33 & 0.38 & 0.46 \\ 0.17 & 0.33 & 0.40 & 0.51 \\ 0.17 & 0.35 & 0.45 & 0.54 \\ 0.19 & 0.39 & 0.60 & 0.47 \end{bmatrix} \tag{17}$$

$$M_u = \begin{bmatrix} 30 & 55 & 83 & 103 \\ 31 & 58 & 88 & 111 \\ 32 & 62 & 94 & 119 \\ 33 & 65 & 94 & 121 \end{bmatrix} \quad M_c = \begin{bmatrix} 0.09 & 0.25 & 0.37 & 0.62 \\ 0.09 & 0.26 & 0.36 & 0.63 \\ 0.10 & 0.25 & 0.32 & 0.64 \\ 0.13 & 0.27 & 0.65 & 1.14 \end{bmatrix} \tag{18}$$

Now that all other criteria have been evened out and disregarded, the value that each alternative has on the remaining non-evened criterion (undulation) represents the all-out ranking value of the alternative. The result is shown in Figure 3.

The highest ranked location, i.e. the location pointed out as the best, is situated about 8 km south-west of the city of Siquirres in Limón Province. Coincidentally, this is the same location where the Reventazón Dam, inaugurated on 16 September 2016, was built. At a height of 130 metres and a structural volume of 9 000 000 m^3, the Reventazón Dam is the largest power station in the country with an installed capacity of 305.5 MW and is expected to provide power for 525 000 homes (Wikipedia, 2017).

Figure 3. The marked grid cells represent the highest ranked locations for building a dam on the Reventazón River in Costa Rica
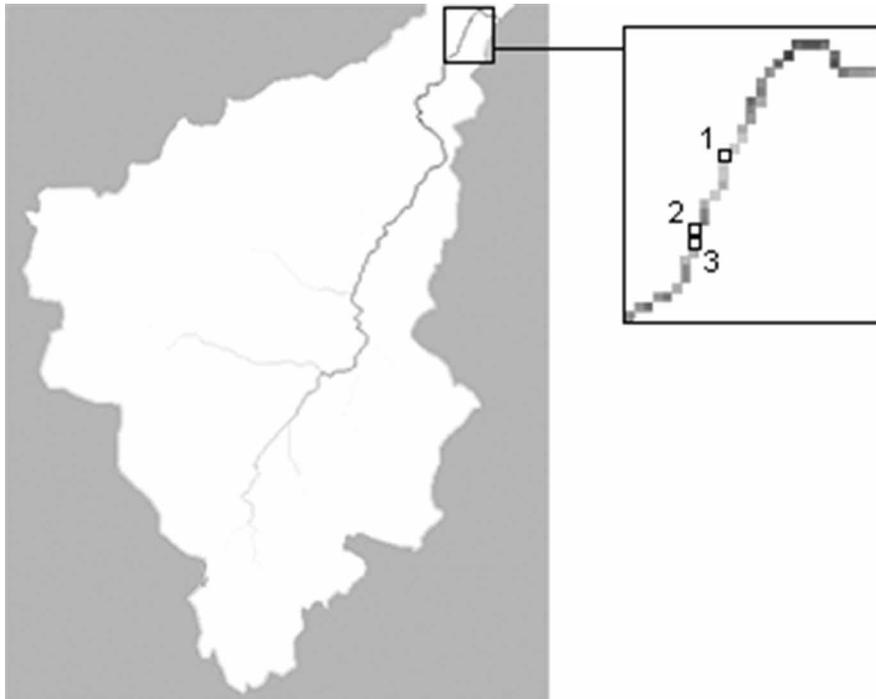
4. DISCUSSION AND CONCLUSION

GISwaps is a method based on the concept of value trade-offs under certainty, when the consequences of each alternative with respect to each criteria or objective are known. Keeney & Raiffa (1976) describe the essence of the issue of trade-offs under certainty as "How much achievement on objective 1 is the decision maker willing to give up in order to improve achievement on objective 2 by some fixed amount?". They point out that the trade-off issue often is a question of personal value, and as such requires the subjective judgement of the decision maker. We believe that our method provides efficient means for the decision maker to express his/her preferences and to translate them into concrete values that form the basis of the decision process. Being a trade-off method based on even swaps, GISwaps is an intuitive and efficient tool to deal with one of the big challenges in the context of decision support systems: how to handle the preferences of the decision maker. When using GISwaps, the decision maker is concerned with concrete values and differences, rather than with the more abstract concept of the relative importance of criteria. Furthermore, the decision maker is able to define non-linear value functions, which makes our method more flexible than the commonly used approach of combining weighting and aggregation methods, e.g. AHP and WLC.

Since results of a decision process are by necessity dependent on the knowledge, the preferences and the judgement of the decision maker, evaluation of a method for decision-making is always difficult. However, the result of the performed case study is encouraging. We have no knowledge of the decision process that led to building the dam at the given location. Nevertheless, the fact that it coincides with the result of our case study is a strong indicator of the reliability of our method.

In order for any trade-off method to be efficient and reliable, it is important that the decision maker is aware of some important bias issues and addresses them accordingly. A number of issues concerning value trade-offs have been pointed out in relevant literature. Tversky & Simonson (1993) discuss the issue of trade-off contrast, suggesting that the decision maker's tendency to prefer x over y is enhanced if he/she encounters other choices in which comparable value on one attribute is associated with a larger difference on another. Another important issue, loss aversion, is discussed in a number of studies (Tversky & Kahneman, 1991; Tversky & Simonson, 1993; Bleichrodt & Pinto, 2002). The loss aversion hypothesis states that losses loom larger than corresponding gains, i.e. that the impact of a difference on a dimension (attribute) is generally greater when the difference is evaluated as a loss as compared to when the same difference is evaluated as a gain (Tversky & Kahneman, 1991). Yet another phenomenon that has impact on value trade-offs is the scale compatibility phenomenon. It has been shown (Tversky et al., 1998; Delquie, 1993) that the weight of an attribute is enhanced when its measuring scale is compatible with the measuring scale of the response attribute. Lahtinen & Hämäläinen (2016) consider loss aversion together with scale compatibility the primary cause of the path dependence phenomenon. They have shown that different paths, i.e. different choices of reference and measuring stick criteria, can lead to different choices, due to the accumulated effect of successive biased even swap tasks, the biases being due to scale compatibility and loss aversion phenomena. Addressing these issues is beyond the scope of this paper and they were not considered during the development of the GISwaps method. However, if and how those issues are manifested in our method is an important question that should be investigated in further research.

Apart from the bias issues, some common mistakes in making trade-offs need to be mentioned. In Keeney (2002) the author outlines a number of mistakes, such as not having measures for consequences or using inadequate measures, trying to calculate "objective" (correct) trade-off values, assessing value trade-offs independently of the range of consequences, etc. He suggests guidelines for avoiding those mistakes and following the guidelines should if not insure, then certainly substantially increase the reliability of any trade-off based decision-making method, and so even GISwaps. A decision support system based on our method gives the decision maker a possibility to work with concrete value trade-offs in form of even swaps when managing continuous multi-criteria problems. This increases the transparency and makes the decision making process more intuitive, which we believe

makes decision making both easier and more efficient. The main features of our method help to address the issue of the role of visualization in GIS decision making. As pointed out by Andrienko and Andrienko (2003), visualization is rarely used in the choice phase of the decision-making process and is usually limited to the initial phase where it plays an important role. This is largely due to the non-interactive nature of traditional methods. GIS decision-making requires more extensive use of visualization and interaction even during the decision process itself, i.e. during making the actual choices. In order to decide whether or not a trade-off to be made is feasible or admissible, the decision maker needs to see how an option is positioned in both the geographical and the attribute space, as well as how it compares to other options (Andrienko and Andrienko, 2003). Jankowski et al. (2001) find that highly interactive depiction of both criteria and decision spaces would be more productive for understanding the structure of the decision situation than static display. Thus, decision procedures should be facilitated by highly interactive visualization. In a forthcoming study, we will propose an interactive visualization framework for GISwaps guided by the above principles, that will help the decision maker to get insight into the complex relationships between the choice of reference and response criteria, the design of the update coefficients, and their effect on the huge amount of alternatives both in attribute space and geographical space.

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APPENDIX

Assume that we have a multi-criteria decision problem with four criteria: A, B, C and D. The decision maker (DM) needs to make six comparisons between criteria. Suppose that DM answers the questions about the importance relations between criteria in the following way (scenario 1):

A and B are equally important
A is 2 times more important than C
D is 3 times more important than A
B is 2 times more important than C
D is 3 times more important than B
D is 6 times more important than C

These answers are used to calculate a priority vector that includes the weight coefficients (Table 11). In this case the matrix is completely consistent and the calculations can be performed directly. However, the matrix is not always consistent and in such a case it is common that an Eigenvalue method is used in order to solve the problem. If DM would have answered in a following way, the matrix would not have been consistent (scenario 2):

A and B are equally important
A is 2 times more important than C
D is 3 times more important than A
B is 3 times more important than C
D is 3 times more important than B
D is 4 times more important than C

A and B are equally important but A is 2 times more important than C, while B is 3 times more important than C. In addition, the fact that DM thinks that D is 3 times more important than A and that A is 2 times more important than C should also mean that the DM thinks that D is 6 times more important than C, which is not the case. Since the comparison matrix is not consistent, the weights are calculated by the Eigenvalue method (Table 12).

The inconsistency is acceptable for CR < 0.1. Consistency ratio CR is calculated as:

$$CR = \frac{CI}{R_i}$$

Table 11. Weight coefficients for A, B, C and D based on scenario 1

	A	**B**	**C**	**D**	**Sum**	**Weight**
A	1	1	2	1/3	4.33	0.18
B	1	1	2	1/3	4.33	0.18
C	1/2	1/2	1	1/6	2.17	0.09
D	3	3	6	1	13.00	0.55
					23.83	1.00

Table 12. Weight coefficients for A, B, C and D based on scenario 2, calculated by the Eigenvalue method

	A	**B**	**C**	**D**	**Sum**	**Weight**
A	1	1	2	1/3	4.33	0.19
B	1	1	3	1/3	5.33	0.21
C	1/2	1/3	1	1/4	2.08	0.09
D	3	3	4	1	11.00	0.51
					22.75	1.00

where CI is consistency index for the matrix, calculated as:

$$CI = \frac{\lambda_{(max)} - n}{n - 1}$$

for n entries, and R_i is random consistency index. In the latter example:

$$\lambda_{(max)} = 4.06228$$

which gives us CI = 0.0208 and CR = 0.0230. The consistency ratio CR is smaller than 0.1, thus we can accept the inconsistency.

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