Exploring the Nexus Between the Shadow Economy, Finance, and Economic Growth in Tunisia:
Asymmetric NARDL Model

Chokri Terzi, Faculty of Economic Sciences and Management of Nabeul and LEGI, EPT, Carthage University, Tunisia*
https://orcid.org/0000-0002-9478-2941
Khalil Mhadhbi, Faculty of Economic Sciences and Management of Nabeul, Carthage University, Tunisia
Faouzi Abdennour, Faculty of Economic Sciences and Management of Nabeul, Carthage University, Tunisia

ABSTRACT

This study aimed to assess the effect of the shadow economy on the finance-growth relationship in Tunisia over the period 1984-2020. The authors used a nonlinear autoregressive distributed lags (NARDL) model to verify the impact of the informal economy as measured by Tanzi’s method on the finance-growth relationship. The results suggest that in the long term, with a positive change at the level of the shadow economy, the effect of financial development on growth becomes negative. The opposite is also true. However, in the short run, asymmetric effect of the shadow economy is only detected on economic growth and not on the financial development-economic growth nexus. Indeed, the level of the informal economy has an important role in the Tunisian economy. The significant and positive impact of financial development on the economy is strongly influenced by the size of the informal economy.

KEYWORDS
Economic Growth, Non-ARDL Approach, Shadow Economy

1. INTRODUCTION

The challenge of achieving sustained economic growth has long plagued developing countries as they strive to meet the aspirations of their people. The obstacles to growth vary between countries, including factors such as natural and human resources, technical knowledge, and capital. However, it is widely recognized that increasing a country’s capital resources is essential for accelerating economic progress. As such, the financial system plays a crucial role in the growth process. This relationship between financial development and economic growth has generated a vast body of literature (King and
Levine, 1993a, b; Thornton, 1994; Gregorio and Guidotti, 1995; Berthelemy and Varoudakis, 1996; Greenwood and Bruce, 1997; Greenwood and Smith, 1997; Blackburn and Hung, 1998; Rajan and Zingales, 1998; Beck et al., 2000; Kirkpatrick, 2000; Craigwell et al., 2001; Fase and Abma, 2003; Beck and Levine, 2004; Ang, 2008a; Fung 2009; Kar et al., 2011; Murinde, 2012; Pradhan, 2013; Hsueh et al., 2013; Herwartz and Walle, 2014; Uddin et al., 2014; Menyah et al., 2014).

Some authors have emphasized the non-linear relationship between financial development and economic growth, but the sources of this non-linearity remain inconclusive (Shen and Lee, 2006; Deidda and Fattouh, 2008; Cecchetti and Kharrouri, 2012; Law and Singh, 2014; Arcand et al., 2015; Ibrahim and Alagidede, 2017; Mhadhbi and Terzi, 2022).

One of the main challenges faced by developing countries is the informal economy, which negatively affects the financial sector (Blackburn et al., 2012; Bose et al., 2012; Capasso and Jappelli, 2013; Straub, 2005; Dabla-Norris et al., 2008). Blackburn et al. (2012) and Capasso and Jappelli (2013) argue that the development of the shadow economy renders the financial system unable of effectively managing economic functions. So, the growth of the shadow economy can impede economic progress through its impact on the financial system. In Tunisia, the informal economy affects all sectors and regions, including the service sector.

This paper aims to study the role of shadow economy asymmetric changes in the relationship between financial development and economic growth, using the nonlinear ARDL for Tunisia. Empirical evidence supports the assumption that a large shadow sector reduces the allocation of the financial sphere to the real sphere.

This paper makes a two-fold contribution. First, this study develops a proxy for measuring the informal economy of Tunisia using the Tanzi method (1980). Second, it investigates the potential asymmetric relationship between the informal economy, financial development, and economic growth. The study adopts the asymmetric co-integration methodology, namely the NARDL model (Shin et al., 2014).

The paper is structured as follows: Section 2 outlines Tanzi’s methodology for calculating a proxy for the shadow economy of Tunisia. Section 3 describes the proxy measures of financial development and economic growth. Section 4 focuses on the econometric methodology used in the study. Section 5 analyzes the empirical results, and Section 6 summarizes the conclusions.

2. MEASURING THE SIZE OF THE SHADOW ECONOMY: APPROACHES AND CHALLENGES

The shadow economy, also known as the non-observed economy, refers to markets where the exchange of goods and services occurs illegally, including the provision of illegal goods to consumers. Due to the hidden nature of these markets, they are difficult to observe and measure. To address this issue, economists have developed various methods to estimate the level of the informal economy in different countries.

There are two broad approaches to measuring the informal economy: indirect and direct. The direct approach relies on microeconomic data to estimate the scale of non-observed activities through surveys or tax audits. On the other hand, indirect approaches use four different methods: national accounts, monetary aggregates, demand for money, and multivariate methods. These methods are not easily comparable and may produce skewed estimations due to the nature of the field of activity. However, these methods are continuously evolving to address the complexity of the issue, and there is a need to complement the direct approach with macroeconomic estimations. The indirect approach is more relevant as it takes into account all available information on producing units and market balances.

The demand for money is one of the most important indirect methods to estimate the size of the informal economy (Gutmann, 1977). It is based on the assumption that individuals tend to use cash for their informal transactions to avoid detection by tax authorities. However, this method also has
its limitations, as it assumes that the demand for cash is stable, which may not always be the case in developing countries where people may switch to other currencies or assets.

2.1. How the Demand for Money Can Shed Light on the Shadow Economy

Tanzi’s contribution to the measurement of the shadow economy is based on the notion that factors other than the tax system and official regulations also affect the demand for liquid assets. Tanzi’s approach considers the cash payment of transactions and takes into account various conventional factors, such as the habits of payments, interest rates, direct and indirect taxes, public regulations, and the complexity of the fiscal system. These factors are accounted for in econometric estimations to isolate the excess demand for money that results from cashless transactions.

Furthermore, Bhattarachya et al. developed a quarterly model of demand for money in the United Kingdom between 1960 and 1984. This model distinguishes between the ratio of demand for money and declared income, as well as the ratio between demand for money and undeclared income. While it is based on the same hypotheses as those of Tanzi, the model excludes the fiscal pressure of explanatory variables. Thus, it presents an estimation of a phenomenon with an indefinite cause.

The consideration of various factors impacting the demand for liquid assets, including the cash payment of transactions, provides a more comprehensive understanding of the shadow economy. By accounting for these factors, the demand for money can be more accurately estimated, and the excess demand resulting from cashless transactions can be isolated. However, while these approaches have made progress in measuring the shadow economy, there is still much work to be done to fully understand the scope and impact of this phenomenon.

2.2. Shadow Economy Contribution in Tunisian Economy: Method of Tanzi

The Tanzi’s (1980) method used in this study involves in estimating a model with \( \frac{\text{Liquid assets}}{M_2} \) ratio as dependent variable. Explanatory variables are the gross domestic product per capita (GDP\(_{pc}\)), \( \frac{\text{Tax}}{\text{GDP}} \) ratio, \( \frac{\text{Total of salaries}}{\text{GDP}} \) ratio and the interest rate on the fixed-term deposits. Among the advantages of this method is that it can solve the problem of data availability. We present the equation object of estimation as:

\[
\ln \left( \frac{LA}{M_2} \right)_t = \alpha_0 + \alpha_1 \ln \left( \frac{\text{GDP}_{pc}}{\text{GDP}} \right)_t + \alpha_2 \ln \left( \frac{T}{\text{GDP}} \right)_t + \alpha_3 \ln \left( \frac{W}{\text{GDP}} \right)_t + \alpha_4 \ln R_t + \varepsilon_t
\]

(1)

Where:

- \( \frac{LA}{M_2} \) : Liquid assets ratio;
- \( \frac{GDP_{pc}}{\text{GDP}} \) : GDP per capita;
- \( \frac{T}{\text{GDP}} \) : Direct taxes reported to GDP;
- \( \frac{W}{\text{GDP}} \) : Total salaries reported to GDP;
- \( R \) : Credit interest rate;
Annual period from 1984 until 2020.

This first equation allows us to calculate liquid assets as follows:

\[
LA_t = e^{\alpha_0 + \alpha_1 \ln(GDP_{pc}) + \alpha_2 \ln\left(\frac{T}{GDP}\right) + \alpha_3 \ln\left(\frac{W}{GDP}\right) + \alpha_4 \ln R + \ln(M_2)}
\]

(2)

Because informal economy transactions are not subject to taxation, liquid assets will have the following formula:

\[
LA_t = e^{\alpha_0 + \alpha_1 \ln(GDP_{pc}) + \alpha_2 \ln\left(\frac{W}{GDP}\right) + \alpha_3 \ln R + \ln(M_2)}
\]

(3)

Empirically, liquid assets should be deduced according to equation (3) after estimation of equation (1). Table 1 presents the estimation results of equation (1) using ordinary least square (OLS) method:

According to these results, liquid assets can be calculated from equation (3) using this formula:

\[
LA_t = e^{1.314 - \frac{0.317}{\ln(GDP_{pc})} - 0.158 \ln\left(\frac{W}{GDP}\right) + 0.041 \ln R + \ln(M_2)}
\]

(4)

By subtracting the liquid assets from the mass of liquid money \(M_1\), we obtain the subterranean currency \(SC = M_1 - LA_{estimated}\).

Table 1. OLS estimation results: Independent variable EG

<table>
<thead>
<tr>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
</tr>
<tr>
<td>(GDP_{pc})</td>
</tr>
<tr>
<td>(\frac{T}{GDP})</td>
</tr>
<tr>
<td>(\frac{W}{GDP})</td>
</tr>
<tr>
<td>(R)</td>
</tr>
<tr>
<td>F-statistic</td>
</tr>
<tr>
<td>Prob(F-statistic)</td>
</tr>
<tr>
<td>(R^2)</td>
</tr>
<tr>
<td>Included observations</td>
</tr>
</tbody>
</table>
According to assumption that the speed \( S \) of circulation of liquid money is the same in the regular economy or the informal economy, then \( S = \frac{GDP_{official
economy}}{M_1} = \frac{GDP_{non-observed
economy}}{SC} \) which means that \( GDP_{non-observed
economy} = S \times SC \).

Descriptive statistics of the calculated values show that the share of the unobserved economy fell from 1985 to 1994, giving way to an upward trend until 2020 with a stagnation between 2002 and 2004 with an actual shadow economy importance accounts for more than 40% of the GDP. This share has seen an increase of 9 points explained essentially by the political environment that is punctuated by multiple crises characterized by instability and insecurity.

During the study period, Tunisian non-observed economy increased from a minimum of 12% of the GDP in 1994 to 44.5% in 2020 with an average of 28.7%. The informal economy presents the following dilemma: on one hand, it has a positive social character represented by its ability to fight poverty, create jobs and income; it cushions social crises and is a guarantor of social peace. On the other hand, it imposes some tax inequity and unfair competition.

### 3. MEASURING FINANCIAL DEVELOPMENT AND ECONOMIC GROWTH

In previous studies, various measures has been used to capture the level of financial development and economic growth. To ensure consistency with widely accepted measures of financial and economic development, we chose variables that closely follow these measures. The bank-based variables are the most commonly used measures in previous studies, including domestic and private credit, as well as monetary aggregates (King and Levine, 1993; Arestit and Demetriades, 1997; Kar et al., 2011; Khan and Senhadji, 2003; Liang and Teng, 2006; Luintel and Khan, 1999; Odhiambo, 2009; Hsueh et al., 2013; among others).

To measure financial development (FD) in our analysis, we followed King and Levine’s (1993) approach and selected two variables: domestic credit to the private sector by banks (DCPSB) as a percentage of GDP, and domestic credit provided by the financial sector (DCPFS) as a percentage of GDP. We excluded monetary variables to avoid the multicollinearity problem, as these variables were used to measure the informal economy. Economic growth (EG) is defined by GDP growth (annual %). To construct our variables, annual data from the World Development Indicators (2021) are used. Our selection of variables is grounded in the belief that they are reliable indicators of financial and economic development, and their use in previous studies lends further support to their validity.

### 4. EMPIRICAL MODELLING

In this study, asymmetry is observed when decrease (negative change) in the level of the shadow economy has a stronger impact on the finance-growth relationship compared to an increase (positive change) in the shadow economy. This asymmetry in the effect of changes in the shadow economy on the finance-growth relationship is an important factor to consider in our analysis.

The main objective of this paper is to analyze the impact of the shadow economy level on the finance-growth relationship in Tunisia over the period 1984-2020. To account for the asymmetries in the short and long run dynamics of the finance-growth relationship, we employ the cointegrating nonlinear ARDL (NARDL) model developed by Shin et al. (2014).

This model is well suited for our analysis as it allows for short and long run asymmetries, which is essential to capture the complex relationship between the shadow economy, finance, and economic growth. The model takes the following form:
\[ EG_t = \varphi_0 + \varphi_1 FD_t + \varphi_2 Shad^+_t + \varphi_3 Shad^-_t + \varepsilon_t \]  \hspace{1cm} (5)

In equation (5), the variable \( EG \) denotes the economic growth, while \( FD \) represents proxy measures for financial development (DCPSB and DCPFS). \( Shad^+_t \) and \( Shad^-_t \) are respectively partial sums of positive and negative changes in the shadow economy \( Shad_t \). \( \varphi = (\varphi_0, \varphi_1, \varphi_2, \varphi_3) \) is a vector of long run parameters to be estimated:

\[ Shad^+_t = \sum_{i=1}^{t} \Delta Shad^+_i = \sum_{i=1}^{t} \max(\Delta Shad_i; 0) \]  \hspace{1cm} (6)

\[ Shad^-_t = \sum_{i=1}^{t} \Delta Shad^-_i = \sum_{i=1}^{t} \min(\Delta Shad_i; 0) \]  \hspace{1cm} (7)

Equation (5) can be reformulated into an autoregressive distributed lag (ARDL) framework following the approach of Pesaran and Shin (1999) and Pesaran et al. (2001), as demonstrated in Shin et al. (2011):

\[ \Delta EG_t = a + b_0 EG_{t-1} + b_1 FD_{t-1} + b_2 Shad^+_t + b_3 Shad^-_t + \sum_{i=1}^{p} c_i \Delta EG_{t-i} + \sum_{i=0}^{q} d_i \Delta FD_{t-i} \]
\[ + \sum_{i=0}^{s} \left( \theta^+_t \Delta Shad^+_t + \theta^-_t \Delta Shad^-_t \right) + v_t \]  \hspace{1cm} (8)

According to the above formulation, \( \varphi_2 \) and \( \varphi_3 \) allows respectively to appreciate the long run relationship between economic growth and shadow economy increases (reduction), which are expected to be negative (positive). Furthermore, we suppose that an increase in the shadow economy will lead to declines and causes long-term changes in economic growth. These changes represented in the equation reflects the asymmetric impact of the shadow economy on economic growth over the long term.

In the equation (8), \( p \), \( q \), and \( s \) represents the lag orders of variables. \( \varphi_2 = -\frac{b_2}{b_0} \) and \( \varphi_3 = -\frac{b_3}{b_0} \) are the measures of the long run impacts of shadow economy increases and reductions on economic growth, respectively. \( \sum_{i=0}^{s} \theta^+_t \) and \( \sum_{i=0}^{s} \theta^-_t \) measures respectively the short-run influences of shadow economy increases (reduction) on economic growth. Therefore, apart from the asymmetric long run relation, this setting also captures the asymmetric short-run influences of changes in the shadow economy level on economic growth.

To examine the impact of positive and negative changes in the shadow economy on the relationship between finance and growth, the benchmark specification defined in equation (8) allows to distinguish between shadow positive specification (equation (9)) and shadow negative specification (equation (10)) defined as below:

\[ \Delta EG_t = a + b_0 EG_{t-1} + b_1 FD_{t-1} + b_2 Shad^+_t + \sum_{i=1}^{p} c_i \Delta EG_{t-i} \]
\[ + \sum_{i=0}^{q} d_i \Delta FD_{t-i} + \sum_{i=0}^{s} \theta^+_t \Delta Shad^+_t + v_t \]  \hspace{1cm} (9)

\[ \Delta EG_t = a + b_0 EG_{t-1} + b_1 FD_{t-1} + b_2 Shad^-_t + \sum_{i=1}^{p} c_i \Delta EG_{t-i} \]
\[ + \sum_{i=0}^{q} d_i \Delta FD_{t-i} + \sum_{i=0}^{s} \theta^-_t \Delta Shad^-_t + \mu_t \]  \hspace{1cm} (10)

The first step in NARDL approach is to check for stationarity. unit root tests are performed to determine the order of variables integration. In the second step, we estimated equations (9) and (10)
using the OLS method, while fixing the lag length based on the information criterion SCI. In the third step, we tested for the presence of co-integration among the variables using Bounds testing procedure. The presence of co-integration allows examining the short and long run asymmetries of the shadow economy on the finance-growth relationship using Wald test.

5. RESULTS AND DISCUSSION

As outlined earlier, Bounds testing procedure requires that variables should be I(0) and I(1) but not I(2). ADF and PP unit root tests are performed for our variables and the results are shown in Table 1 with the presence of unit root as null hypothesis.

Unit root tests results show that economic growth \(EG\) is stationary at level. However, shadow \(Shad\) and the two measures of financial development \(DCPSB\) and \(DCPF\) are stationary at their first differences.

According to this, no variable is I(2) and Bounds testing procedure can be applied. Schwarz information criterion (SIC) is used to select the optimal lag length.

The results of Bounds co-integration test for three models are presented in Table 2. Pesaran et al. (2001) assume that the long run relation is confirmed when the calculated F statistics are greater than the upper critical value Bounds².

This assumption is verified for the three models at 1%, which allow us to study the dynamic relation between economic growth, financial development and shadow economy (asymmetry of positive and negative changes).

Before presenting, the results for our three dynamic specifications estimation (Table 3) and as robustness check, tests performed show the following: for all specifications, J-B normality test prove that error terms are normally distributed. LM and ARCH tests confirm respectively the absence of residuals autocorrelation and heteroscedasticity.

Table 2. ADF and PP unit root tests result

<table>
<thead>
<tr>
<th>Variables</th>
<th>Level</th>
<th>First Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ADF</td>
<td>PP</td>
</tr>
<tr>
<td></td>
<td>Intercept</td>
<td>Trend and Intercept</td>
</tr>
<tr>
<td>EG</td>
<td>-4.037***</td>
<td>-4.781***</td>
</tr>
<tr>
<td>Shad</td>
<td>-0.406</td>
<td>-2.699</td>
</tr>
<tr>
<td>DCPSB</td>
<td>-0.907</td>
<td>-1.546</td>
</tr>
<tr>
<td>DCPF</td>
<td>-0.798</td>
<td>-0.214</td>
</tr>
</tbody>
</table>

Note: ***Rejection of null hypothesis of unit root at the 1% level.

Table 3. Results of bounds test for non-linear co-integration

<table>
<thead>
<tr>
<th>Dependent Variable EG</th>
<th>F-Statistic</th>
<th>95% Lower Bound</th>
<th>95% Upper Bound</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark specification</td>
<td>13.03680</td>
<td>3.29</td>
<td>4.37</td>
<td>Cointegration</td>
</tr>
<tr>
<td>Shadow positive specification</td>
<td>13.48238</td>
<td>3.65</td>
<td>4.66</td>
<td>Cointegration</td>
</tr>
<tr>
<td>Shadow negative specification</td>
<td>17.71157</td>
<td>3.65</td>
<td>4.66</td>
<td>Cointegration</td>
</tr>
</tbody>
</table>
Both the short and long term estimation results mentioned in Table 4 show a negative and significant effect of the positive change in the shadow economy (DShad+) on the economic growth. This effect is rather positive and significant for the negative change in the shadow economy (DShad-).

Table 4. Short and long-term NARDL results

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.236298*</td>
<td>0.197784</td>
<td>0.171287</td>
</tr>
<tr>
<td>$EG(-1)$</td>
<td>-2.148564***</td>
<td>-2.238995***</td>
<td>-2.015025***</td>
</tr>
<tr>
<td>$DCPSB(-1)$</td>
<td>0.621021**</td>
<td>-0.881045**</td>
<td>1.066895***</td>
</tr>
<tr>
<td>$DCPFS(-1)$</td>
<td>-0.614002***</td>
<td>-0.785284***</td>
<td>0.867945***</td>
</tr>
<tr>
<td>$SHAD^+(-1)$</td>
<td>-0.083045*</td>
<td>-0.045408***</td>
<td></td>
</tr>
<tr>
<td>$SHAD^-(1)$</td>
<td>0.024639***</td>
<td></td>
<td>0.118200***</td>
</tr>
<tr>
<td>$DEG(-1)$</td>
<td>0.735494***</td>
<td>0.953324**</td>
<td>0.698464***</td>
</tr>
<tr>
<td>$DEG(-2)$</td>
<td>0.631402***</td>
<td>0.606254***</td>
<td>0.440006***</td>
</tr>
<tr>
<td>$DEG(-3)$</td>
<td>0.142718*</td>
<td>0.191887*</td>
<td>0.110399*</td>
</tr>
<tr>
<td>$DDCPSB(-1)$</td>
<td></td>
<td>-0.284700</td>
<td>-0.474084</td>
</tr>
<tr>
<td>$DDCPSB(-2)$</td>
<td>-0.170953</td>
<td>-0.477850*</td>
<td>-0.403084</td>
</tr>
<tr>
<td>$DDCPSB(-3)$</td>
<td></td>
<td>0.203424**</td>
<td>0.337469***</td>
</tr>
<tr>
<td>$DDCPF(-1)$</td>
<td></td>
<td>0.228448***</td>
<td>0.353141***</td>
</tr>
<tr>
<td>$DDCPF(-2)$</td>
<td></td>
<td>0.228448***</td>
<td>0.353141***</td>
</tr>
<tr>
<td>$DSHAD^+(1)$</td>
<td>-0.671381***</td>
<td>-0.340646***</td>
<td>0.203310***</td>
</tr>
<tr>
<td>$DSHAD^+(2)$</td>
<td>-0.210926**</td>
<td>-0.224223**</td>
<td></td>
</tr>
<tr>
<td>$DSHAD^-(1)$</td>
<td>0.556167***</td>
<td></td>
<td>0.256374***</td>
</tr>
<tr>
<td>$DSHAD^-(2)$</td>
<td>0.034163*</td>
<td>-0.01480</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.782809</td>
<td>0.774963</td>
<td>0.770669</td>
</tr>
<tr>
<td>D-W</td>
<td>2.083845</td>
<td>2.156945</td>
<td>2.123750</td>
</tr>
<tr>
<td>J-B</td>
<td>3.709126</td>
<td>0.772702</td>
<td>2.421198</td>
</tr>
<tr>
<td>LM(1) F-statistic</td>
<td>0.592959</td>
<td>1.168270</td>
<td>0.810389</td>
</tr>
<tr>
<td>LM(2) F-statistic</td>
<td>0.424995</td>
<td>0.555777</td>
<td>0.465191</td>
</tr>
<tr>
<td>ARCH(1) F-statistic</td>
<td>0.080967</td>
<td>0.000474</td>
<td>0.039605</td>
</tr>
<tr>
<td>ARCH(2) F-statistic</td>
<td>0.056937</td>
<td>0.005570</td>
<td>0.222913</td>
</tr>
</tbody>
</table>

The superscripts “*” and “−” denote positive and negative partial sums, respectively. *Significance at the 10% level. **Significance at the 5% level. ***Significance at the 1% level.
This allows providing evidence of asymmetry for all specifications. Concerning the financial development, the effect of the two measures is respectively opposite, negative and positive for benchmark, positive and negative specifications.

According to Shin et al. (2014), the cointegrating NARDL estimation results (equation 5) allows to study the effects of long run dynamics for three specifications as presented in Table 5.

This table shows that financial development effect on the economic growth depends on shadow economy variation. As expected above, the coefficient of negative component is positive and significant to explain economic growth. Concerning the positive component of shadow economy, the coefficient is negative and statistically insignificant. The coefficients of the two measures of financial development are opposite and insignificant.

Compared to benchmark specification, coefficients are statistically significant allows to observe that coefficients of the two financial development measures change from negative upon shadow positive specification to positive with shadow negative specification. This result involves that shadow economy effect on finance-growth relationship depend on its level.

To deepen the analysis of short and long run asymmetry of the shadow economy on finance-growth relationship, Wald test is used. Table 6 presents results of the asymmetric test for the benchmark specification.

In Table 6, the null hypothesis that the variables are symmetric is rejected, meaning that both in the short and long run, the positive and negative partial sum are significantly different from each other and support asymmetric behavior. Thus, shadow economy differently influence economic growth in both short and long-run and with different levels of positive and negative effects. This triggered their further analysis to find their impact on growth.

6. CONCLUSION

The shadow economy has had a significant impact on Tunisia, especially after the 2011 revolution. This study aimed to assess the effect of the shadow economy on the finance-growth relationship in

### Table 5. Long-run results

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.109980***</td>
<td>0.088336***</td>
<td>0.085005***</td>
</tr>
<tr>
<td>DCPSB</td>
<td>0.289040</td>
<td>-0.393500***</td>
<td>0.529470**</td>
</tr>
<tr>
<td>DCPFS</td>
<td>-0.285773</td>
<td>-0.350731**</td>
<td>0.430737*</td>
</tr>
<tr>
<td>SHAD$^+$</td>
<td>-0.038651</td>
<td>-0.020281**</td>
<td></td>
</tr>
<tr>
<td>SHAD$^-$</td>
<td>0.011468*</td>
<td></td>
<td>0.058659**</td>
</tr>
</tbody>
</table>

### Table 6. Wald test asymmetry results

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>Probability (Long Run)</th>
<th>Probability (Short Run)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-statistics</td>
<td>0.0022</td>
<td></td>
</tr>
<tr>
<td>F-statistics</td>
<td>0.0022</td>
<td>0.1122</td>
</tr>
<tr>
<td>Chi-square</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>
Tunisia. To achieve this, a nonlinear ARDL model was used to capture both long run and short-run asymmetric effects of the informal economy on the relationship between finance and economic growth.

The results showed that the relationship between financial development and economic growth in Tunisia is highly sensitive to the informal economy. In the long term, the two financial development indicators had a positive impact on economic growth, but the component of the informal economy had an expected effect. However, when the component of the informal economy move from negative to positive, the impact of financial development on the real economy change from positive to negative. In contrast, in the short-run, this asymmetry appears only between the informal economy and economic growth and not between shadow economy and the finance-growth relationship.

In summary, the study revealed new insights into the relationship between financial development and economic growth in Tunisia. It highlighted that the significant and positive impact of financial development on the economy is strongly influenced by the size of the informal economy. Thus, policy makers in Tunisia should consider this issue when ensuring the control of the share of the informal economy in the country. Implement strategies to reduce high levels of shadow economy in Tunisia would make the role of financial development more efficient in economic growth.
REFERENCES


**ENDNOTES**

1. The semi-logarithmic specification of Tanzi was selected because it remains the best among estimated model.

2. We used $k = 4$ for benchmark specification and $3$ for shadow positive and shadow negative specification. Critical values are recommended by Shin *et al.* (2011).