Review of Solution Approaches for the Symmetric Traveling Salesman Problem

Georgios K. D. Saharidis, Department of Mechanical Engineering, University of Thessaly, Volos, Greece

ABSTRACT

In this paper, the main known exact and heuristic solution approaches and algorithms for the symmetric Traveling Salesman Problem (TSP), published after 1992, are surveyed. The paper categorize the most important existing algorithm to 6 main groups: i) Genetic algorithms, ii) Ant colony methods, iii) Neural Methods, iv) Local search algorithms and Tabu search, v) Lagrangian methods and vi) Branch and bound and branch & cut algorithms.

INTRODUCTION

One of the most well-known computational optimization problems is the traveling salesman problem (TSP). Consider a salesman who has to visit \( n \) cities, the TSP asks for the shortest tour through all the cities such that no city is visited twice and the salesman returns at the end of the tour back to the starting point. Let \( G=(V,A) \) be a graph where \( V \) is a set of \( n \) vertices and \( A \) a set of arcs. Let \( C=(c_{ij}) \) be a distance matrix associated with \( A \). The TSP consists of determining a minimum distance circuit passing through each vertex once and only once. Such a circuit is known as a tour or Hamiltonian cycle (or tour). In several application of TSP \( C \) could be a cost, travel time, fuel consumption, emissions, environmental externalities etc. In some case the matrix \( C \) is symmetrical \((c_{ij} = c_{ji})\) and in some other case asymmetrical \((c_{ij} \neq c_{ji})\). The Euclidean TSP is a special case of symmetric TSP when \( V \) is a set of point in \( \mathbb{R}^2 \) and \( c_{ij} \) is the straight line distance between \( i \) and \( j \).

TSP formulations and solution approaches are used for computer wiring, wallpaper cutting, hole punching, job sequencing, dartboard design, crystallography etc. There are many theoretical results concerning the optimal solution of the TSP. Some of them talk about the properties of the optimal solution and some other for novel formulations which in some cases can provide the optimal solution faster than other approaches.

Warren (1992) presents 3 theorems about the optimality of a selected arc based on the structure of the length matrix of the TSP. These theorems give the conditions under which an
arc is part of the optimal tour in general, when the matrix is symmetric and when it contains only 0-1 value. Codenotti and Margara (1992) present a local search heuristics for solving general optimization problems which satisfies a simple difference equation. If this equation is satisfied then the solutions obtained by the local search heuristics have interesting structural properties. The authors prove that the TSP with certain notions of neighborhood satisfies the above-mentioned difference equation and that its solution applying local search heuristics has certain structural properties. The main structural properties are that: a) the average cost over all the feasible solutions depends on the cost of a specific feasible solution $x$ and on the costs of the solutions which belong to its neighborhood, b) the average cost of the solutions belonging to the neighborhood of any solution $x$ is linearly related to the cost of $x$ and c) the average cost of the solutions belonging to the neighborhood of any solution $x$ $(A\text{VE}(x))$, the average cost over all the feasible solutions $(A\text{VE})$, and the cost of $x$ $(COST(x))$ satisfy one and only one of the following 3 relations:

$$COST(x) < A\text{VE}(x) < A\text{VE} \quad \text{or}$$

$$COST(x) = A\text{VE}(x) = A\text{VE} \quad \text{or}$$

$$COST(x) > A\text{VE}(x) > A\text{VE}.$$ 

Warburton (1993) presents, a tight worst-case performance analysis of some convex hull insertion heuristics for the Euclidean TSP. Insertion algorithms are based on the idea that between two selected points constructing a subtour, a third point will be selected based on a criterion and inserted between the two original points (ex. select to insert the point which is nearest to the starting point). Specifically, Warburton studies the cheapest and nearest insertion algorithm. The main theoretical result of the paper is that starting these insertion algorithms with the convex hull subtour rather than with a single city subtour may considerably enhance the overall performance of an insertion heuristic.

van Dal et al. (1993) analyze two existing algorithms applied to two TSP problems, a small TSP and a large TSP. The main purpose of the paper is to show the relationship between the subtour elimination algorithm of Gilmore and Gomory (1964) for the large TSP, and Gabovich’s (1970) algorithm for the small TSP. Some theoretical results are presented and proved concerning the complexity of each solution approach as well as some conditions for optimality of TSP, using these algorithms.

A new bound for the symmetric TSP is proposed by Cowling and Maffioli (1995) using a matroid theoretic formulation of the problem. The resulting bound is shown to present a link between the well-studied 1-tree (Held & Karp, 1970) and 2-matching (Pekny & Miller, 1994) bounds. Another approach for finding upper bounds for the solution of the TSP is discussed by Logani (2000). The new algorithm referred to as Minimal-cost-first algorithm (MCFA) is compared to the Quick Tour Construction Algorithm (QTCA) (Tucker, 1995). MCFA finds upper bounds with three main steps that are simpler than QTCA. It is shown that the obtained bounds by MCFA are very close to QTCA.

Arthanari and Usha (2000) present an alternate formulation of the symmetric TSP. The new formulation has a constraint set of polynomial size. The authors compare the polytope defined by this new formulation with the standard subtour elimination polytope (SEP) showing that the new polytope is smaller than the SEP. The advantage of this formulation is that a solution approach can start with this formulation and add progressively all other facet-defining inequalities to its constraint set.

Monnot et al. (2002) show that the well-known 2-opt algorithm approximately solves TSP within differential approximation ratio bounded above by $1/2$. They analyze the approximation behavior of 2-opt algorithm when used to approximately solve TSP in bipartite graphs and prove that it achieves differential approximation ratio bounded above by $1/2$ also in this case. They also prove that, for any $\epsilon > 0$, it is NP-hard to differentially approximate metric TSP within better than $649/650 + \epsilon$ and TSP with distances 1 and 2 within better than $741/742 + \epsilon$. 
Motivators to Adopt Green Supply Chain Initiatives
[www.igi-global.com/article/motivators-to-adopt-green-supply-chain-initiatives/129688?camid=4v1a](www.igi-global.com/article/motivators-to-adopt-green-supply-chain-initiatives/129688?camid=4v1a)

[www.igi-global.com/chapter/selection-evaluation-3pl-providers/69249?camid=4v1a](www.igi-global.com/chapter/selection-evaluation-3pl-providers/69249?camid=4v1a)