Mathematical Model for Designing Supply Chains

Jairo R. Montoya-Torres
Universidad de La Sabana, Colombia

INTRODUCTION

According to well-known definitions available in the literature (Simchi-Levy, Kaminsky, & Simchi-Levy, 2004; Melo, Nickel, & Saldanha-da-Gama, 2009), Supply Chain Management (SCM) is “the process of planning, implementing and controlling the operations of the supply chain in an effective way. SCM spans all movements and storage of raw material, work-in-process inventory, and finished goods from the point-of-origin to the point-of-consumption.” Part of the planning process in SCM aims to find the best possible configuration of the supply chain. In addition to the generic facility location problem, other areas (e.g., procurement, production, inventory, distribution, routing) has to be considered as well (Cordeau, Pasin, & Solomon, 2006). These areas can be considered either as an integrated decision-making model or as independent, yet interconnected, models. The former approach drives to a very complex model, while the latter is easier to understand and to implement in real practice, yet requiring consistency between solutions of each decision level. When applying a hierarchical decision-making approach, three decision-making levels are usually distinguished depending on the time horizons: strategic, tactical and operational (Vidal & Goetschalckx, 1997; Bender, Hennes, Kalesics, Melo, & Nickel, 2002). In this chapter, we focus on the strategic level. It deals with decisions that have a long-lasting effect on the firm such as the number, location and capacities of warehouses and manufacturing plants (Simchi-Levi et al., 2004). In strategic decisions due to the large amounts of investments, facilities that are located now are expected to operate for a long-term horizon (Melo et al., 2009). The design of the distribution network and the definition of material flows has become a major challenge for organizations as they simultaneously aim to decrease costs and increase responsiveness in today’s competitive market. In this regard, operational aspects such as lead times and material flows have been identified as issues which are best treated by considering them once the network design is established.

In this chapter, we consider a decision-making problem related to simultaneously define the location of facilities (warehouses) and the flow of products through the supply network. When only two stages (manufacturing plants and warehouses) are taken into account and warehouses are considered to be of infinite capacity, this problem is known in the literature as the Two-Echelon Uncapacitated Facility Location Problem (TUFLP). This problem belongs to the family of multi-item (or multiple products) production-distribution problems, which is one of the most important problems in Facility Location Theory (Daskin, 1995). In a general case, the problem is defined as follows. A firm may have relatively few products and a number of plants. Products are shipped from plants to markets (or customers) via a set of warehouses. The key issues we are concerned with are:

1. How many warehouses to have,
2. Where to locate warehouses, and
3. How the products should flow through the system.

Implicit in the product flow decision are other decisions about which products should be pro-
duced at which plants for which markets. Figure 1 is a schematic of such a system.

The problem can be formalized using mathematical programming, but it is known to be NP-hard (Cornuejols, Nemhauser, & Wolsey, 1990), which means that optimal solutions for large-sized instances are not easy to find in reasonable amount of time. This chapter presents a well-known mathematical model based on mixed integer linear programming. The reader must note that the NP-hardness of this problem suggests the use of approximate algorithms to obtain efficient solutions. Some heuristics and meta-heuristics algorithms are also studied in this book (Montoya-Torres, 2013).

**OVERVIEW OF RELATED LITERATURE**

As stated before, we consider a strategic decision-making problem regarding the design of a supply chain with two echelons. Warehouses have infinite capacity, and the design decision consists of both their location and how manufactured products flows from production plants to markets through the selected warehouses. In Complexity Theory, this problem belongs to the class of NP-hard. Hence, various solution approaches have been considered in the scientific literature reported in state-of-the-art surveys of various variants of the problem. Vidal and Goetschalckx (1997) present a review of models focusing on the impact in the management of the global supply chain. Chen (2004) classifies existing models to solve production-distribution problems into five classes based on level of decisions, structure of production-distribution integration, and problem parameters. Most of the resolution approaches are based on mixed-integer linear programming (MILP) for small to medium size instances.


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**Figure 1. Representation of the TUFLP**

![Figure 1. Representation of the TUFLP](image)