ABSTRACT

A mathematical tool, namely the Kelvin transformation, has been employed in order to derive analytical expressions for important hydrodynamic quantities, aiming to the understanding and to the study of the blood plasma flow past a Red Blood Cell (RBC). These quantities are the fluid velocity, the drag force exerted on a cell and the drag coefficient. They are obtained by employing the stream function $\psi$ which describes the Stokes flow past a fixed cell. The RBC, being a biconcave disk, has been modelled as an inverted prolate spheroid. The stream function is given as a series expansion in terms of Gegenbauer functions, which converge fast. Therefore we employ only the first term of the series in order to derive simple and ready to use analytical expressions. These expressions are important in medicine, for studying, for example the transportation of oxygen, or the drug delivery to solid tumors.

Keywords: Blood Plasma Flow, Kelvin Transformation, Mathematical Model, RBC, velocity, drag coefficient

1. INTRODUCTION

The main function of the blood as it circulates through the body is to transport oxygen, nutrients and waste. The blood flow in large arteries is considered as the flow of a homogeneous viscous fluid where its particular composition is disregarded (Yamaguchi et al., 2006). However, when the blood flows through capillaries, the flow may be described as creeping or Stokes flow. This assumption is justified by considering the values of the physical characteristics of the fluid (velocity, density, viscosity) and also by taking into account the dimensions of the RBC. Some indicative values are, for the density of the blood plasma $\rho = 1$ gr/cm$^3$, the velocity $v \leq 0.1$ cm/sec, the viscosity $\eta \approx 10^{-2}$ g/(cm⋅sec) and the characteristic length $S \approx 10^{-3}$ cm. These imply that
the Reynolds number, Re, becomes Re \ll 10^{-2},
which confirms the creeping flow assumption,
according to which the viscous forces dominate
over inertial forces. Stokes flow has also been
employed for describing the flow of many other
bio-fluids, (Davis et al. 2003; Marhefka et al.

Aiming in the better understanding of the
blood behaviour, many studies consider the
blood as a multiphase fluid, namely the blood
plasma, where three kinds of cells are suspending
in it. These are the red blood cells, RBCs,
the white blood cells and the platelets. Since the
RBCs occupy about the 45% of the fluid, the
study of the blood plasma flow around RBCs
is significant.

Furthermore, the study of the blood flow
at a cellular level needs also the mathematical
modelling of another parameter, which is the
geometrical shape of the RBC. As the blood
flows into the vessels the normal RBCs, are
deformable into different shapes depending
on the hydrodynamic stresses that act on them.
RBCs have been considered in the literature
either as rigid or solid elastic spheres (Quinlan
& Dooley 2007; Wang & Skalak 1969), truncated
ellipsoids (Fitz-Gerald 1972), rigid pistons (Zien
1969) or more realistically, as a biconeave disc
(Bagchi 2007; Noguchi & Gompper 2005; Mc-
Whirter et al. 2009; Dassios et al. 2012),
that at rest are having the major diameter of about
8\mu m and thickness of at least 2 \mu m (Sugii et al.
2005). Another category of studies consider that
the normal RBCs are deformable into different
shapes, as the blood flows into the vessels,
depending on the hydrodynamic stresses that
act on them. Different numerical approaches
for solving this problem can be found in the
literature. Some of them are the boundary
element method (Youngren \& Acrivos 1975;
Pozrikidis 2001; Pozrikidis 2003), the immersed
boundary methods (Bagchi 2007; Eggleton \&
Popel 1998), particle methods (Tsubota et al.
2006) etc. Experimental works regarding flow
visualization techniques in microcirculation
can be also found (Lima et al. 2009). All these
methods and techniques although describe ad-
equately the process, their utility is sometimes
limited due to some mostly technical factors
(spatial resolution etc.)

Aspiring to address these difficulties, we
provide analytical expressions for the quantities
of interest. In this manuscript, the problem of
the blood plasma flow around a red blood cell
is considered as Stokes flow around a rigid
inverted prolate spheroid (Dassios et al. 2012).
The stream function $\psi_\alpha (r)$ has been analyti-
cally obtained by employing the Kelvin inver-
sion (Dassios \& Kleinman 1989a; Dassios \&
Kleinman 1989b; Baganis \& Hadjinicolaou
2009; Baganis \& Hadjinicolaou 2010; Dassios
2009) and the concept of the semiseparation of
variables in PDEs (Dassios et al. 1994). It is
given through a series expansion of Gegen-
bauer functions which converge fast (Lebedev
1972). The Kelvin inversion is an effective
transformation method that has also been em-
ployed for solving analytically, exterior poten-
tial problems with Dirichlet and with Neumann
boundary conditions, (Dassios \& Kleinman
1989a; Dassios \& Kleinman 1989b; Baganis
\& Hadjinicolaou 2009; Baganis \& Hadjinicola-
ou 2010). According to this method, one can
obtain the solution of the exterior potential or
Stokes flow boundary value problems by trans-
forming through Kelvin inversion the solutions
of the equivalent interior problems and vice
versa.

In the present work, we exploit these two
methods in order to calculate the velocity com-
ponents of the fluid, the drag force $F_z$ exerted
on the RBC and the dimensionless drag coef-
efficient (Happel \& Brenner 1991). These quan-
tities are expected to be utilized in the study of
transport phenomena related to the blood flow,
such as the transportation of oxygen or drug
delivery. In order to express mathematically
the flow field we employ the stream function
$\psi$ obtained in (Dassios et al. 2012), which is
given through a fast converging series expan-
sion. Consequently, we restrict our calculations
only to the first term of the series, which seems
to be adequate for gaining simple, “closed form”
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