A Column Generation Procedure for the Split Delivery Vehicle Routing Problem Using a Route-Based Formulation

Joseph Hubert Wilck IV, Department of Engineering, East Carolina University, Greenville, NC, USA

Tom M. Cavalier, The Harold and Inge Marcus Department of Industrial and Manufacturing Engineering, Pennsylvania State University, University Park, PA, USA

ABSTRACT

The Split Delivery Vehicle Routing Problem (SDVRP) allows customers to be assigned to multiple routes. A column generation procedure using a route-based formulation is developed for the SDVRP and computational results are given for thirty-two data sets from previous literature. The SDVRP route-based formulation appears tighter (stronger) linear relaxation than a comparable SDVRP flow formulation. With respect to the total travel distance and computer time, the column generation procedure compares favorably versus previously published methods. The result is important for real-world since both the tractability and solution quality of larger (e.g., many vehicle routes, many customers) problems are both key issues when making vehicle routing decisions. The column generation method, along with the route-based formulation, provides an improved solution procedure that would be more appropriate for real-world problems than the previously published methods.

Keywords: Column Generation, Integer Programming, Split Delivery, Vehicle Routing

1. INTRODUCTION AND LITERATURE REVIEW

The primary research findings of this paper include combining a route-based formulation and a column generation procedure for the split delivery vehicle routing problem (SDVRP), including computational results compared to previously published methods. The Vehicle Routing Problem (VRP) is a prominent combinatorial optimization problem in distribution planning. Various forms of the VRP are studied for both theoretical and practical purposes. The solution quality of these problems is essential to the logistics industry when considering transportation costs. Dantzig and Ramser (1959) first exhibited these cost savings by providing a formulation and solution approach for gasoline distribution. Toth and Vigo (2002a) estimate that computer-generated solutions for distribution

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problems reduce transportation costs by 5%-20%, and this is a significant decrease since transportation costs represent between 10%-20% of the final cost of a delivered product or unit. Ellis et al. (2010) depicted this decrease to be approximately 10% within a manufacturing facility. Review literature for the VRP includes a survey by Toth and Vigo (2002b) for exact solution procedures, whereas Cordeau et al. (2002, 2005) survey heuristic procedures. Recent papers include Gutiérrez-Jarpa et al. (2009) and Jabali et al. (2009) which depict the single vehicle routing problem and time-dependent vehicle routing, respectively.

Dror and Trudeau (1989, 1990) formally introduced the SDVRP. The primary motivation to split a customer’s demand over multiple routes is to reduce the travel distance and the number of vehicle routes (or total number of vehicles). If each vehicle has the same capacity, then the minimum number of routes is the total demand divided by the vehicle capacity rounded up to the nearest integer. They proved, given that the distance between nodes follows the triangle inequality, there exists an optimal solution where no two routes can have more than one split demand point in common, and that there exists an optimal solution with no \( k \)-split cycles (for any \( k \)). Based on these proofs, Dror and Trudeau (1990) devised a heuristic to solve the SDVRP given an initial capacitated VRP (CVRP) solution by splitting a node’s demand to fill the routes to capacity. Dror et al. (1994) extended the formulation of Dror and Trudeau (1989, 1990) with additional constraints, and developed a constraint relaxation branch and bound algorithm. The results from Dror and Trudeau (1989, 1990) and Dror et al. (1994) show that the percent reduction in travel distance, when compared to the CVRP, is most prominent among problems where customers have high demands (i.e., more than 10% of the vehicle capacity). The results showed that the SDVRP solution used fewer routes than the CVRP solution; however, the SDVRP solution did not always use the minimum number of routes.

A graphical representation for an SDVRP solution is provided in Figure 1. In this example, there are three customers with demands of 100 units apiece and the vehicle capacity is 150 units. In the CVRP, the solution would be to visit each customer once and return to the depot. Whereas, in the SDVRP a single customer’s demand can be split over two vehicle routes; thus, saving a vehicle route.

Frizzell and Giffin (1992, 1995) solved the SDVRP with time windows on grid networks and developed heuristics to generate solutions. Archetti et al. (2006a) and Aleman and Hill (2009) developed tabu search procedures for the SDVRP. Archetti et al. (2006b) analyzed the worst-case properties of the SDVRP. Archetti et al. (2008) discussed when demand splitting is most beneficial. Lee et al. (2006) developed a dynamic programming model for the vehicle routing problem with split pick-ups with an uncountable (infinite) number of state and action spaces. Belenguer et al. (2000) performed a polyhedral study on the SDVRP to produce lower bounds through formulating the problem with undirected arcs by assuming symmetric distances. Using a cutting-plane algorithm in conjunction with a relaxed formulation, they
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