Chapter 8

Chaotic Attractor in a Novel Time-Delayed System with a Saturation Function

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ABSTRACT

From the viewpoint of engineering applications, time delay is useful for constructing a chaotic signal generator, which is the major part of diverse potential applications. Although different mathematical models of time-delay systems have been known, few models can exhibit chaotic behaviors. Motivated by attractive features and potential applications of time-delay models, a new chaotic system with a single scalar time delay and a nonlinearity described by a saturation function is proposed in this chapter. Nonlinear behavior of the system is discovered through bifurcation diagrams and the maximum Lyapunov exponent with the variance of system parameters. Interestingly, the system shows double-scroll chaotic attractors for some suitable chosen system parameters. In order to confirm the correctness and feasibility of the theoretical model, the system is also implemented with analog electronic circuit. Finally, a practical application of such circuit is discussed at the end of this chapter.

1. INTRODUCTION

After the discovery of the first classical chaotic attractor in 1963 (Lorenz, 1963), chaos theory has received a great deal of attentions (Anishchenko, 1993; Grebogi, 1997; Davies, 2004; Banerjee, 2011). Although the fact that there is not an universal definition of chaos, three remarkable characteristics of a chaotic system are: dynamical instability, topological mixing and dense periodic orbits (Hasselblatt,
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2003). Dynamical instability is known as the “butterfly effect”, which means that a small change in initial conditions of system can create significant differences. In other word, this vital characteristic makes the system highly sensitive to initial conditions (Lorenz, 1963; Strogatz, 1994). Topologically mixing is refers as stretching and folding of the phase space, which means that the chaotic trajectory at the phase space will evolve in time so that each given area of this trajectory will eventually cover part of any particular region. Dense periodic orbits means that the trajectory can come arbitrarily close ev-ery possible asymptotic state. Hence, the future behaviors of a chaotic system seem to be arbitrary and unpredictable. In fact, chaos appears naturally in weather and climate, biology, sociology, and physics etc. (Strogatz, 1994; Holland, 1998; Hilborn, 2000).

From the view point of engineering, chaotic systems with their complex characteristics have also been successfully utilized in diverse applications, ranging from secure chaotic communications (Cuomo, 1993), information encryption (Volos, 2013), economics (Medio, 2009), robotics (Volos, 2012), to liquid shakers (Zhang, 2007) and so on. One of the most interesting applications is chaotic cryptography (Kocarev, 2001), where chaotic systems provide alternative techniques capable of enhancing cryptographic features (Koc, 2009). The structural relationship between cryptography and chaos seem quite natural (Alvarez, 2006). It has been known that main properties of cryptography like confusion, diffusion, deterministic pseudo randomness and algorithm complexity connect directly with their analogous properties of chaos like ergodicity, sensitivity to initial conditions or system parameters, deterministic dynamics and structural complexity, respectively (Shannon, 1949; Alvarez, 2006; Kocarev, 2011). By now, a great majority of chaos-based cryptographic systems has been developed (Zhang, 2005; Tong, 2009; Wang, 2012; Seyedzadeh, 2012). Recent obtained results of such cryptographic systems can be found in (Kocarev, 2011).

Time delays have become the subject of active research due to the fact that they almost present in any real dynamical systems, few examples are optical bistable resonator (Ikeda, 1980; Ikeda, 1987), neural networks (Murray, 1990), control systems (Pyragas, 2006), or biological systems (Mackey, 1997; Sun, 2006). Delay differential equations differ from ordinary differential equations in that the evolution of dependent variables at a certain time depend on their values at previous times (Driver, 1977). Therefore, delay differential equation has used widely to study properties of such systems with delays (Kuang, 1993; Balachandran, 2009; Lakshmanan, 2011).

As it has already been established that delay can be exploited for chaos control (Pyragas, 1992). Time-delayed feedback control has reported as an alternative method besides the classical OGY method (Ott, 1990). This effective control method does not require the prior knowledge about the model and can stabilize the chaotic behavior to one of unstable fixed points or unstable periodic orbits embedded within chaotic attractor (Pyragas, 2006). Because the feedback signal is proportional to the difference of output signal and its delayed version, there are two control parameters: the feedback gain and the feedback time delay. It has been known that the feedback time delay is often different from the intrinsic delay of the considered time-delay system which needs to be controlled. In contrast, the presence of delay in dynamical systems may induce instability and complex phenomena (Xia, 2009). Oscillatory instabilities are frequently observed in systems described by delay differential equations. These unexpected oscillations lead to limitations to the performance of practical systems, especially mechanical machines (Hu, 2002). On the opposite, time delays may also be useful to generate chaos in the so-called time-delayed chaotic system which constitutes an importance class of chaotic systems (Lu, 1996). Time-delayed chaotic systems have been discovered in both theory (Lakshmanan, 2011) and realistic applications, i.e. secure communications (Prokhorov, 2008; Kwon, 2011), economic process (Salarieh, 2009), road traf-