Chapter 18

On Stability Analysis of Switched Linear Time-Delay Systems under Arbitrary Switching

Marwen Kermani
National Engineering School of Monastir, Tunisia

Anis Sakly
National Engineering School of Monastir, Tunisia

ABSTRACT

This chapter focuses on the stability analysis problem for a class of continuous-time switched time-delay systems modelled by delay differential equations under arbitrary switching. Then, a transformation under the arrow form is employed. Indeed, by using a constructed Lyapunov function, the aggregation techniques, the Kotelyanski lemma associated with the M-matrix properties, new delay-dependent sufficient stability conditions are derived. The obtained results provide a solution to one of the basic problems in continuous-time switched time-delay systems. This problem ensures asymptotic stability of the switched time-delay system under arbitrary switching signals. In addition, these stability conditions are extended to be generalized for switched systems with multiple delays. Noted that, these obtained results are explicit, simple to use, and allow us to avoid the problem of searching a common Lyapunov function. Finally, two examples are provided, with numerical simulations, to demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

A switched system is composed of an indexed family of subsystems described by continuous or discrete-time dynamics and a rule orchestrating the switching between them. As a special class of hybrid systems, switched systems have strong engineering background in various areas and are often used as a unified modeling tool for a great number of real-world systems such as power electronics chemical processes,
mechanical systems, automotive industry, aircraft and air traffic control and many other fields (Morse, 1997; Liberzon & Morse, 1999; Liberzon, 2003; Sun & Ge, 2005, Sun & Ge, 2011; Shorten et al., 2003).

There has been an increasing research interest in stability analysis and control design for switched systems during the past two decades. Therefore, it is well known that stability of switched systems depends not only on the dynamics of every subsystem but also on the property of the switching signal. Thus, it is commonly recognized that there are mainly located in three basic types of problems considering the stability and stabilisation issues of switched systems (Lin & Antsaklis, 2009; Yang, Xiang & Lee, 2012; Hu et al., 2013): (i) asymptotical stability of the switched system with arbitrary switching, (ii) stability for certain useful classes of switching sequences, and (iii) construction of asymptotically stabilizing switching signals for a switched system. Specifically, stability analysis under arbitrary switching problem (i) which will be focused in this article is fundamental in the analysis and design of switched systems (Liberzon & Morse, 1999; Sun & Ge, 2011; Shorten et al., 2003; Zhang & Yu, 2008; Kim et al., 1999; Shorten et al., 2006; Liberzon & Tempo, 2004, Shorten et al., 2007). This problem deals with the case that all the subsystems are stable. Indeed, they exist many examples where all subsystems are stable but inappropriate switching rules can make the whole system unstable. In addition, if we know that a switched system is stable under arbitrary switching, then we can consider higher control specifications for the system. In this framework, it is well known that the existence of a common Lyapunov function for all subsystems is a sufficient condition for such systems to be asymptotically stable under arbitrary switching (Shorten et al., 2003; Shorten & Narendra, 2003; Narendra & Mason, 2003; Sun et al., 2006; Liberzon & Tempo, 2004). However, this method is usually very difficult to apply even for continuous-time switched linear systems (Liberzon, 2003; Shorten et al., 2007).

Frequently, to avoid the conservatism related to the existence of a common Lyapunov function, some attention has been widely interested for seeking conditions that guarantee stability of the switched systems under restricted switching problems (ii) and (iii). Although many efficient approaches and important results have been proposed for this alternative such as the multiple Lyapunov function approach (Branicky, 1998) and average dwell time method (Ishii, 2002; Hespanha, 1999), stability under arbitrary switching which is considered in this work remains most preferable for practical systems. Indeed, it offers great flexibility and it allows us to achieve other performances for designing a control law along stability maintained.

On the other hand, time-delay phenomenon is very common in various practical systems. The presence of delay may degrade systems performance and lead to instability. Indeed, many engineering systems always involve time delay phenomenon. Therefore, switched time-delay systems have various applications in practical engineering systems for instance, power systems (Meyer et al., 2004), power electronics (Sun & Ge, 2002) and networked control systems (Zhang & Yu, 2008). Hence, considerable discussions on delays and their impact on performance as well as on the analysis and design problems in control systems have attracted the interest of numerous investigators in recent years. It is generally advised to incorporate the delays to avoid design flaws and incorrect analysis conclusions. The available results can be categorized into two broad categories: delay-independent and delay-dependent. In the former category, the solutions of stability and stabilization problems are feasible irrespective of the size of the delay. In the latter category, methods are developed to take information about time delays into consideration in the process of controller design. Generally, delay-dependent methods are regarded as more practical and yield less conservative designs.