Chapter 81

Uncertainty Modeling Using Expert’s Knowledge as Evidence

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ABSTRACT

In this paper we discuss the uncertainty modeling using evidence theory. In practice, very often availability of data is incomplete in the sense that sufficient amount of data which is required may not be possible to collect. Therefore, uncertainty modeling in that case with this incomplete data set is not possible to carry out using probability theory or Monte Carlo method. Fuzzy set theory or any other imprecision based theory is applicable in this case. With a view to this expert’s knowledge is represented as the input data set. Belief and plausibility are the two bounds (lower and upper) of the uncertainty of this imprecision based system. The fundamental definitions and the mathematical structures of the belief and plausibility fuzzy measures are discussed in this chapter. Uncertainty modeling using this technique is illustrated with a simple example of contaminant transport through groundwater.

INTRODUCTION

The theory of evidence, also called Dempster-Shafer (D-S) theory or belief functions theory, has been introduced by Shafer (1976) as a new approach for representing uncertainty. Nowadays, this formalism is considered as one of the most interesting alternatives to Bayesian networks and fuzzy sets. Evidence theory is often misunderstood because its theoretical foundations can be quite confusing. This is due to the great number of models and justifications that can be found in the literature. The first part of this chapter aims at clarifying and making an overview of the theoretical background of evidence theory. First, some basic concepts will be addressed and that will be necessary for the rest of the chapter. This begins with the notions of ignorance, uncertainty and imprecision, which, even if they are rather intuitive, have a precise meaning. Then the standard aspects of reasoning process are presented, since all the models presented in this report share the same basis. D-S theory was initiated because of the limitations of the classical probability models. In the second section of this chapter, we retrace the successive steps of its history to discover the
foundations of evidence theory. This is quite essential since almost all the D-S derived models share the same background ideas.

The D-S theory is an especially interesting methodology because of its applicability in areas where information (evidence) must be combined and can be considered as generalization of classical probability theory and also as a generalization of possibility theory. Possibility theory (Dubois et al., 2000; Klir & Wierman, 1998) first introduced by Zadeh is usually chosen to handle epistemic uncertainty (Datta, 2009). Possibility theory uses fuzzy measures to describe the possibility or membership grade by which a certain event can be plausible or believable (Dubois et al., 2000). Contrary to the classical probability theory, possibility theory is usually used to quantify only epistemic uncertainty. Besides possibility theory, interval analysis can be applied when the information is available in the form of an interval (lower bound, upper bound).

The D-S theory is more general than probability and possibility theories (Ayyub & Klir, 2006). It uses plausibility and belief (Dubois et al., 2000) to measure the likelihood of event, without making additional assumptions. When the belief and plausibility measures are equal, the general evidence theory reduces to the classical probability theory. Therefore, the classical probability theory is special case of evidence theory. Moreover, evidence theory can combine empirical evidence from different experts to construct coherent picture of reality.

**BACKGROUND**

It is always required to have some concept behind the evolution of evidence theory. As pointed out in the introduction section that belief and plausibility are the two extreme bounds of uncertainty in the field of evidence theory based uncertainty modeling, one always requires to know why such kind of bounds exist? In order to proceed further in detail, we feel to introduce some fundamental concepts which are background behind this theory. Literature survey (Dubois et al., 2000; Klir & Wierman, 1998; Shafer, 1976; Smets, 1991) provides the atoms pertaining to the notion of evidence theory and some glimpses are presented here in this chapter for the sake of completeness of the reasons to use the evidence theory for uncertainty modeling.

**REASONING WITH IGNORANCE**

Reasoning under uncertainty is a quite vague notion. What does mean reasoning? What is uncertainty? After a short introduction on classical reasoning methods, this chapter focuses on the notion of ignorance and introduces the common components of the reasoning models presented in this chapter.

**Principles of Reasoning Methods**

A reasoning process can be divided into three main parts: a static one where knowledge is stored, a dynamic one where knowledge is entertained, and a decision one where a final hypothesis or action is chosen.

**Static Part**

It is necessary to have a way to store already available information, in order to use them later to make deductions. For classical logical models, this can be the set of induction rules and already proved formulas. In probability formalism, this will be the set of already known probabilities or conditional probabilities, etc.

**Dynamic Part**

If a system can only store knowledge, its role is reduced to a big dictionary. To be useful, a system has to provide a way to analyze and use new data