Chapter 7
Elasto–Plastic Frames

ABSTRACT

The goal of this chapter is to describe how the concepts presented in Chapters 5 and 6 can be included in the mathematical models for the elastic plastic analysis of frame structures. The numerical implementation of such an analysis is described in Chapter 8. The models presented in this chapter cover applications for reinforced concrete frames, shear walls, wide beams, and dual systems, as well as steel structures. Both cases, planar and tridimensional analyses, are considered. However, this chapter does not yet describe the numerical and computational analysis of elasto-plastic structures; this is the subject of the next chapter.

7.1 ELASTO-PLASTIC CONSTITUTIVE MODEL FOR A SLENDER ELEMENT OF A PLANAR FRAME

Buildings in seismic zones must be designed so that there is no structural damage at all under service loads or earthquakes of small intensity; the latter are defined as those events that may occur frequently during the life time of the structure. In mathematical terms, no structural damage means that the structures must behave elastically under such conditions. In the case of earthquakes of intermediate intensity, i.e. those that are expected to occur at most once or twice during the life time of the structure, it is allowed some structural damage provided that it is repairable. In mathematical terms, repairable structural damage means a limited amount of plastic deformations or, the formation of a small number of plastic hinges in the structure. Taking into account that an earthquake of intermediate intensity is considered an event of high probability during the expected life time of the structure, a plastic analysis of such kind of structures should be mandatory.

Earthquake loading is the main cause of structural damage in the practice; however, frame structures may also enter into the plastic range in the case of other kind of overloads. For instance, those due to displacements of its supports (because of the failure of the soil underneath) or impacts or explosions.

DOI: 10.4018/978-1-4666-6379-4.ch007
7.1.1 Lumped Plasticity Model

As mentioned in Chapter 4, distributed forces on the elements are used to characterize service loads; they are assumed to be small compared with exceptional overloads such as earthquake forces, settlement of foundations, explosions or others; these exceptional overloads can be represented as nodal forces or displacements. As aforementioned, a structure only under service loads does not need inelastic analysis; it is therefore assumed that nodal forces are the main cause of plastic deformations in the elements. Typically, the moment distribution in these cases (small distributed forces and relatively large nodal forces) presents maximums at the ends of the elements such as indicated in Figure 1.

The simplest way to consider the presence of plasticity in a frame structure under these conditions is the lumped plasticity model that consists in assuming that a slender frame element can be represented as the assemblage of an elastic beam-column and two plastic hinges at the ends $i$ and $j$ such as shown in Figure 2.

In the lumped plasticity model, the deformations of a frame member can be decomposed into two terms:

$$\{\Phi\}_b = \{\Phi^e\}_b + \{\Phi^p\}_b$$  \hspace{1cm} (7.1.1)

where $\{\Phi^e\}_b$ is the matrix of generalized deformations of the elastic beam-column and $\{\Phi^p\}_b$ is called matrix of plastic deformations; the latter contains the rotations and elongations of the plastic hinges:

$$\{\Phi^p\}_b = \begin{bmatrix} \phi_i^p \\ \phi_j^p \\ \delta^p \end{bmatrix}$$  \hspace{1cm} (7.1.2)

**Figure 1.** Typical distribution of moments in a frame with large nodal forces and small distributed forces on the elements

**Figure 2.** Lumped plasticity model of a slender frame member