Chance Single Sampling Plan for Variables

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ABSTRACT

This paper considers the design of single sampling plan for variables when the experimental values are treated as observations on independently and identically distributed normal random variables with fuzzy mean value. The design makes use of Liu's (2008) model IV of Chance Theory. Sampling Plans determined by the sample sizes and acceptance numbers are constructed for situations involving imprecise parameter on using chance theory. The process of determining sample sizes and acceptance threshold values has been carried out on assuming the observed values have hybrid normal distribution. Optimal chance sampling plans for variables are also determined by using minimum angle criterion for different choices of fuzzy risks.

Keywords: Chance Distribution, Credibility Distribution, Hybrid Normal, Minimum Angle Criterion, OC Curve, Optimal Sampling Plan, Single Sampling Plan for Variables

1. INTRODUCTION

Statistical Quality Control aims at drawing reliable inferences about the quality of a manufactured product by making use of appropriate statistical tools. Statistical Process Control and Acceptance Sampling are two branches of statistical quality control differentiated based on the time of inspection. The main objective of acceptance sampling is to examine whether the manufactured product meets the desired quality level. The quality standards can be studied in two different ways. If the manufactured product is classified into one of well-defined categories (like ‘defective’ and ‘non defective’) then the sampling is called sampling for attributes. On the other hand, if quality requirements are assessed through measurements (like radius, length etc) then the sampling is referred to as sampling for variables. Sampling plans are nothing but the tools used in performing acceptance sampling. Researchers have made wide variety of contributions in designing acceptance sampling plans. For detailed description on various types of sampling plans, one can refer to Schilling (1982). These sampling plans are meant for situations free from any kind of vagueness in the environment being...
studied except randomness induced in the sampling process. Designing of these sampling plans takes into consideration the values of quantities like Consumer’s risk, Producer’s risk and so on. Evaluation of the characteristics of the designed sampling plans is possible only if certain parametric values like, proportion of non-confirming units in the lot are fully known. In real life situations, it is extremely difficult to get good estimates for these quantities. The introduction of Fuzzy set theory paved way for seeking alternate solutions. Tools of Fuzzy set theory become relevant for a given situation involving impreciseness. Designing of Fuzzy sampling plans have been considered by several researchers including Arnold(1996), Ohta and Ichihashi(1988), Kanagawa and Ohta(1990), Tamaki, Kanagawa and Ohta(1991), Grzegorzewski (1998,2001,2002), Hryniewicz(2008), Sampath(2009), Jamkhaneh, Sadeghpour and Yari(2011), Tong and Wang (2012), Sampath and Deepa(2013) etc. It is to be mentioned that majority of these works are related to sampling for attributes and they assume the presence of fuzziness in the parameters related to the underlying distributions. While some of these works considered fuzziness in producer’s risk and consumer’s risk, others considered fuzziness in the submitted lot quality level. To study the characteristics under imprecise situation Liu and Liu(2002) and Liu(2004) introduced a theory called Credibility theory parallel to fuzzy set theory. Sampath(2009), Jamkhaneh, Sadeghpour and Yari(2011), Tong and Wang (2012), Sampath and Deepa(2013) etc. have applied Chance Theory in the theory of acceptance sampling for designing fuzzy acceptance sampling plan for attributes. In this paper, it is proposed to design the acceptance sampling plan for variables using the Chance Theory. The section wise organization of the paper is as follows. Section 2 gives a brief description on Chance theory and also introduces Hybrid normal distribution which is to be used in the later sections. Section 3 introduces Chance single sampling plan for variables using the hybrid normal distribution and discusses about the determination of sampling plan using a conventional method. Section 4 considers the fuzzy analogue solution of the approach pursued by Soundararajan and Christina (1997) for the determination of optimal chance single sampling plan for variables. The Concluding section gives direction for further scope. Appendix contains a table which is similar to the standard normal tables available in Probability theory. The table can be used for computing chance values of desired sets under what is known as a $\varepsilon$ standard hybrid normal distribution described in Section 2.

2. CHANCE THEORY

The introduction of Chance theory requires an understanding of the Credibility theory that provides the foundation for the introduction of fuzzy variables and Probability theory.

2.1. Credibility Theory

Let $\Theta$ be a nonempty set and $P$ be the power set of $\Theta$. Each element of $P$ is called an event. For every event $A$, we associate a number denoted by $Cr\{A\}$, which indicates the credibility that $A$ will occur and that satisfying the following four axioms.

- **Axiom 1 (Normality):** $Cr(\Theta) = 1$
- **Axiom 2 (Monotonicity):** $Cr(A) \leq Cr(B)$ whenever $A \subset B$
- **Axiom 3 (Self duality):** $Cr(A) + Cr(A^c) = 1$ for any event $A$
- **Axiom 4 (Maximality):** $Cr\left(\bigcup_i A_i\right) = \sup_i Cr(A_i)$ for any events $\{A_i\}$ with $\sup_i Cr(A_i) < 0.5$

- **Credibility Measure:** The set function $Cr$ is called a credibility measure if it satisfies the normality, monotonicity, self-duality and maximality axioms.
- **Credibility Space:** Let $\Theta$ be a nonempty set, $P$ be the power set of $\Theta$ and $Cr$ a