Bounded and Semi Bounded Inverse Theorems in Fuzzy Normed Spaces

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ABSTRACT

In this paper, the author introduces the notion of the complete fuzzy norm on a linear space. And the author considers some relations between the fuzzy completeness and ordinary completeness on a linear space, moreover a new form of fuzzy compact spaces, namely b-compact spaces, b-closed space is introduced. Some characterization of their properties is obtained. Also some basic properties for linear operators between fuzzy normed spaces are further studied. The notions of fuzzy vector spaces and fuzzy topological vector spaces were introduced in Katsaras and Liu (1977). These ideas were modified by Katsaras (1981), and in (1984) Katsaras defined the fuzzy norm on a vector space. In (1991) Krishna and Sarma discussed the generation of a fuzzy vector topology from an ordinary vector topology on vector spaces. Also Krishna and Sarma (1992) observed the convergence of sequence of fuzzy points. Rhie et al. (1997) Introduced the notion of fuzzy α-Cauchy sequence of fuzzy points and fuzzy completeness.

Keywords: B-Closed Spaces, B-Compact Spaces, Bounded Inverse Theorems, Bounded Operator, Fuzzy Normed Linear Space, Fuzzy Normed Space, Fuzzy Topology Spaces, Semi Bounded Inverse Theorem

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1. INTRODUCTION AND PRELIMINARIES

The notions of fuzzy vector spaces and fuzzy topological vector spaces were introduced in Katsaras and Liu (1977). These ideas were modified by Katsaras (1981), and in (1984) Katsaras defined the fuzzy norm on a vector space. In (1991) Krishna and Sarma discussed the generation of a fuzzy vector topology from an ordinary vector topology on vector spaces. Also Krishna and Sarma (1992) observed the convergence of sequence of fuzzy points. Rhie et al. (1997) Introduced the notion of fuzzy Cauchy sequence of fuzzy points and fuzzy completeness.

Throughout this paper $X$ and $Y$ means fuzzy topological spaces (fts). The notions $\text{Cl}(A)$ will stand for the fuzzy closure of a fuzzy set $A$ in a fts $X$. Support of a fuzzy set $A$ in $X$ will be denoted by $S(A)$. The fuzzy sets in $X$ taking on respectively the constant value 0 and 1 are denoted by $0_x$ and $1_x$ respectively.

In this paper, we first observe a type of the convergence of sequences as an analogy of Bag and Samanta (2003) in a fuzzy normed linear space. Secondly, we introduce the notion of a complete fuzzy norm, using the convergence of a sequence of a linear space. And we consider some relations between the fuzzy completeness and the ordinary completeness on a linear space.

**Definition 1.1:** [4] For two fuzzy subsets $\mu_1$ and $\mu_2$ of $X$, the fuzzy subset $\mu_1 + \mu_2$ is defined by

$$
(\mu_1 + \mu_2)(x) = \bigvee \{\mu_1(x_1) \land \mu_2(x_2) \mid x = x_1 + x_2\}.
$$

And for a scalar $t$ of $K$ and a fuzzy subset $\frac{1}{t}$ of $X$, the fuzzy subset $t\frac{1}{t}$ is defined by

$$
(t\mu)(x) = \begin{cases}
\mu \left(\frac{x}{t}\right) & \text{if } t \neq 0 \\
0 & \text{if } t = 0 \text{ and } x \neq 0, \\
\bigvee \{\mu(y) \mid y \in X\} & \text{if } t = 0 \text{ and } x = 0.
\end{cases}
$$

**Definition 1.2:** [2] $\mu \in I^X$ is said to be

1. Convex if $t \mu + (1 - t) \mu \subseteq \mu$ for each $t \in [0,1]$
2. Balanced if $t \mu \subseteq \mu$ for each $t \in K$ with $|t| \leq 1$
3. Absorbing if $\bigvee \{t \mu(x) \mid t > 0\} = 1$ for all $x \in X$.

**Definition 1.3:** [2] Let $(X, \tau)$ be a topological space and

$$
\omega(\tau) = \{f : (X, \tau) \to [0,1] \mid f \text{ is lower semicontinuous}\}
$$

then $\omega(\tau)$ is a fuzzy topology on $X$. This topology is called the fuzzy topology generated by $\tau$ on $X$. The fuzzy usual topology on $K$ means the fuzzy topology generated by the usual topology of $K$.

$$
n \geq M \text{ implies } \frac{t}{2} \rho(x_n - x) > 1 - \varepsilon \\
\Rightarrow n \geq M \text{ implies } P_{1-\varepsilon}(x_n - x) \leq \frac{t}{2} < t.
$$
Sign Language Recognition for Bengali Characters
www.igi-global.com/article/sign-language-recognition-for-bengali-characters/133123?camid=4v1a