ABSTRACT

The bimatrix game theory is concerned with how two players make decisions when they are faced with known exact payoffs. The aim of this paper is to develop a simple and an effective bilinear programming method for solving bimatrix games in which the payoffs are represented by intervals. Because the payoffs of the bimatrix game are intervals, the values of the bimatrix game for both players are intervals as well from the viewpoint of logic. Based on the definition of the values of the bimatrix game for players, the game values may be regarded as functions of payoffs belonging to the payoff intervals, which are proven to be monotonic non-decreasing. A pair of auxiliary bilinear programming models is formulated to obtain the upper bound and the lower bound of the interval-type values of the interval-valued bimatrix game by using the upper bounds and the lower bounds of the payoff intervals, respectively. A real example of the engineering project management problem is used to illustrate the applicability and effectiveness of the proposed models and method.

INTRODUCTION

Bimatrix games are a type of two-person nonzero-sum and non-cooperative games in which both players have finite pure strategies. A bimatrix game is usually expressed with a pair of payoff matrices whose elements are represented by real numbers, which indicate that the payoffs are precisely known a priori. This assumption is reasonable for clearly defined games, which have many successful applications such as management and finance (Hladík, 2010; Li, 2011a, 2015; Owen, 1982). In reality, however, there are some cases in which the payoffs are not crisp values known and have to be estimated even though two players do not change their strategies due to lack of adequate information and/or imprecise information (Nan & Li, 2013). In general, imprecision and uncertainty may result in different types of games such as stochastic games, fuzzy games and fuzzy stochastic games as well as interval-valued games. This paper focuses on intervals. Interval-valued matrix games have been extensively studied. Interval computing has been a well established field (Moore, 1979) and successfully applied to some areas (Li et al, 2012; Sengupta & Pal, 2000; Sun et al, 2010). Recently, Nayak and Pal (2009) constructed a pair of interval linear programming models for the interval-valued
matrix game. Li (2011b) derived and suggested bi-objective linear programming models, which was solved by the lexicographic method. Based on the defined interval inequality relations and the fuzzy ranking index, Li et al. (2012) derived a pair of bi-objective linear programming models from the constructed auxiliary interval programming models for the interval-valued matrix game. Li (2011a) formulated a pair of auxiliary linear programming models for obtaining the upper bound and the lower bound of the interval-type value of the interval-valued matrix game by using the upper bounds and the lower bounds of the payoff intervals, respectively. The above studies focus on the interval-valued matrix game while this paper mainly concerns interval-valued bimatrix games.

Hladěk (2010) investigated support set invariances for interval-valued bimatrix games when perturbing the payoffs in the intervals. Sufficient and necessary conditions were proposed for three types of support set invariances by means of a series of systems of linear equalities and inequalities. The work (Hladík, 2010) essentially dealt with all instances of the interval-valued bimatrix game and hereby had a large amount of computation. In additions, the work (Hladík, 2010) cannot obtain the interval-type values of the interval-valued bimatrix game for both players. However, the values of the interval-valued bimatrix game for players should be intervals from the viewpoint of logic. To the best of our knowledge, there is no effective method for computing the interval-type values of the interval-valued bimatrix games for both players. The aim of this paper is to develop a simple and an effective bilinear programming method for determining the interval-type values of the interval-valued bimatrix games. The rest of this paper is organized as follows. Firstly, the definition and notations are briefly reviewed for the classical bimatrix games. Secondly, the interval-valued bimatrix game is formulated and the monotonicity of its values is discussed. To obtain the lower and upper bounds of the interval-type values of the interval-valued bimatrix game for both players, a pair of auxiliary bilinear programming models is derived from the definition of the values of the bimatrix game and the monotonicity. Thirdly, the proposed method is illustrated with a real example of the engineering project management problem. Conclusion is made in the final of this paper.

BIMATRIX GAMES AND BILINEAR PROGRAMMING MODELS

Assume that $S_1 = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ and $S_2 = \{\beta_1, \beta_2, \ldots, \beta_n\}$ are sets of pure strategies for two players I and II, respectively. The payoff matrices for players I and II are represented by $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$, respectively. The vectors $y = (y_1, y_2, \ldots, y_m)^T$ and $z = (z_1, z_2, \ldots, z_n)^T$ are mixed strategies for players I and II, respectively, where $y_i$ ($i = 1, 2, \ldots, m$) and $z_j$ ($j = 1, 2, \ldots, n$) are probabilities in which players I and II choose their pure strategies $\alpha_i \in S_i$ and $\beta_j \in S_2$, respectively; the symbol “$T$” is the transpose of a vector/matrix. Sets of all mixed strategies for players I and II are denoted by $Y$ and $Z$, respectively, i.e,

$Y = \{y \mid \sum_{i=1}^{m} y_i = 1, y_i \geq 0 (i = 1, 2, \ldots, m)\}$

and

$Z = \{z \mid \sum_{j=1}^{n} z_j = 1, z_j \geq 0 (j = 1, 2, \ldots, n)\}$ .

Thus, a two-person nonzero-sum finite game
Related Content

A Computational Model for Texture Analysis in Images with Fractional Differential Filter for Texture Detection

[www.igi-global.com/chapter/fuzzy-logic-based-approach-supporting/24349?camid=4v1a](www.igi-global.com/chapter/fuzzy-logic-based-approach-supporting/24349?camid=4v1a)
Rough Fuzzy Set Theory and Neighbourhood Approximation Based Modelling for Spatial Epidemiology
Balakrushna Tripathy and Sharmila Banu K. (2016). Handbook of Research on Computational Intelligence Applications in Bioinformatics (pp. 108-118).
www.igi-global.com/chapter/rough-fuzzy-set-theory-and-neighbourhood-approximation-based-modelling-for-spatial-epidemiology/157484?camid=4v1a

Combining Artificial Neural Networks and GOR-V Information Theory to Predict Protein Secondary Structure from Amino Acid Sequences
www.igi-global.com/article/combining-artificial-neural-networks-gor/2393?camid=4v1a