Chapter 11
Stochastic Processes for the Risk Management

Gamze Özel
Hacettepe University, Turkey

ABSTRACT

The financial markets use stochastic models to represent the seemingly random behavior of assets such as stocks, commodities, relative currency prices such as the price of one currency compared to that of another, such as the price of US Dollar compared to that of the Euro, and interest rates. These models are then used by quantitative analysts to value options on stock prices, bond prices, and on interest rates. This chapter gives an overview of the stochastic models and methods used in financial risk management. Given the random nature of future events on financial markets, the field of stochastic processes obviously plays an important role in quantitative risk management. Random walk, Brownian motion and geometric Brownian motion processes in risk management are explained. Simulations of these processes are provided with some software codes.

INTRODUCTION

Risk is an inherent ingredient of financial business in the sense of the possibility of losses. Financial business is becoming more and more complicated, resulting in greater risks due to the development of financial services and financial products. In addition, the financial industry is getting more involved in the economy, both nationally and globally. The financial market has an influence on economy to a certain extent, e.g. the gloom of the financial market could lead to an economic recession.

The measurement of risk has become a competitive factor for the financial institutions. How can we quantify risk? Risk management deals with this problem. Risk management can be defined as a discipline for “Living with the possibility that future events may cause adverse effects” (Kloman, 1999). In the context of risk management in financial institutions such as banks or insurance companies these adverse effects usually correspond to large losses on a portfolio of assets. Risk management can be seen as a core competence of an insurance company or a bank. Specific examples include losses on a portfolio.
of market-traded securities such as stocks and bonds due to falling market prices; losses on a pool of bonds or loans, caused by the default of some issuers or borrowers (credit risk); losses on a portfolio of insurance contracts due to the occurrence of large claims (Eberlein et al., 2007). A financial institution can take on risks and manage them by various techniques such as diversification, hedging, or repackaging risks and transferring them back to markets etc.

It is important to understand the character of stochastic process that underlines the dynamic of financial variables for successful modeling of the financial series. To give some insights into the financial market, we present finance as a stochastic process, where psychology of people is the most important element. To define the stochastic process first, we would say that a stochastic process is a variable that evolves over time in a way that is at least partly random. Weather is a stochastic process. Namely, variation in temperatures is partly deterministic and predictable, we can expect to have warm summers and cold winters and that temperatures rise during the day and fall at night, and partly random and unpredictable.

However, if we can predict what the temperature of the January 1 will be over the next two years or even what temperature we will have tomorrow, it is impossible to predict the price of a stock of Google over the next two years. Why is that? This is because weather is a (weakly) stationary process, which means that statistical properties of the variable are constant over time, while asset prices are non-stationary, meaning that the expected value can grow, or fall, without bounds. Precisely this stochastic nature of asset prices is what makes opportunities to individuals for increasing their wealth, but also threats of losing it. Therefore, individuals are taking risks.

In this chapter an introduction to some of the stochastic aspects of financial risk management is given. Firstly, random walk process is explained for the risk management. Random walk process is used in many fields including finance and economics for modeling fluctuations in the stock market. It says that any shock to stock price is permanent and there is no tendency for the price level to return to a trend path over time. Secondly, Brownian motion process is introduced. In the modeling of financial market, especially stock market, Brownian motion plays a significant role in building a statistical model. Finally, we focus on the geometric Brownian motion process. Geometric Brownian motion is a employed to analyze risk for stocks and other financial assets. It is also the model that underpins the famous Black-Scholes option pricing formula.

**BACKGROUND**

Definitions of risk vary based on context. The International Organization for Standardization (ISO) in the ISO Guide 73 defines risk as the “effect of uncertainty on objectives” (ISO, 2009). The reason for a risk is uncertainty. Uncertainty is “the state of being uncertain.” Uncertain means “not able to be relied on; not known or definite.” Imagine that you would like to purchase a property in San Francisco, which is known for its susceptibility to earthquakes. We know that there is a risk of an earthquake occurring at any time, but we cannot say if there will be an earthquake during the next three years or not. It is uncertain.

Risk can be negative or positive. Earthquakes and diseases are classic examples of negative risk. In contrast, when somebody plays roulette in a casino, he or she hopes for a positive risk. In a way, this is counterintuitive to the manner in which the word risk is used in our everyday language. We often talk about risk as something inherently bad, for example, health risks (e.g., smoking increases the risk of cancer), natural disasters like earthquakes and floods, and sudden economic downturns. But, in fact, the basis of our market economy is actually taking risks. A merger or takeover of a company is often pursued because the acquiring party expects that it is worth