An OR Practitioner’s Solution Approach for the Set Covering Problem

Yun Lu, Department of Mathematics, Kutztown University, Kutztown, PA, USA
Francis J. Vasko, Department of Mathematics, Kutztown University, Kutztown, PA, USA

ABSTRACT

The set covering problem (SCP) is an NP-complete problem that has many important industrial applications. Since industrial applications are typically large in scale, exact solution algorithms are not feasible for operations research (OR) practitioners to use when called on to solve real-world problems involving SCPs. However, the best performing heuristics for the SCP reported in the literature are not usually straightforward to implement. Additionally, these heuristics usually require the fine-tuning of several parameters. In contrast, simple greedy or even randomized greedy heuristics typically do not give as good results as the more sophisticated heuristics. In this paper, the authors present a compromise; a straightforward to implement, population-based solution approach for the SCP. It uses a randomized greedy approach to generate an initial population and then uses a genetic-based two phase approach to improve the population solutions. This two-phase approach uses transformation equations based on a Teaching-Learning based optimization approach developed by Rao, Savsani and Vakharia (2011, 2012) for continuous nonlinear optimization problems. Empirical results using set covering problems from Beasley’s OR-library demonstrate the competitiveness of this approach both in terms of solution quality and execution time. The advantage to this approach is its relative simplicity for the practitioner to implement.

Keywords: Operations Research Practice, Population-Based Solution Approaches, Set Covering Problem, Teaching-Learning-based Optimization

INTRODUCTION AND BACKGROUND

As a young mathematician working in the research department of a major US steel company, one of the authors was charged with designing optimal ingot sizes to be used to produce structural steel products. He formulated the problem as a pre-emptive goal programming problem with the first goal involving the solution of a large-scale minimum cardinality set covering problem. Specifically, taking into account numerous steelmaking and rolling mill constraints, he was able to generate a large number of feasible candidate ingot sizes. Based on steel deformation principles, these ingot sizes were mapped into finished structural steel products. In other words, he identified which structural steel products could be produced from each candidate ingot size.

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Each ingot size was associated with these structural steel products. The set covering part of the problem was to cover all the structural steel products with a minimum number of ingot sizes. After consulting the literature and communicating with some top researchers on the SCP, he decided to develop his own heuristic approach to solve this important steel industry application. For more information on optimal ingot design, see Vasko et al. (1987).

Unfortunately, many operations research (OR) practitioners do not have the time to develop their own algorithms because management usually wants answers quickly and algorithm development and implementation requires time. Since the SCP is essentially a consolidation problem with many industrial applications (both inside and outside the steel industry), practitioners should have a solution approach that is straightforward to code and implement, requires few (or no) parameters to fine-tune, consistently give optimal or near-optimal solutions and requires reasonable (or little) computer time. In this paper, we demonstrate such a solution approach.

In the next section, we will formally define the SCP and then discuss our population-based solution approach. This will be followed by empirical results using problems from Beasley’s OR-library. A summary and observations will conclude the paper.

THE SET COVERING PROBLEM AND PROPOSED SOLUTION APPROACH

In this section, we will introduce the set covering problem which is a discrete NP-complete (Karp, 1972) combinatorial optimization problem that has many industrial applications. Two important applications from the steel industry are ingot mold selection (Vasko, Wolf and Stott, 1987) and metallurgical grade assignment (Vasko, Wolf and Stott, 1989).

The set covering problem (SCP) is the problem of covering the rows of an m-row, n-column, zero-one matrix \( (a_{ij}) \) by a subset of the columns at minimum cost. A mathematical formulation for the SCP is:

Minimize

\[
\sum_{j=1}^{n} c_j x_j
\]  

subject to

\[
\sum_{j=1}^{n} a_{ij} x_j \geq 1, \quad i = 1, \ldots, m
\]

\[
x_j \in \{0, 1\}, \quad j = 1, \ldots, n
\]

where \( x_j \) is one if column \( j \) is in the solution and zero otherwise. Constraint set (2) ensures that each row is covered by at least one column and constraint set (3) ensures that the \( x_j \)’s take on only the values zero or one.

Since the SCP is NP-complete and industrial applications typically involve large problem instances, many heuristic solution procedures have been developed over the years. For a discussion of recent heuristic and metaheuristic approaches for solving the SCP, we suggest you consult Ren, Feng, Ke and Zhang (2010).

Our equations used to transform solutions in the population are based on a Teaching-Learning Based Optimization (TLBO) methodology developed by Rao, Savsani and Vakhria (2011, 2012) for solving continuous nonlinear optimization problems and has been successfully applied to several mechanical design problems (See Togan (2012) and Rao and Patel (2013)). The TLBO is a population-based procedure that requires no parameter fine-tuning other than the population.
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