Towards Ultrametric Modeling of Unconscious Creativity

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ABSTRACT

Information processed by complex cognitive systems is characterized by the presence of various closely connected hierarchic structures. The most natural geometry for the representation of such structures is geometry of trees and the corresponding topology is the ultrametric topology of on these trees. And the p-adic trees provide the simplest model for representation of mental hierarchies. Moreover, p-adic trees can be endowed with the natural arithmetic reminding the usual arithmetic of real numbers. Therefore it is natural to start from the p-adic models of brain’s functioning. In this note the authors apply this model to demonstrate the ability of the “p-adic brain” to process adequately the objects of the physical Euclidean space in the p-adic tree representation. This study also leads to p-adic modeling of brain’s creativity and its ability to create abstract images. The authors’ model is about unconscious processing of information by the brain. Therefore the authors can say about elements of coming theory of unconscious creativity.

Keywords: Brain, Euclidean Plane, P-adic Brain, P-adic Trees, Theory of Unconscious Creativity, Ultrametric Topology

1. INTRODUCTION

The p-adic\(^1\) and more generally ultrametric approach to mathematical modeling of brain’s functioning was started in the works of one the authors of this paper (Khrennikov, 1997, 1998, 2002, 2004a, 2004b, 2007) and his collaborators (Albeverio et al., 1997; Dubischar, 1999). This approach was initially used merely for modeling of functioning of human subconsciounsness and, in particular, for development of a mathematical model for Freud’s psychoanalysis (Freud, 1900, 1923). Recently the ultrametric approach to the mathematical representation of some essential features of human unconscious mind was also explored in the works of other authors (Lauro-Grotto, 2007; Murtagh, 2012a, 2012b; Contreras & Murtagh, 2012). We also remark that our ultrametric model of information processing by the brain matches well the general program of development of Cognitive Informatics and Cognitive Computing (Wang et al., 2010).

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Applications of p-adics to cognitive science and psychology were stimulated by its applications to theoretical physics: *string and superstring theory*, cosmology, quantum mechanics and quantum field theory, spin glasses (Vladimirov et al., 1994; Khrennikov, 1997). As was mentioned in footnote 1, in many biological applications it is not important to restrict encoding of information to the prime number quantization, \{0,1,...,p-1\}. The use of quantization based on an arbitrary natural number, \{0,1,...,m-1\}, can be useful as well. In general the problem of a proper selection the prime or natural basis of the model has the contextual nature. It depends on the class of cognitive or biological phenomena. In image analysis the choice of the prime \(p\) (or more generally \(m\)) may depend on the class of images and it can be justified in the process of learning by using a database of images. The biological brain can represent a huge variety of trees. It may be able to select a tree having the structure matching a problem.

Information processed by complex cognitive systems and especially by human beings (but already a single cell is a complex information processor) is characterized by the presence of various closely connected hierarchic structures. The most natural geometry for the representation of such structures is geometry of trees and the corresponding topology is the ultrametric topology of on these trees. And the p-adic trees provide the simplest model for representation of mental hierarchies (Khrennikov, 1997; Khrennikov & Nilsson, 2004).

We remind that a p-adic tree \(Q_p\) is *the homogeneous tree* such that there are \(p\) branches leaving each vertex. As was discussed in the footnote 1, the use of the prime \(p\) is not so important in applications to cognition and psychology. However, the p-adic trees with the prime \(p\) are more convenient for mathematical modeling. They can be endowed with the natural arithmetic structure reminding the usual arithmetic structure of real numbers. They are number fields. Therefore, at the first stage, it is natural to proceed with the p-adic models of brain’s functioning. Here one can use the methods of p-adic analysis. Of course, some of these methods can be generalized even to the case of general m-adic trees (denoted as \(Q_m\)), or even of ultrametric spaces (Khrennikov & Nilsson, 2004). (We remark that, for a nonprime \(m\), \(Q_m\) is not a number field, but, nevertheless, it is a ring, i.e., the operations of addition, subtraction, and multiplication are well defined, but not the operation of division.) However, in such general situations they are not as powerful as in the p-adic case. Our main mathematical tool is theory of *p-adic dynamical systems* which serve as the base for simulation of the processing of mental information. Theory of such dynamical systems is well developed only in the p-adic case (Khrennikov & Nilsson, 2004). However, in this paper we shall use only a very simple dynamics, based on the shift map. Here we can proceed with m-adic numbers, where \(m>1\) is an arbitrary natural number.

In this note we apply the p-adic model to demonstrate the ability of the “p-adic brain” to process adequately the objects of the physical Euclidean space by using the p-adic tree representation. This study also leads to p-adic modeling of *brain’s creativity and its ability to create abstract images*. To justify the p-adic (ultrametric) model of processing information by the brain, it is important to describe coupling of this purely informational model, so to say brain’s software, with the physical processing of information by neurons and more generally by neuronal networks. Such a coupling was established in a series of works (Khrennikov, 2004a, 2004b; Khrennikov & Nilsson, 2004) and it is based on consideration of *hierarchic chains of neurons as the basic computational elements*, i.e., a bit of mental information is represented not by a single neuron, but by a chain of hierarchically coupled neurons. Each such a chain represents the information which encoded by a branch of the p-adic tree (or more generally an ultrametric tree). In the p-adic case each branch can be encoded by a number. Thus we propose the special way to *encode mental states by numbers* (but special numbers, the p-adic numbers or more generally the m-adic numbers).
Feature Reduction with Inconsistency
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