Chapter 2
Graspable Mathematics: Using Perceptual Learning Technology to Discover Algebraic Notation

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ABSTRACT
The goal of this chapter is to describe a process of touch screen technology development, beginning from basic cognitive research and resulting in an applied educational intervention for algebra. To that end, we introduce Graspable Mathematics (GM), a novel approach to algebra instruction that integrates strong theory and evidence from perceptual learning, cognitive science and mathematics education. We present a number of concrete examples of how this framework can be applied to technology-based interventions that aim to engage students in mathematics. This chapter will also describe many of the methodological, theoretical, and practical considerations that guided our iterative design and development, as well as address many of the challenges and lessons learned when implementing the intervention in classroom settings.

INTRODUCTION
This chapter begins with a literature review from cognitive science and mathematics education that explores the potential benefits of using perceptual training, guided discovery-learning, and learning technologies in algebra instruction. Building on this literature, we describe the theoretical framework of Graspable Mathematics, an approach that we have developed that applies touch screen interfaces to align the content of algebraic transformations with core cognitive and perceptual systems.

The second part of the chapter focuses on how touch-screen technologies that emphasize the Graspable Mathematics framework can successfully get students to engage with and explore algebraic concepts in novel ways. We describe a number of concrete examples of how this framework can be applied to

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technology-based interventions that aim to engage students in mathematics. We introduce From Here to There, a touch-screen application that embeds this approach and provides users with opportunities to explore algebraic concepts through discovery-based puzzles.

The third part of this chapter describes many of the methodological, theoretical, and practical considerations that guided our iterative design and development, as well as address many of the challenges and lessons learned when implementing the intervention in classroom settings. We hope that the presentation of our process and asking questions will help lead to new advances in the development and use of research-tested technology in mathematics education. Future directions are discussed.

BACKGROUND

Perceptual Learning and Algebra Instruction

Formalisms play a crucial role in algebraic reasoning. Students are expected to develop a fluent procedural understanding of the structure, arrangement, and legal transformations of basic algebraic expressions. The U.S. Common Core Standards for Mathematical Practice (CCSS-M 2010) state that students should “look for and make use of structure” and “look for and express regularity in repeated reasoning” (CCSS-M, 2010, 5). This includes noticing mathematical patterns, looking for shortcuts, and recognizing that algebraic expressions can be seen as “single objects or as being composed of several objects”. Explicitly teaching algebraic structure can contribute to improved student understanding of algebraic expressions. However, the current state of algebra instruction leaves much to be desired: few students can successfully read, write and interpret the syntax of basic symbolic expressions (NMAP, 2008), or master the notation universally used to express even relatively straightforward algebraic claims (Koedinger, Alibali, & Nathan, 2008; Clement, Lochhead, & Monk, 1981; Christianson, Mestre, & Luke, 2013; Landy, Brookes, & Smout, 2014).

The U.S. Common Core standards also require students to identify and generate equivalent expressions through transformation (CCSS-M, 2010). Transforming algebraic expressions requires fluent and robust facility at determining which transformations are appropriate in a given context, and applying those transformations correctly. However, many students have difficulty creating and interpreting standard algebraic notation, and transforming expressions into equivalent forms. Often, algebraic properties and procedures are taught through memorization of abstract and sometimes seemingly arbitrary rules. As a result, when faced with a novel problem, students often over generalize rules along patterns of overall visual similarity (Marquis, 1988), or make frequent procedural errors such as combining unlike terms, ignoring order of operations, and making arithmetic errors (Ottmar & Landy, 2012).

Substantial empirical work has demonstrated that reading and transforming algebraic notation involves not just memorizing rules, but also learning appropriate perceptual processes (Goldstone, Landy, & Son, 2010; Kirshner, 1989; Kirshner & Awtry, 2004; Landy & Goldstone, 2007; Landy & Linkenauger, 2010; Jacob & Hochstein, 2008; Kellman, Massey, & Son, 2009; Maruyama et al., 2012; Landy, Goldstone, & Jones, 2008; Jansen, Marriott, & Yelland, 2003). Recent findings in cognitive science and mathematics education suggest that facility with algebraic notation involves perceptual training, or learning to see expressions and equations as structured objects (Kellman et al., 2008; Landy & Goldstone, 2007). Research by our lab and others suggest that despite the emphasis teachers and textbooks place on rules,