Complex Role Inclusions with Role Chains on the Right are Expressible in SROIQ

Michael Compton, CSIRO, Cygnet, Australia

ABSTRACT

The syntax of both SROIQ and OWL 2 allow complex role inclusion axioms where the role composition occurs on the left of the subsumption relation, but not where the role composition occurs on the right. This paper proves that, despite the lack of syntax, SROIQ, and consequently OWL 2, can already express such complex role inclusions. This is shown by proving that complex role inclusion axioms where the role composition occurs on the right of the subsumption relation can already be expressed in terms of complex role inclusions on the left. The resulting syntactic restrictions are similar to those already present in SROIQ and OWL 2.

Keywords: Axioms, Complex Role Inclusion, OWL 2, SROIQ, Syntax

INTRODUCTION

The Description Logic SROIQ has syntax for role subsumptions of the form:

\[ S_1 \circ \ldots \circ S_n \subseteq R \tag{1} \]

but not of the form:

\[ R \subseteq S_1 \circ \ldots \circ S_n \tag{2} \]

Such axioms, called complex role inclusion axioms, show a subsumption relation between role \( R \) and the role composition, or chain, \( S_1 \circ \ldots \circ S_n \).

Given an interpretation \( \mathcal{I} \), role compositions have the standard meaning:

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\[(R \circ S)^I = \{ (x, z) \mid \exists y, (x, y) \in R^I \land (y, z) \in S^I \}\]

and thus (1) has the semantics:

\[
\forall y_0, \ldots, y_n \left[ \left( y_0, y_1 \right) \in S^I_1 \land \ldots \land \left( y_{n-1}, y_n \right) \in S^I_n \right] \rightarrow \left( y_0, y_n \right) \in R^I
\]

In general, such axioms result in undecidability, but, decidability can be regained by enforcing restrictions on the relationships between roles. \textit{SROIQ}, for example, requires a strict partial ordering of roles. Recently, complex role inclusions with role chains on the left have been studied in \textit{SHIQ}, \textit{RIQ}, \textit{SRIQ} and \textit{SROIQ}.

Excepting a part solution for \textit{RIQ} (Mosurivč & Krđavc, 2011), no description logics that allow role chains on the right of the subsumption relation have been published.

**Contribution**

This paper shows that complex role inclusions on the right, that is, of the form of (2), with semantics:

\[
\forall y_0, y_n \left[ (y_0, y_n) \in R^I \rightarrow \exists y_1, \ldots, y_{n-1} \left[ \left( y_0, y_1 \right) \in S^I_1 \land \ldots \land \left( y_{n-1}, y_n \right) \in S^I_n \right] \right]
\]

are definable in terms of complex role inclusions on the left, and thus, despite not being directly permitted in the syntax, can already be expressed in \textit{SRIQ}, \textit{SROIQ} and, consequently, OWL 2.¹

Both soundness and completeness are proved for the method. Soundness showing that the definitions given imply the desired role compositions, and completeness demonstrating that all models with the intended semantics are admitted: i.e., the definitions do not imply ‘hidden’ restrictions on satisfying models.

A set of syntactic restrictions constrain the valid complex role inclusion axioms in \textit{SROIQ}. The definitions given here result in syntactic restrictions for complex role inclusions on the right; these restrictions are a consequence of the construction in terms of role chains on the left and do not add any further restrictions to the logic.

The restrictions are similar to the existing \textit{SROIQ} restrictions:

- Roles on the right of the subsumption in a complex role inclusion are not ‘simple’, \(S_1, \ldots, S_n\) in (2); and
- The global partial ordering on roles requires either \(R \prec S_1\) or \(R \prec S_n\).

**Paper Outline**

First, motivating examples and related work are given. Then, \textit{SROIQ} syntax, semantics and restrictions on role compositions are reviewed.
Refactoring and its Application to Ontologies
www.igi-global.com/chapter/refactoring-its-application-ontologies/52870?camid=4v1a

Property Clustering in Linked Data: An Empirical Study and Its Application to Entity Browsing
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