Chapter 2
The Method of Probabilistic Nodes Combination in Decision Making and Risk Analysis: Data Extrapolation in Planning

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ABSTRACT
Proposed method, called Probabilistic Nodes Combination (PNC), is the method of 2D curve interpolation and extrapolation using the set of key points (knots or nodes). Nodes can be treated as characteristic points of data for modeling and analyzing. The model of data can be built by choice of probability distribution function and nodes combination. PNC modeling via nodes combination and parameter $\gamma$ as probability distribution function enables value anticipation in risk analysis and decision making. Two-dimensional curve is extrapolated and interpolated via nodes combination and different functions as discrete or continuous probability distribution functions: polynomial, sine, cosine, tangent, cotangent, logarithm, exponent, arc sin, arc cos, arc tan, arc cot or power function. Novelty of the paper consists of two generalizations: generalization of previous MHR method with various nodes combinations and generalization of linear interpolation with different (no basic) probability distribution functions and nodes combinations.

INTRODUCTION
Probabilistic modeling is still a developing branch of economic and computer sciences: operational research (for example probabilistic model-based prognosis) (Lorton, Fouladirad & Grall, 2013), decision making techniques and probabilistic modeling (Pergler & Freeman, 2008), artificial intelligence and machine learning. Different aspects of probabilistic methods are used: stochastic processes and stochastic model-based techniques, Markov processes (Cocozza-Thivent, Eymard, Mercier & Roussignol, 2006), Poisson processes, Gamma processes, Monte Carlo methods, Bayes rule, conditional probability and
many probability distributions. In this paper the goal of a probability distribution function is to describe the position of unknown points between given interpolation nodes. Two-dimensional curve is used to represent the data points and extrapolation of the unknown values enables analyzing risk and then decision making.

The chapter clarifies the significance and novelty of the proposed method compared to existing methods (for example polynomial interpolations and Bézier curves). Previous published papers of the author were dealing with the method of Hurwitz-Radon Matrices (MHR method). Novelty of this paper and proposed method consists in the fact that calculations are free from the family of Hurwitz-Radon Matrices. Problem statement of this paper is: how to reconstruct (interpolate) missing points of 2D curve and how to anticipate (extrapolate) unknown values or data having the set of interpolation nodes (key points) and using the information about probabilistic distribution of unknown points. For example the simplest basic (uniform) distribution leads to the easiest interpolation – linear interpolation. Apart from probability distribution, additionally there is the second factor of proposed interpolation method: nodes combination. The simplest nodes combination is zero. Thus proposed curve modeling and extrapolation is based on two agents: probability distribution and nodes combination. First trial of probabilistic modeling in MHR version was described in (Jakóbczak, 2013). Significance of this chapter consists in generalization for MHR method: the computations are done without matrices in curve fitting and data anticipation, with clear point interpolation formula based on probability distribution function (continuous or discrete) and nodes combination. The paper also consists of generalization for linear interpolation with different (non-uniform) probability distribution functions and nodes combinations. So this chapter answers the question: “Why and when should we use Probabilistic Nodes Combination (PNC) method in extrapolation and interpolation?”

Curve interpolation (Collins, 2003) represents one of the most important problems in mathematics and computer science: how to model the curve (Chapra, 2012) via discrete set of two-dimensional points (Ralston & Rabinowitz, 2001)? Also the matter of shape representation (as closed curve - contour) and curve parameterization is still opened (Zhang & Lu, 2004). Operational research in planning and scheduling, also decision making systems in risk analysis, solve the problems which are based on data modeling and extrapolation via the choice of key points. So interpolation and extrapolation is not only a pure mathematical problem but important task in economic and artificial intelligence. The paper wants to approach a problem of curve modeling by characteristic points. Proposed method relies on nodes combination and functional modeling of curve points situated between the basic set of key points and outside of this set. The functions that are used in calculations represent whole family of elementary functions with inverse functions: polynomials, trigonometric, cyclometric, logarithmic, exponential and power function. These functions are treated as probability distribution functions in the range [0; 1].

An important problem in operational research and computer sciences (Ballard, 1982) is that of appropriate shape representation and reconstruction. Classical discussion about shape representation is based on the problem: contour versus skeleton. This paper is voting for contour which forms boundary of the object. Contour of the object, represented by contour points, consists of information which allows us to describe many important features of the object as shape coefficients (Tadeusiewicz & Flasiński, 1991). In the paper contour is dealing with a set of curves. Curve modeling and generation is a basic subject in many branches of industry and computer science, for example in the CAD/CAM software.

The representation of shape has a great impact on the accuracy and effectiveness of object recognition (Saber, Yaowu & Murat, 2005). In the literature, shape has been represented by many options including curves (Sebastian & Klein, 2003), graph-based algorithms and medial axis (Liu & Geiger, 1999).