On the Incremental Union of Relations:  
A Key Property of General Systems Explained

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ABSTRACT

Relations are one of the most important conceptual models and mathematical entities in logic, discrete mathematics, computer science, software science, system science, and formal semantics. However, some fundamental and indispensable operations on formal relations were overlooked in traditional studies. This paper presents an extended relation theory with a set of novel algebraic operators on relations beyond classic operations. The algebraic operators on formal relations known as the incremental union and decremental disunion are formally elaborated. The property of relational gains is mathematically modeled, which explains the dynamic mechanism of relations generated by associations of static sets of objects in physical or abstract systems.

Keywords: Behavioral Relation, Decremental Disunion, Denotational Mathematics, Discrete Mathematics, Hierarchical Relation, Incremental Union, Parallel Relation, Relation Composition, Relation Theory, Serial Relation, Structure Relation, System Composition

1. INTRODUCTION

Relations are both a pervasive phenomenon in the natural world [Rapoport, 1962; Tomassi, 1999; O’connor & Robb, 2003; Wang, 2003, 2007b, 2009c, 2015h; Wang & Berwick, 2012, 2013; Wang et al., 2009c] and a fundamental mathematical concept in the abstract world [Russell, 1903; Artin, 1991; Kolman et al., 1996; Bender, 1996; Lipschutz & Lipson, 1997; Zadeh, 1965, 1999; Harel et al., 2000; Gallian, 2002; Gowers, 2008; Sibley, 2009; Wang, 2007a, 2007b, 2008a,c,d, 2009a,b, 2010a, 2011, 2012a-d, 2013a,b, 2014a, 2015a,b,f; Wang, Zadeh, et al., 2009a]. It is traditionally perceived that a relation is a subset of a Cartesian product between sets of objects [Kolman et al., 1996; Lipschutz & Lipson, 1997]. Actually, a relation as a pair or tuple is more fundamental than Cartesian products, because the latter is defined by the former. A set of generating functions and mechanisms of relations is recognized such as pairs, tuples,

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sets, Cartesian products, bi-Cartesian products, combinations, permutations, and system fusions, as well as contemporary algebraic systems in denotational mathematics [Balakrishnan, 1995; Sibley, 2009; Wang, 2007a, 2008a,d, 2012a,d, 2014d, 2105b,g].

Relations are one of the most important models in a wide range of science and engineering fields such as discrete mathematics, computer science, software science, programming theories, system science, and formal semantics. Theories of system science are fundamentally underpinned by contemporary relation theory, which explains the mechanism of system gains [Rapoport, 1962; Klir, 1992; Wang, 2008d, 2015b]. Relations play an indispensable role in modeling neuroinformatics, human memory, and internal knowledge representation in brain science and cognitive sciences as well as neurology [Marieb, 1992; Dayan & Abbott, 2005; Wang, 2003, 2013b; Wang & Fariello, 2012; Wang et al., 2006, 2009b]. The basic property and mechanism of natural intelligence are essentially based on neural synaptic relations embodied by the nervous systems [Marieb, 1992; Wang, 2013b, 2014b, 2015g; Wang et al., 2006]. The modern cognitive knowledge bases [Debenham, 1989; Ullman & Widom, 1997; Wang, 2014c, 2015d, g] are relational and content-addressed knowledge systems. Relation theories in linguistics are studied in formal syntaxes and semantics [Chomsky, 1956; Montague, 1974; Keenan, 1975; Wang & Berwick, 2012, 2013]. A set of relational models and operations on formal semantics is developed in concept algebra [Wang, 2008c, 2010b, 2015e], semantic algebra [Wang, 2013a; Wang et al., 2011], and inference algebra [Wang, 2011, 2012b]. Seventeen behavior-oriented relational operators is rigorously modeled in real-time process algebra (RTPA) [Wang, 2002, 2008b, 2014a], which reveals that the relational rules of system behavioral processes may be formally described by a set of relational process operators known as those of sequential, parallel, and iterative relations [Wang, 2007a]. Therefore, to a certain extent, one of the focuses of denotational mathematics [Wang, 2008a, 2012a,d, 2015a] is on formal relations of abstract objects such as concept, semantics, behavior, knowledge, perception, inference, induction, probability, and systems.

It is recognized that many fundamental and essential operations on relations were overlooked in traditional relation theories [Russell, 1903; Kolman et al., 1996; Bender, 1996; Lipschutz & Lipson, 1997; Sibley, 2009; Wang, 2008a, 2011, 2012a,b,d, 2015a]. In studies of hyperstructures, formal semantics, and complex systems, it is discovered that a fundamental operation is the incremental union of relations in order to explain the nature of relations and its formal manipulations [Wang, 2008d, 2015b]. The mathematical operations of incremental union and its inverse counterpart known as decremental disunion reveal a fundamental property of general systems towards a rigorous explanation of the formal mechanisms of systems and information fusions.

This paper presents novel operations on formal relations known as the incremental union and decremental disunion of relations. In the remainder of this paper, the mathematical models of primitive and complex relations are formally described in Section 2. The incremental union of relations as a formal model of system fusion is rigorously explained in Section 3. Inversely, the decremental disunion of relations as a formal model of system decomposition is elaborated in Section 4. The mathematical theory of formal relations reveals how the dynamic creation and fusion gains of relations are generated on static sets of objects in physical and abstract systems.
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