Chapter 2

NK Simulation Modeling

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ABSTRACT

Launched and developed primarily by Kauffman from the end of sixties, NK simulation modelling candidates for capturing networks dynamics. Grounded in reference to biological networks, it has aroused a great and durable interest in economics and management sciences too. This methodology is split into a version focused on studying proper Boolean networks dynamics, whose trajectories are substantially conditioned by Boolean functions, and a (much more frequented) version focused on systems co-evolutionary paths driven by the search for optimizing its fitness value. Besides the unquestionable value of Kauffman’s work for the theoretical implications on evolutionary biology and the strong interest for economics and management sciences, in this chapter failures and limitations of both NK modelling versions are discussed. In particular, it is shown that as applications try to be more realistic, this modeling becomes hardly treatable from a computational point of view. On the other hand, it is underlined that, especially the fitness landscape version, NK simulation modelling is very useful to show general aspects of system’s dynamics, and the impossibility to find general optima (excepted for very special and unrealistic cases). This result sounds a sharp criticism to general economic equilibrium, and it is perfectly consistent with Simon’s contributions.

INTRODUCTION

A way to perform network dynamic analysis is through Boolean networks (BNs) methodology, which here is treated in one of its most important formulations, proposed by Kauffman in the sixties (1969a, 1969b) and seventies (1971a, 1971b), and then developed and refined in the eighties (1984, 1986, 1988) in two directions: one focused on the self-organizing dynamics followed by a single BN, and the other focused on a system’s (possibly also a BN) potential adaptive evolution within a fitness landscape. BNs theory and modelling is rooted in various research streams developed in the 60s: the theory of dynamic systems (Brian, 1984; Klir, 1991; Weisbuch, 1991; Wuensche, 1994, 1998); automata studies and cellular automata theory (Gill, 1962; Hanson, 2009; Ilachinski, 2001; Shannon & McCarthy, 1953; Sutner, 2009; Trakhtenbrot & Barzin, 1973; Waldrop, 1992; Wolfram, 2002; Wuensche & Lesser, 1992), cyber-
NK Simulation Modeling

netics (Ashby, 1956; Ashby & Walker, 1966; Glushkov, 1966; Kauffman, 1984, 1993; Trappl, 1983, von Foerster, 1982, 2003), and complexity science (Arrow et al., 1988; Arthur et al., 1997; Casti, 1989, 2004; Khalil & Boulding, 1996; Mainzer, 1994; Mitleton-Kelly, 2003; Strogatz, 2001, 2003; Sulis & Trofimova, 2000)\(^2\). In fact, networks are discrete systems, and so they have much in common with the theory of dynamic systems and complexity science. Further, besides its roots in cybernetics and systems science, cellular automata can be seen as a BN sub-group (Wuensche, 1994). From the same scientific milieu developed in the Santa Fè Institute (Waldrop, 1992), an important stream has been founded and brought forth either in economics (Arrow et al., 1988; Arthur et al., 1997; Arthur, 2010; Blume & Durlauf, 2006; Lane et al., 2009) or in management and organization sciences (McKelvey, 1999, 2004; Schneider & Somers, 2006; and the Special Issue of Organization Science, 1999)\(^3\).

Following the exposition made by Kauffman in his two books “At home in the universe” and “The origins of order”, the first part of this chapter is dedicated to Boolean network modelling, which will be labelled as NK-BN, and the second part to fitness-driven network evolution, which will be called NK-FL. In fact, for current literature does not distinguish the two models, and calls NK model also those based on FL, to avoid confusion this specification is made. Kauffman links the two models into his main book “The origins of order” (pp. 209-219), when FL is a Boolean network space. The separation of the two models is accompanied by two disciplinary orientations: authors in the field of biology, mathematics and computer science focus on improving mostly NK-BN, while those in the field of economics and management focused almost exclusively on NK-FL models. Consistently with the setting of this volume, both models will be applied in the second part (see chapter 9 and 10).

As it will be clear later on, the separation of the two models hides important issues, because the relevance of self-organizing properties of biological (and social) systems (networks) rests more into the RBN rather into the FL model, while vice versa for the role of selection. Thus, its separation addresses to the incomplete view of a self-organizing but not selective or a selective but not self-organizing world. On the contrary, it seems that the evolution of biological and especially social systems results from the conflicting forces of self-organization and selection (with its connected devices of mutation and retention).

PART I: THE NK MODEL AS “PURE” BOOLEAN NETWORK DYNAMICS

Built on the theory of random Boolean networks (Bollobas, 1985; Dorogovtsev & Mendes, 2003; Gershenson, 2002, 2004a; Serra & Villani, 2006; Somogyvari & Payrits, 2000), the idea of the standard version of NK modelling is that the dynamics of a topologically invariant system (or network) - that is a system that does not mutate neither its size nor its links distribution - is defined deterministically (or stochastically) by the interaction of its nodes according to (as well invariant) interaction rules. A brief description of this methodology, as it has been proposed by Kauffman and applied by him and some others, can be done as follows:

- Given a certain binary network topology, it is assumed that nodes can have a number of activation states, which are usually reduced to two: on or off (active or inactive);
- The (active) edges going to each node\(^4\) can be combined according to Boolean functions, which can be called (dis)activation rules;
- As output of each combination, a node receiving edges will be active or inactive, thus, determining a distribution of active/inactive nodes at the whole network level;

\(^1\) Glushkov, 1966

\(^2\) In fact, networks are discrete systems, and so they have much in common with the theory of dynamic systems and complexity science.

\(^3\) From the same scientific milieu developed in the Santa Fè Institute.

\(^4\) The (active) edges going to each node can be combined according to Boolean functions, which can be called (dis)activation rules.