Chapter 19
Planning of a Project with Imprecise Activity Time

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ABSTRACT
A project where activity times are characterized by intervals or triangular fuzzy numbers is considered in this chapter. Classical project evaluation and review technique (PERT) cannot be directly applied to such projects. Comparison of two intervals or two triangular fuzzy numbers plays an important role in this problem. Ranking methods based on some fuzzy mathematical techniques are discussed and two algorithms for finding the critical path for such a project are given. An illustrative example is also provided.

INTRODUCTION
Project evaluation and review technique (PERT) is a method to analyze different tasks required in completing a given project, especially the time needed to complete each task, and to find out the minimum time needed to complete the total project. Any individual task which utilizes resources and which has a start and an end is called an activity. The activities are generally inter-related in such a way that some activities cannot be started before the completion of some others. Thus a combination of inter-related activities in a certain order constitutes the whole project. An event represents a point in time signifying the completion of some activities and the beginning of some new ones. The inter-dependencies and the precedence relationship among the activities of a project reveal efficiently when it is represented as a network diagram. Usually, the events of a project are represented by the set of nodes \( V = \{ v_1, v_2, \ldots, v_n \} \) of a network \( N = (V, E) \), where \( E \) is the set of edges \( e_{ij} \) between two nodes \( i \) and \( j \); \( e_{ij} \) determines the estimated time of completion of an activity within the events given by the nodes \( i \) and \( j \).

In classical project management, the estimated completion time of each activity is taken as crisp real numbers. But, as time fluctuates with traffic conditions, payloads and so on, it is not practical to

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consider each activity time as a deterministic value. For example, suppose the natural completion time for the construction of a building is 17 days provided the availability of a certain number of workers and instantaneous availability of the raw materials. But, if we take into account the possibility of bad weather, the allotted time to complete the work may be say, 20 days, being there is a chance of completing it earlier. So, it will be justified to consider the time as an interval number, say, [15, 20] days or a triangular fuzzy number as (15, 17, 20) days or some other way. Different kinds of uncertainties may occur for different problems.

Management of a project where the activity times are imprecise in nature requires specialized techniques for aggregation and comparison of quantities describing the imprecision. Dubois & Prade (1980) were the first to analyze the problem for fuzzy activity time by using the extended addition/ subtraction and fuzzy maximum/ minimum for the aggregation and comparison of fuzzy quantities. The main drawback of their proposal was that the path length determined may not correspond to the original one in the network. Also the double inclusion of the uncertainties of the fuzzy durations resulted in a more fuzzy quantity during the determination of the path. This is due to the backward traversal after the extended addition has been applied in the forward calculations. Another approach to fuzzy PERT by introducing the convolution law to aggregate the fuzzy duration times was analyzed by Mares & Horak (1983) and Mares (1989). They used enumeration to examine all possible paths individually to obtain the critical path. The overall fuzzy duration times were determined in terms of maximum/ minimum elements in the supports, the modes or the median elements depending upon the decision maker’s attitude of pessimism or optimism so that the computational time can be tremendously high. Since then a lot of research in this direction have been carried out by a number of researchers by considering different kind of imprecision and different types of aggregation and comparison operations. Nayeem & Pal (2008) have studied the problem using ranking method based on ‘acceptibility index’ and deconvolution operation.

In fuzzy optimization theory, the most important fuzzy ranking methods are based on the possibility and necessity measures. The possibility theory was proposed by Zadeh (1978) and developed by many researchers such as Dubois & Prade (1988). Liu (2002) introduced the credibility measure of a fuzzy event as an average of the possibility and necessity measure. But, in reality, most decision-makers are neither absolutely optimistic/ optimistic, nor absolutely neutral. To balance between the optimism and pessimism, a convex combination of the possibility measure and the necessity measure is introduced by Yang & Iwamura (2008). It gives scope to the decision maker to set the degree of optimism/ pessimism. Thus $m_\lambda$ measure is a generalization of the credibility measure. Liu (2004) shown that the expected value of a fuzzy variable can be defined in terms of the credibility measure. In a similar manner, the $\lambda$-expected value of a fuzzy variable can be defined in terms of the $m_\lambda$ measure (Nayeem, 2012).

In this chapter, our aim is to review the problem of project management where activity times are imprecise using a combination of the above stated fuzzy mathematical tools.

**ARITHMETIC AND RANKING OF IMPRECISE NUMBERS**

In this section, we give the arithmetic and ranking methods of intervals and triangular fuzzy numbers. An interval number is defined as $A = [a, b] = \{x: a \leq x \leq b\}$ where, $a$ and $b$ are real numbers called the left end point and the right end point of the interval $A$. 
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