Chapter 13
Rough Approximations on Hesitant Fuzzy Sets

D. Deepak
National Institute of Technology Calicut, India

Sunil Jacob John
National Institute of Technology Calicut, India

ABSTRACT

Introduction of hesitant fuzzy rough sets would facilitate the use of rough set based techniques to hesitant fuzzy environment. Hesitant fuzzy rough sets deal with the lower and upper approximations in a hesitant fuzzy domain. For this purpose concepts of hesitant fuzzy relations are discussed first to create a theoretical framework to study hesitant fuzzy rough sets. The concepts of equivalence relations are discussed. Hesitant fuzzy rough sets and the properties of the approximations are discussed. The dual nature of the lower and upper approximations is proved. This chapter introduces the model of a hesitant fuzzy rough set which approximates a hesitant fuzzy set using a hesitant fuzzy relation.

1. INTRODUCTION

Characterization of uncertainty is a major research area in Soft Computing. Rough set based techniques to characterise uncertainty have been used efficiently in many areas including reasoning from imprecise data. Rough set theory, introduced by Pawlak (1982), consists of the approximation of the set by its lower and upper approximations. It uses the notion of an indiscernibility relation to partition the universal set to equivalence classes called granules. The lower approximation consists of all those granules which completely belong to the set whereas the upper approximation consists of elements with non empty intersection. The difference between the upper approximation and the lower approximation forms the boundary region thereby characterizing the uncertain elements. But the notion of an indiscernibility relation is very much restrictive when it comes to applications and further scope of the area. It is here that the importance of hybrid structures involving rough sets comes to play. Pawlak (2004) and Pawlak (1991) discuss concepts regarding rough sets and reasoning about data in great detail.

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Zadeh (1965) introduced fuzzy sets which helped in characterising uncertainty by allotting membership values to elements in the set. Various other developments to these concepts have been experimented with great success. Atanassov (1986) introduced intuitionistic fuzzy sets which could additionally account for non-membership values of the elements. Hai-long Sheng-gang (2009) discusses the intuitionistic fuzzy relations in great detail. Hesitant fuzzy sets introduced by Torra (2010), Torra and Narukawa (2009) assigns a set of possible membership values to every element in the set. The set of possible membership values arises because of difficulties in assigning a certain membership value to the element. The difficulties might arise due to many reasons. It may be the lack of knowledge or for example in decision making problems; situations may arise where the decision makers are not able to arrive at a mutually acceptable decision. Hesitant fuzzy sets opened up new areas of research by allowing scope for hesitancy in the membership values of a set.

Hybrid structures involving fuzzy sets and rough sets have also been studied in great detail by many researchers. Dubois and Prade (1990) studies rough fuzzy sets and fuzzy rough sets effectively. Nanda and Majumdar (1992) discuss various theoretical aspects regarding fuzzy rough sets. Wei et al. (2012) have analysed the relationships among rough approximations of some of the generalized rough set models for hybrid data. Variable precision fuzzy rough set model, studied by Mieszkowicz and Rolka (2008), is one of the suitable tools for analyzing information systems with crisp or fuzzy attributes. Hassanien (2007) introduced a hybrid scheme that combines the advantages of fuzzy sets and rough sets in conjunction with statistical feature extraction techniques the use of which has been demonstrated for breast cancer detection (by classifying the breast cancer images). Some shortcomings of traditional rough set attribute reduction, dealing with noise and real-valued attributes, have been addressed to a certain extend by Jensen and Shen (2004) by presenting a fuzzy-rough method for attribute reduction. Intuitionistic fuzzy rough relations and their properties have been studied in detail by Mukherjee and Das (2013). Degang and Suyun (2010) emphasised the use of local reduction with fuzzy rough sets for decision systems. Petrosino and Ferone (2009) have effectively used the properties and operations of rough fuzzy sets for the introduction of a new coding/decoding scheme for image compression.

This chapter discusses approximations in hybrid structures involving hesitant fuzzy sets and rough sets. In place of an indiscernibility relation in rough set theory, we have a hesitant fuzzy relation. Section 2 and section 3 are mainly based on the results by Deepak and John (2014). The chapter first probes the notion of a hesitant fuzzy relation to form a theoretical basis to introduce hesitant fuzzy rough sets. The notion of an equivalence hesitant fuzzy relation is studied. The chapter then goes on to reflexive and symmetric kernel and their formulae. The last section discusses hesitant fuzzy rough sets and then moves on to prove that the hesitant fuzzy rough approximation operators are dual to each other and discusses their properties.

2. PRELIMINARIES

Basic Definitions

In this section we discuss some of the basic definitions regarding hesitant fuzzy sets. Some new conjunction and disjunction operations on hesitant fuzzy sets are proposed. Properties of operations on hesitant fuzzy sets are studied to lay the theoretical framework for further studies in this area.