Chapter 3
Performance Analysis of an M/G/1 Feedback Retrial Queue with Two Types of Service and Bernoulli Vacation

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ABSTRACT
This chapter is concerned with the analysis of a single server retrial queue with two types of service, Bernoulli vacation and feedback. The server provides two types of service i.e., type 1 service with probability $\theta_1$ and type 2 service with probability $\theta_2$. We assume that the arriving customer who finds the server busy upon arrival leaves the service area and are queued in the orbit in accordance with an FCFS discipline and repeats its request for service after some random time. After completion of type 1 or type 2 service the unsatisfied customer can feedback and joins the tail of the retrial queue with probability $f$ or else may depart from the system with probability $1-f$. Further the server takes vacation under Bernoulli schedule mechanism, i.e., after each service completion the server takes a vacation with probability $q$ or with probability $p$ waits to serve the next customer. For this queueing model, the steady state distributions of the server state and the number of customers in the orbit are obtained using supplementary variable technique. Finally the average number of customers in the system and average number of customers in the orbit are also obtained.

INTRODUCTION
Queueing system has a wide range of applications in our day to day life, such as manufacturing industries, airports, traffic-congestion, retail shops, medical shops etc. Queueing system has a prominent place among the modern analytic techniques of Operations Research. From the past decades retrial queueing system plays a vital role in queueing models. Basically it consists of three main components such as

DOI: 10.4018/978-1-5225-0044-5.ch003
arriving customer, orbit (retrial queue) and server. Retrial queueing system is characterized by the phenomenon that an arriving primary customer finds the server busy is supposed to leave the service area and repeat its request for service after some random time. Meanwhile the blocked primary customers are said to be in orbit. These retrial queueing models have applications in many real life situations such as auto-repeat facilities in telephone systems, random access protocols in computer networks, telecommunication networks etc.

In a standard queueing system it is assumed that the arriving primary customer who finds the server busy either leaves the service area or joins in a waiting line. However in practical, the real behavior of unsatisfied customer is that they repeat their request after some random time which shows the vitality of retrial concept in queueing system. The concept of retrial queues was first introduced by Cohen (1957) in which he analyzed the basic problems of telephone traffic theory through the influence of repeated calls. Interested readers may refer the survey papers by Yang and Templeton (1987) and Falin (1990). Earlier studies on retrial queues have considered the classical retrial policy which have been seen detailed in Yang and Templeton (1987) and Falin (1986) where the intervals between successive repeated attempts are exponentially distributed with rate $n\theta$, when the number of customers in the orbit is $n$. However in recent trends there are queueing situations, where the retrial rate is independent of the number of customers in the orbit i.e., the retrial rate is $\alpha(1-\delta_{n,0})$. This retrial policy is called constant retrial policy which was introduced by Fayolle (1986) in the investigation of telephone exchange model. Later Artalejo and Gomez-Corral (1997) have introduced linear retrial policy in which time intervals between successive repeated attempts are exponentially distributed random variables with parameter $\nu_n = \alpha(1-\delta_{n,0}) + n\nu$, where $n$ denotes the number of customers in the orbit and $\delta_{n,0}$ denotes the Kronecker delta. Also we have general retrial time policy in which each job in the orbit generates a repeated attempt that are independent of the jobs in orbit and the server state. For detailed study readers may refer Kapyrin (1977) who first introduced the concept of general retrial times in his work. Later Falin (1986) objected this policy but Yang et.al (1994) have established an approximation method for Kapyrin (1977). However in recent years, several retrial models have been analyzed with general retrial times, which have been found in Krishnakumar and Arivudainambi (2002), Krishnakumar et.al (2002) Atencia and Moreno (2004), Atencia and Moreno (2005), Moreno (2004), Wang and Zhao (2007), Choudhury (2009), Ke and Chang (2009), Choudhury and Ke (2012).

A new class of service model is two types of service provided by a single server which have notable applications in automobile stations, post offices, banks, call centers, computer centers etc. Madan et al. (2005) have discussed a batch arrival queue with single server providing two types of heterogeneous service in which before the service starts the customer may choose either type of service.

Another important and unavoidable concept in this field is server absence which is termed as server vacation. In many real life queueing situations the server may not be available for a period of time due to break, secondary jobs, maintenance activities, programmed interruptions etc. which in-turns termed as vacation. A survey on vacation queues have been analyzed by Doshi (1986). Keilson and Servi (1986) have investigated oscillating random walk models for $GI/G/1$ vacation system with Bernoulli schedules in which he was the first who introduced the concept of Bernoulli vacation. Krishnakumar and Arivudainambi (2002) have investigated a $M/G/1$ retrial queue with Bernoulli schedule and general retrial times. In this system, the single server takes a Bernoulli vacation i.e., after each service completion, the server takes a vacation with probability $q$, and with probability $1-q$, it waits to serve the next customer. Atencia and Moreno (2005) have analyzed a single server retrial queue with general retrial times and Bernoulli schedule in which he considered the retrial group (orbit) in accordance with an FCFS discipline.