Chapter 7
A Stochastic Inventory Model with Multiple Vacations and N – Policy:
A Stochastic Inventory Model with Multiple Vacations

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ABSTRACT

We consider a perishable inventory system under continuous review at a service facility in which a waiting area for customers is of finite size. We assume that the replenishment of inventory is instantaneous. The items of inventory have exponential life times. The service starts only when the customer level reaches a prefixed level, starting from the epoch at which no customer is left behind in the system. The arrivals of customers to the service station form a Poisson process. The server goes for a vacation of an exponentially distributed duration whenever the waiting area is zero. The service process is subject to interruptions, which occurs according to a Poisson process. The interrupted server is repaired at an exponential rate. Also the waiting customer independently reneges the system after an exponentially distributed amount of time. The joint probability distribution of the number of customers in the system and the inventory levels is obtained in steady state case. The results are illustrated with numerical examples.

INTRODUCTION

Research on queueing systems with inventory control has captured much attention of researchers over the last decades. Many researchers considered, customers arrive at the service facility one by one and require service. In order to complete the customer service, an item from the inventory is needed. A served customer departs immediately from the system and the on-hand inventory decreases by one at the moment of service completion. This system is called a queueing - inventory system. Berman and Kim (1999) analysed a queueing - inventory system with Poisson arrivals, exponential service times and

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zero lead times. The authors proved that the optimal policy is never to order when the system is empty. Berman and Kim (2004) addressed an infinite capacity queueing - inventory system with Poisson arrivals, exponential distributed lead times and service times. The authors identified a replenishment policy which maximized the system profit.

Krishnamoorthy and Anbazhagan (2008) analyzed a perishable queueing-inventory system with $N$ policy, Poisson arrivals, exponential distributed lead times and service times. Joint probability distributions of the number of customers in the system and the inventory level were obtained in the steady state case. Two other papers where an inventory model with service time is considered, are by Krishnamoorthy et.al (2010, 2012). Krishnamoorthy et.al 2010, considered an inventory model with instantaneous replenishment and the service process is subject to interruptions. In (Krishnamoorthy et.al, 2012), they have studied an inventory model with positive lead time, server interruptions and an orbit of infinite capacity, where no waiting space is provided for customers, other than for the one whose service gets interrupted. Recently, Jeganathan and Periyasamy (2014), studied a perishable inventory system with repeated customers and server interruptions. They have assumed that an arriving primary customer finds the waiting room full is permitted to enter into orbit otherwise the customer waits for his service in the waiting hall and the inventory is replenished according to an $(s, S)$ ordering policy. Also the joint probability distribution of the number of customers in the waiting area, the number of customers in the orbit and the inventory level is obtained for the steady state case. Some important system performance measures in the steady state are derived. Several numerical examples are presented to illustrate the effect of the system parameters.

In many real world queueing systems, server(s) may become unavailable for a random period of time when there are no customers in the waiting line at a service completion instant. This random period of server absence, often called a server vacation can represent the time of server’s performing some secondary task. This has been extensively investigated (Tian & Zhang, 2006), (Takagi, 1991, 1993) and (Doshi, 1986, 1990). (Daniel & Ramanarayanan, 1987), have first introduced the concept of server vacation in inventory with two servers. In (Daniel & Ramanarayanan, 1988), they have studied an inventory system in which the server takes a rest when the level of the inventory is zero. (Sivakumar, 2011), analyzed a retrial inventory system with multiple server vacations. (Narayanan et al., 2008), considered an inventory system with random positive service time. Customers arrived to the service station according to a Markovian arrival process and service times for each customers had phase-type distribution. Padmavathi et.al (2013) discussed a retrial inventory system with single and modified multiple vacations for server. Periyasamy (2013) analyzed a finite source perishable inventory system with retrial demands and multiple server vacation.

In this chapter, the author consider a continuous review perishable inventory system at a service facility with $N$ policy, multiple vacations and server interruptions. The limiting probability distribution of the inventory level and the number of customers in the system is obtained in the steady state case. Various system performance measures are derived and the total expected cost rate is calculated.

**MATHEMATICAL MODEL**

Consider a continuous review perishable inventory system at a service facility with the maximum capacity for $S$ units. The waiting hall space is limited to accommodate a maximum number $M$ of customers including the one at the service point. The arrival of customers is assumed to form a Poisson process.