Chapter 15
Utility Maximization and Optimal Portfolio Selection

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ABSTRACT

Utility maximization and optimal portfolio selection with or without consumption/transaction cost based on stochastic models of prices of securities with stochastic volatility are discussed.

1. INTRODUCTION

Markowitz (1952) has proposed the theory of portfolio optimization in a single period in which he has used the mean-variance estimates of portfolios to judge investment strategies and has employed linear regression to quantify risk and return of the portfolios of stocks and bonds. Tobin (1958) has formulated a separation theorem which has showed a way to identify which efficient portfolio should be held by an individual investor by dividing his/her funds between a safe liquid asset such as cash (or treasury bills) and a risky asset (equity portfolio). He has established that, if portfolio theory based on mean and variance of return distribution should hold, either the distribution of asset returns should belong to the two-parameter family or the utility function should be quadratic. Samuelson (1969) has given a strong foundation for multiperiod models of portfolio selection problems in discrete time by formulating and solving many-period models corresponding to life-time planning of consumption and investment decisions. Merton (1969, 1971) has examined the combined problem of dynamic optimal portfolio selection and consumption rules for an investor in a continuous-time model where his/her income is generated by returns on assets and these returns are stochastic processes. Magill and Constantinides (1976) formalized the portfolio selection problem in the presence of transactions costs and highlighted the effect of transactions costs on the general theory of the capital market. Constantinides (1986) has formulated a two-asset inter-temporal portfolio selection model incorporating transaction costs proportional to the assets and iso-elastic utility of consumption and showed that the demand for the assets was sensitive to transaction costs. Karatzas et al. (1987) have characterized optimum consumption and portfolio decisions for

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a small investor on a finite horizon. Karatzas (1989) has computed explicitly the optimal portfolio and consumption rules that maximize the total expected utility of both consumption and terminal wealth under very general conditions on the utility functions of the agent in a market model with constant coefficients. Davis and Norman (1990) have studied optimal consumption and investment problems for an investor who has availed a bank account paying a fixed rate of interest and a stock whose price is a log-normal diffusion. Jeanblanc-Picque and Pontier (1990) have studied the problem of selecting a portfolio to maximize the utility of both consumption and terminal wealth by assuming a financial market in which the evolution of asset prices is described by a continuous-time stochastic process. Schweizer (1991) has modeled price process as a semi-martingale and analyzed a mean-variance option hedging problem in incomplete markets. Cvitanic and Karatzas (1996) have proposed a martingale approach for hedging and portfolio optimization under transaction costs. Sethi (1997) has considered the problem of optimal consumption and investment of a single agent endowed with some initial wealth who seeks to maximize the total expected discounted utility of consumption. The author has explicitly introduced a bankruptcy value/penalty in the consumption/investment model and provided a useful frame for deepening the understanding of the consumption and portfolio selection behavior of individuals. Kallsen and Muhle-Karbe (2010) have showed that duality approach of stochastic control theory and martingales can actually be used for both deriving a candidate solution and verification in Merton’s problem with logarithmic utility and proportional transaction costs. Li and Ng (2000) have considered the problem of optimal portfolio selection in a multi-period mean-variance formulation and obtained analytical optimal portfolio policy and analytical expression of the mean-variance efficient frontier. Zhou and Li (2000) have formulated a bi-criteria optimal portfolio selection problem and by putting weights on the two criteria, they have obtained a single objective stochastic control problem. They have embedded this single problem into a class of auxiliary stochastic linear-quadratic (LQ) problems and obtained an efficient frontier in a closed form for the original portfolio selection problem. Cvitanic et al. (2001) have given a fairly complete solution of the utility optimization problem on incomplete markets for a class of general utility functions not containing the exponential one. Watcher (2002) has examined the portfolio selection problem in a complete financial market by assuming that stock market returns are mean reverting and using the martingale method, he has provided an exact form of the optimal portfolio solution that takes the form of a weighted average. Guasoni (2002) has studied the problem of minimizing the risk of a position at a fixed rate, trading in a market with a risky asset with incomplete information, proportional transaction costs and constraints on strategies. Karguine (2003) has obtained the optimal portfolio rules with the assumptions that asset returns follow a stochastic multi-factor process with time-varying conditional expectations and investments are linear functions of factors. Hugonnier and Kramkov (2004) have studied the problem of optimal investment with random endowments in incomplete markets by maximizing the expected utility of an agent. Techranchi (2004) has proved some Holder-type inequalities for products of certain functionals of correlated Brownian motion processes and using them, he has obtained explicit solutions for the problem of optimal portfolio selection in incomplete markets when the investor’s utility is of the form \( U(X, Y) = g(X)h(Y) \), where \( g(X) \) is exponential, power or logarithmic utility function and \( h(Y) \) is a function of a random variable \( Y \) not correlated with the market. Hu et al. (2005) have considered the problem of utility maximization for a small trader on incomplete financial markets. In their paper, the period of trading is a finite time interval \([0, T]\) and the trader invests in risky stocks and a risk-free bond and aims at maximizing the utility he/she draws from his/her final